Kinematics of Machines

Prof. A. K. Mallik

Department of Civil Engineering

Indian Institute of Technology, Kanpur

Module 13- Lecture - 3

In our last lecture we have seen that an epicyclic gear train has 2 degrees of freedom.

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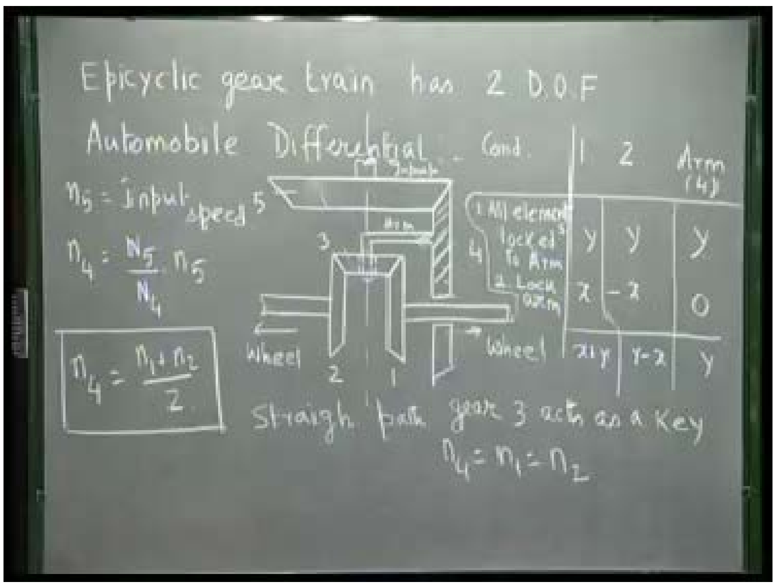

Because it has 2 degrees of freedom it can create two output speeds from single input speed, and that has lot of application. For example, very commonly used epicyclic gear train for the same purpose, that it can create two output speeds from one input speed is in automobile differential, we know that when an automobile goes in a straight path both the rear wheels rotate at the same speed. If the car wants to take a turn, then the input speed may not change, but the outer wheel must rotate faster than the inner wheel. From the same input speed, I must now create two different output speeds when the car wants to take a turn, and that is possible because the input shaft and the wheels are connected by what we call an automobile differential which is nothing but an epicyclic gear train having 2 degrees of freedom.

Let me draw the differential sketch. These are the two shafts of the wheels, (Refer Slide Time: 02:45) say to the one of the rear wheels and this to the other. This is a bevel gear connected to another bevel gear. But this V wheel gear is carried by an arm. These are bevel gears connected to the two rear wheels. This is another bevel gear which carries this arm carries this arm that means this bevel gear rotates about this arm, there is a revolute pair here that is what is called a planetary gear. And this bevel gear is driven by this bevel gear, this is the input.

The input comes to this gear which drives this bevel gear, this bevel gear carries this arm, and this bevel gear is carried on that arm so there can be relative rotation between this arm, and this bevel gear. This bevel gear connects these two bevel gears, which are connected to the wheel. Let me say this gears we name 1, 2 and this is 3, this is the arm which is same rigid body as this gear which is 4, and this is 5. So n_5 is the input speed, this small n I refer to the rpm we call it as input speed. This n_5 decides the rotation of 4, so n_4 we can find that will be N_5 by N_4 into n_5, where N_5 and N_4 are the number of teeth on these two gears. Depending on the direction of this rotation, we can find what is the direction of this rotation for 4? And 4 is nothing but the arm. To analyze this epicyclic gear train, we follow the same tabular method which I said earlier we talk of condition, and we are interested in gear number 1, gear number 2 and arm. This gear 1, gear 2 and arm or gear number 4 they are the same.

First, we say everything is locked to the arm all these elements this gear, this gear everything is locked to the arm. All elements locked to arm, and then the arm is given some y revolution, because 1, 2 and 3 everything is locked to the arm this whole thing rotates like a rigid body, so 1 and 2 also get the same revolution y. We say the second condition. Second condition is we lock the arm that means, we hold this fixed. Arm is locked so no revolutions to the arm. Then these two gears are of same dimension they have the same number of teeth only if the arm is locked. This gear is rotated, the arm is not moving so this is like a simple gear train. This motion is transmitted to this gear through this bevel gear, and the number of teeth on this gear is an important it was the intermediate gear, and because these two gears are of same size only thing they rotate in opposite direction. If gear 1 is given, x revolution gear 2 gets minus x revolution. The resultant is x plus y, y minus x and y. If the arm rotates, we can say speed of 4 which is y is nothing but n_1 plus n_2 by 2, so whatever maybe the values of x and y, the average of n_1 and n_2 will be the rotation of gear 4.

When the car is moving on a straight path, there is no relative rotation between this gear 3 and the gear arm, there is no epicyclic action this acts like a teeth. So, n_4 just drives n_1 and n_2 in the same speed and this gear 3 does not rotate, this does not have any gearing action this just acts like a solid connection then n_1 is same as n_2 is same as n_4. So, straight track, if the car is moving on a straight path there is no relative rotation of gear 3 with respect to the arm there is any epicyclic action, gear 3 does not revolve about its own axis. Gear 3 acts as key, and n_4 is same as n_1 is same as n_2 whereas, if the car takes a turn then this gear starts rotating about its own axis and average of n_1 and n_2 becomes n_4. If it is x plus y and y minus x, the speeds are different. One is y plus x, other is y minus x. This is like how an automobile differential can create two different output speeds if necessary because of this epicyclic gear train. We shall now solve 1 or 2 more problems to show this tabular method for analyzing epicyclic gear train.

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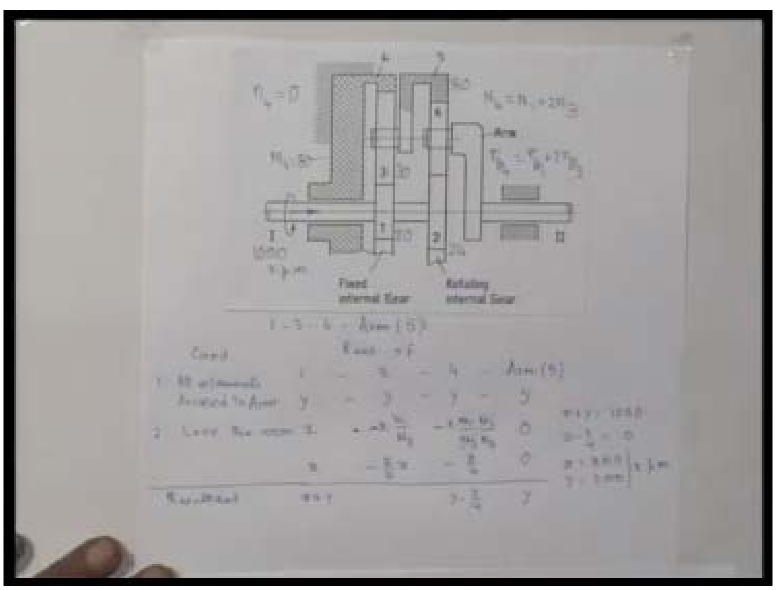

As I said, let me now solve this example of an epicyclic gear train. This is an epicyclic gear train, where the input shaft 1 is connected to this compound gear 1 and 2, that is the speed of this shaft is the same as that of speed of gear 1 and gear 2. Gear 1 is connected to this planet gear, gear 3. Gear 3 is carried by this arm that means there is a revolute pair hear and this arm is this gear 5 which is an internal gear. This 5 is an internal gear which is this arm, for this planet gear 3, and the gear 3 is also connected to this internal gear 4 which is fixed. This internal gear 4 is not rotating, this gear 3 is rotating in space, moving in spaced axis of this gear as this gear rotates the axis of this gear 3 is carried in space. Gear 2 is connected to this planet gear 6, which is carried by this arm there is a revolute pair between the arm and gear 6 here, and gear 6 is also connected to this internal gear 5 which is rotating. Our objective is if the number of teeth is specified, input speed is specified we have to find the output speed.

For example, let us say the input speed is given to be in these direction 1000 rpm, what is the output speed? What is the second output speed? Because we are specifying one input speed, it is an epicyclic gear train so we must specify two input speed the other input speed is specified as this gear 4 is fixed that is speed of gear 4 is 0, so the other is speed of gear 4 is 0. Now two input speeds are specified 0 and 1000, we have to find out this output speed, what is n_{II}? So that is the question. Obviously the number of teeth on the various gears should be specified, let me say number of teeth on gear 1 is given to be 20, number of tooth on gear 3 is 30, number of teeth on gear 2 is 24, and number of teeth on gear 5 is 80. This is an intermediate gear the number of teeth is not given and not necessary either. Here also number of teeth on gear 3 though it is given is not necessary for any speed calculation, but we notice that the number of teeth on gear 4 which will be necessary, because that is not an intermediate gear, so that can be calculated from this value. What is number of teeth on gear 4 that we need, for calculation of various speeds?

Here we can calculate the number of teeth on gear 6 from this 80 and 24, but we will never need it. So, this is the problem the epicyclic gear train is specified two speeds are specified 1000 and 0 we have to find out this output speed. Let me now solve this problem, to solve this problem first of all let us see that there are two epicyclic gear trains in this whole gear train. There are two epicyclic parts because there are two arms, one arm for this, if this is the arm which is nothing but gear 5 and for this portion, this is the arm which is carrying gear 6, and this arm 5 is carrying gear 3. Let me first consider one epicyclic gear train consisting of gear 1, 3, 4 and its arm which is nothing but gear 5. Number of teeth on 1 is 20; number of teeth on 3 is 30, so we can find out what is the number of teeth on gear 4.

For that, we write pitch circle radius of gear 4 as we see this is the pitch circle radius, pitch circle radius of gear 1 is r_{p1} plus pitch circle diameter of gear 3 that is twice r_{p3} because all the gears have the same modules pitch circle radius is also proportional to the number of teeth. From here, I can write number of teeth on gear 4 is number of teeth on gear 1 plus twice the number of teeth on gear 3, n_1 is 20 n_3 is 30 so N_4 is 80 so that gives N_4 is 80. To consider this gear train as before we make the tabular method we write condition and revolutions of gear number 1, 3, 4 and the arm which is nothing but gear 5 so first conditions is lock all the elements to this arm. All elements locked to arm. After locking all the elements to the arm we give y revolution to the arm because everything is locked so we get the same number of revolutions in all the elements. We lock the arm, no revolution to the arm and give any revolution x to say 1. If I give x to 1, then 3 rotates in the opposite direction and minus x into N_1 by N_3 because this is an external gearing so that we are taking opposite direction and inversely proportional to the number of teeth. Now, rotation of 3 will cause rotation of 4, but this is an internal gearing so they rotate in the same direction.

So, 3 has a minus sign, 4 will also have the minus sign and rotation of 3 N_1 by N_3 into N_3 by N_4. As I said earlier, this intermediate gear number of teeth is not relevant so it cancels. I am writing this line again x, this is minus N_1 by N_3, so it is 2 by 3 x minus x , 20 by 80 is 4 and this is 0, this line I am just repeating. Let me talk of the resultant motion, we add these two motions so here I get x + y here I do not need 3, because 3 is not given so I do not write it, 4 I get y minus x by 4 and arm I get y. Input speed is given 1000 rpm which is the speed of one that is x plus y. I get x + y is 1000, I am taking this direction as my positive direction as the direction which is shown which is counter clockwise so this is the axis this is the positive direction, x + y is 1000 and n_4 is 0 so y - x by 4 is 0, which if solve these two equations I can get x is equal to 800 and y is equal to 200 rpm and arm 5 which has the speed y that is speed of arm 5 is 200 rpm.

When I analyze this part of the epicyclic gear train I know the speed of arm 5 which is nothing but 200 rpm and gear 2 again the speed will be known because that is same as shaft one gear 2 and gear 1 are one compound gear integral to the shaft 1. In this epicyclic gear train I will know two speeds that was speed of 2 is 1000 rpm and speed of 5 is 200 rpm and I should be able to solve for the speed of the arm as I will show you just now.

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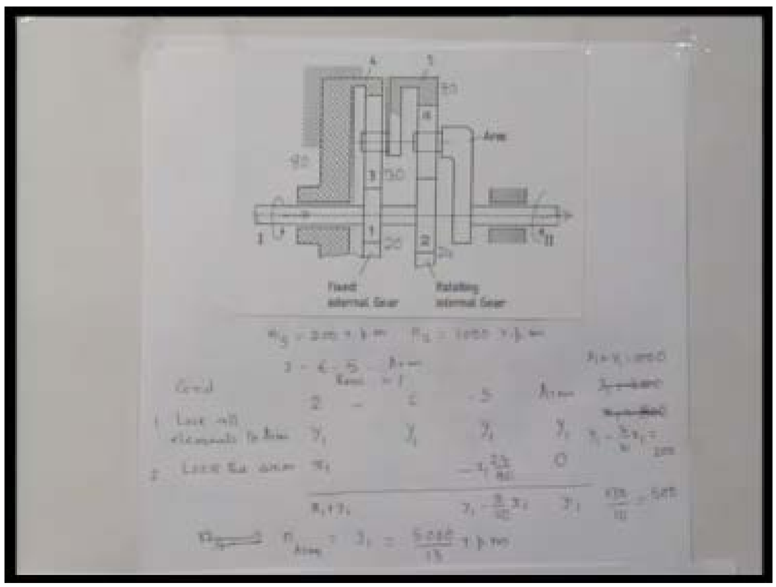

Let me now analyze the second epicyclic gear train consisting of gear 2, gear 6, gear 5, and this arm this arm carries gear 6. Remember this gear 5 was nothing but the arm of the previous train which I have just now solved. We have already seen that n_5 was obtained as 200 rpm and n_2 is given as 1000 rpm, and our objective is to find the rpm of this arm which is the output shaft which is connected to the output shaft so gear train is 2, 6, 5 and the arm. As before we write condition and revolutions of 2, 6, 5 and arm. First lock all elements to arm. All these elements are locked to the arm and give y revolution to the arm, so all the elements get y revolution, then lock the arm, no revolution and let me call it as y_1 so that we do not confuse it x and y we used earlier let me call it as y_1 and lock the arm and give say x_1 revolution to gear 2.

Gear 5 rotates in the opposite direction because this intermediate gear makes it opposite direction 6 and 5 rotate in the same direction so minus x_1 into number of teeth on this gear is not needed so 24 by 80. The resultant is $x_1 + y_1$, y_1 and y_1 - 3 by $10x_1$. The speeds given at n_2 and n_5, n_2 is $x_1 + y_1$ which is 1000 n_2 and, n_5 which is y_1 - 3 by $10x_1$ and that is equal to 200. We have to find what is the speed of the arm n_{arm}? That is the output speed which is nothing but y_1. If we solve for y_1 from this what we get, we eliminate x_1, I multiply by 3 by 10 and add so I get $13y_1$ by 10. 3 by 10 I have multiplied and adding so, x_1 getting canceled 3 by 10 plus one gives me 13 by 10, and this is 3 by 10 that is 300, so 300 plus 200 is 500, y_1 is 5000 by 13 rpm, because it is positive that means it is in the same direction. We get the output speed as 5000 by 13 rpm in the same direction. We shall solve one more problem involving epicyclic gear train.

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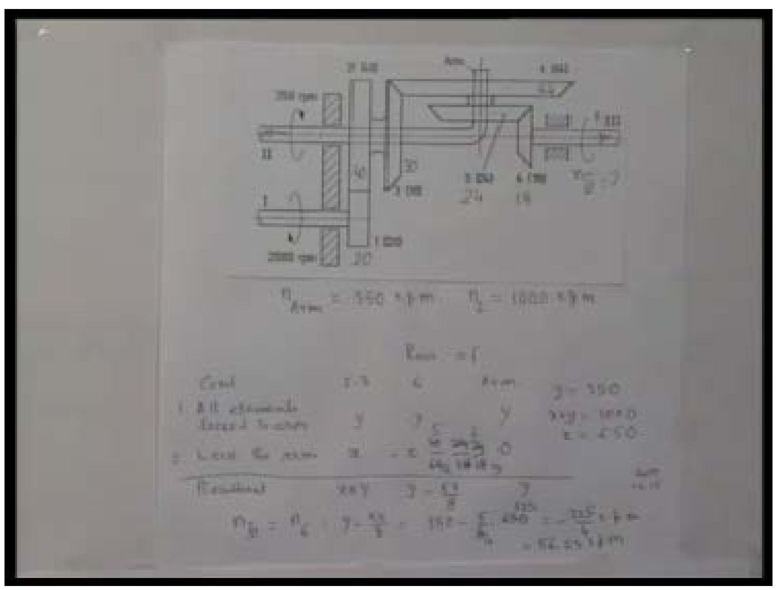

Let me now solve one more problem of epicyclic gear train which is shown in this sketch. This gear 1 is mounted on shaft 1 which is rotating at 2000 rpm in the direction shown. The number of teeth on gear 1 is 20. Gear 1 is connected to gear 2 numbers of teeth on which is 40. Gear 2 and gear 3 forms a compound gear that means they are the same rigid body and number of teeth on gear 3 is 30. This shaft 2 which rotates at 350 rpm carries this arm; this shaft is the arm as it rotates this takes these two bevel gears with it. There is a revolute pair here between the arm and these two bevel gears. This bevel gear 3 is connected to bevel gear 4, number of teeth on which is 64.

This gear 4 and 5 are compound gear again they have the same rigid body, and there is a revolute pair between the arm and these two gears which is the same rigid body 4 and 5, and the number of teeth on 5 is 24. Gear 5 is connected to this bevel gear 6 number of teeth on which is 18, and gear 6 is connected to the output shaft. These two input speeds are specified that of the arm and that of gear 1 and we have to find out what is the output speed in 3? The thing to note in this particular gear train is that, gear 1 decides the speed of gear 2, because this 20 and 40, so if this rotates at 2000 rpm gear 2 rotates at 1000 rpm opposite to this that is in this direction gear 2. n_2 is known if I take this as my positive direction, then what is given is n_{arm} I take this direction as positive n_{arm} is 350 rpm, and n_2 which is decided by n_1 which is 2000 here so that is 1000 rpm opposite to this that is the same direction as this.

We can forget about gear1, and we have an epicyclic gear train that means this gear 1 is not a part of this epicyclic gear train which is obvious because, when we lock all the gears to the arm I cannot lock this gear because then this gear cannot rotate. This is mounted here in the gearing, but if I remove this gear from this epicyclic gear train rest of it is a simple epicyclic gear train, only thing speed of two is decided by that of speed of one that is the only difficult point in this problem. Once we have done this stage this epicyclic gear train is simple. What it consist of? We can write the same table revolutions of gears 2 and 3 which is same then 4 and 5 we are not interested that is intermediate gear so we write gear 6. Gear 6 is rotating about the same axis like the gear 2 and 3 and also the arm which is rotating about this axis so we write arm.

We write condition, first condition all elements locked to the arm, and if arm is given y revolution then gear 6 gets y revolution, 2 and 3 also gets y revolution. We are not writing these two bevel gears because as we see the rotation of this gears allow with the arm and their rotations above the arm are about different axis. These two gears rotate with respect to the arm about this axis, whereas the arm is rotating about the horizontal axis, so we cannot add those kinds of things, and we are not interested, because these are intermediate gears so we do not write it here. Here, we write lock the arm, arm is given no revolution and if I give x revolution to 3, then 4 rotates due to gearing action which is 30 by 64 into x. If it rotates in this direction then it goes in this direction so this gear will rotate in opposite direction. These two gears due to this bevel action rotates in opposite direction, and their rotation is decided by the number of teeth on this and this which is minus x into 30 by 64 and these two rotate in the same direction, then ratio here is… rotation of this I get [as minus x not minus because their rotation is different direction so I am not talking of that] 6 is rotating due to this 24 by18. This has 24 teeth and this has 18 teeth, so it is not 24 by 18 this will rotate more, so it is alright 24 by 18. This is rotating 30 by 64 this is rotating 24 by 18.

We get the resultant so it is x + y and this is y, and this is y minus. If I cancel 18 and 35 and if I cancel 8, 3, 8, 3 and 3 cancels, so y minus 5x by 8 out of which we are given n_{arm} y is 350 and 2 is 1000, x + y is 1000. We get x = 650 and we have to find the revolution of 6 which is $n_{3,}$ so n_3 which is same as n_6 which is y - 5x by 8 which is y is 350 - 5 by 8 into 650. We should calculate this, this is 325 into this is 4 so 4 into 350 is 1400 and 5 into 325 is minus 625 so it is minus 225 by 4. If this rotates in this direction this is minus so it is in this direction, this is this axis, and the value is 225 by 4, which is 56.25 rpm. The output speed is 56.25 rpm opposite to this direction. That is how we solve this epicyclic gear train, only thing to note that this gear does not belong to the epicyclic gear train and if we have bevel gears, then we should not write it in this table because then we cannot add simply like this because all this rotations are about this axis whereas bevel gear with the arm rotates about this axis, but there gearing action make them rotate about a different axis.

We cannot add rotations about two different axes algebraically, so this is how we solve the problems of epicyclic gear trains. Now that we have explained how to analyze these epicyclic gear trains so for the kinematics is concerned, that is how to obtain the speeds of different gears and what is the difference between gear train and gear box. ?

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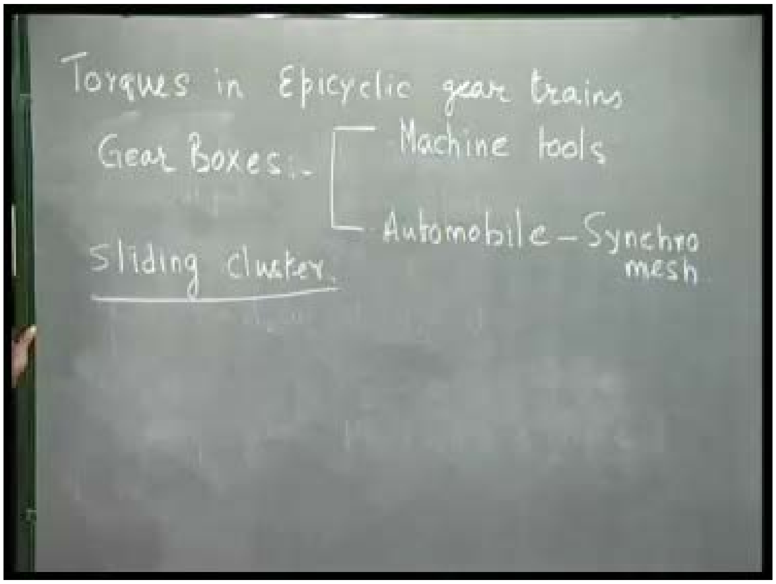

Gear box as we know, there is one input speed, but I may require to get various different output speeds and accordingly, we change the gears within the gear box the gear engagement we change, and using different set of gears within the same gear box I can create different output speed. As we do in automobile from the same input speed I can go to first gear, second gear, third gear as we want to increase the speed of the car, but there are two kinds of gear boxes, one is where the gears are changed that is output speed is changed after bringing the entire system to rest. We change the gear engagement within gear box by sliding or something, and then get switch on the machine and get a different output as we do in the case of machine tools.

The motor of the lathe machine rotates at a constant speed, but I want to get different output speed at the head stock or I want to rotate the job of the machine at different speed depending on the diameter of the job. In machine tools we use a kind of gear box, where we bring the entire system to rest change the gear engagement and then again switch on the machine. Whereas, in automobile we need to change the gears while the car is in motion or the input motion is on. We cannot switch of the car and then change the gears. We have to change the gear while it is in motion for that this gear boxes are more complicated, and we have to use what is known as synchromesh. This is a complicated system where a clutch action is needed such that gears which are going to engage are speeded up, and get into the same speed or proper speed before they get into engagement. Whereas, this machine tool gear boxes which are easier to design and easier to analyze uses, what to be known as sliding clusters. We shall end this course by having a little discussion one this sliding cluster gear boxes, which are used in machine tools that is from one input speed I want to get several output speeds by changing the gear engagement, I will show you through figures now such a sliding gear box which are used in machine tools.

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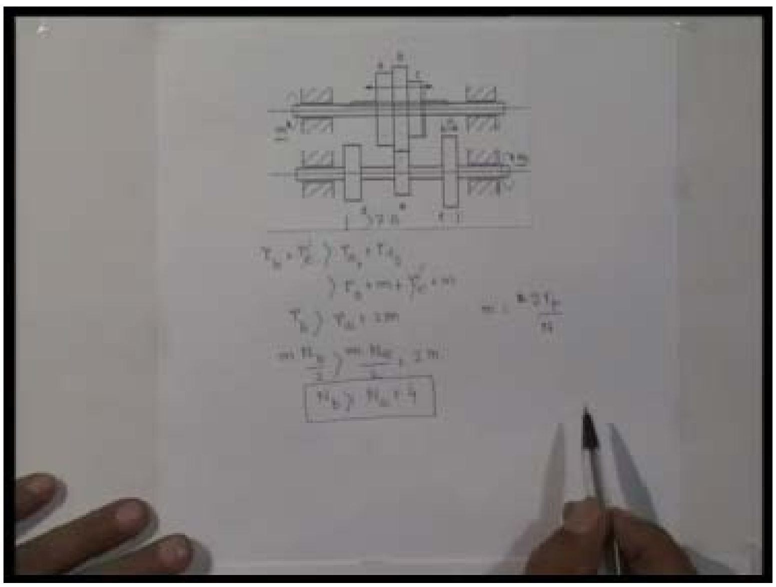

This picture shows what I just now said is a sliding cluster gear boxes, which are used in machine tool application. This is the say the input shaft and this is the output shaft. For the same speed of the input shaft omega$_i$, I may like to get three different output speed omega$_o$, to do that I have a compound gear, this is the same rigid body and this whole cluster can be moved along this shaft, this shaft is a splined shaft and on this splined shaft this whole cluster can be moved, right now this gear b is in engagement with this gear e so it transmits a particular speed ratio. If I want a different speed ratio, I stop the entire machine then slide this cluster such that this gear c comes in engagement with this gear f and depending on the number of teeth, they will transmit a different output. Again if I want to have another third different speed I will stop the machine and move this entire cluster such that this gear comes in contact with this engagement with this, and depending on their number of teeth they will transmit a particular speed. This is a three speed gear box using 6 gears and this is the sliding cluster, this is one rigid body but number of teeth is different on these three gear bodies a, b, and c.

The thing to note is that before this gear comes in engagement with this gear, the gear which was already in engagement must be totally clear. It must leave this engagement before its starts engagement. Similarly, when I move in this way before this gear comes in engagement with this gear, this engagement must be fully clear. Suppose, if the gear face widths of all these gears are same and I call it as B. If every gear the width is B then this total distance as we see is B plus B, because if it has to clear that movement is B so before it comes into engagement so B plus B plus B plus B plus B again this gap must be B and B, so this whole distance becomes at least 7B. It should be more than 7B, one more thing to note if that the number of teeth difference, the difference in the number of teeth between these two gears must be at least 4. Otherwise, while moving in this direction this gear will foul with this gear.

As we see pitch circle radius of this gear b plus pitch circle radius of this gear r_e that is the centre distance but when this gear is sliding over this, this must be greater than the outer radius sum of the outer radius of a and b. Otherwise, a will interfere with e the outer radius of a plus outer radius of e, the center distance should be more than that, such that this should not interfere with this when I am sliding it to the right. If we remember the outer radius is pitch circle radius plus the module, so outer radius is pitch circle radius plus the module so if I cancel pitch circle radius of this so pitch circle radius of b must be greater than pitch circle radius of a plus 2m, and if we remember the number of teeth module is nothing but number of teeth pitch circle diameter by number of teeth this is the module. So r_b I can write, module into number of teeth on gear b divided by 2 should be greater than module into number of teeth on gear a divided by 2 plus 2 m, where m is the module. If we cancel m I get N_b should be greater than N_a plus 4, so for designing such sliding cluster gear boxes one has to ensure that the minimum number of teeth between these two adjacent gears on a cluster is at least 4, should be more than 4 and this width should be sufficient such that total disengagement takes place be from this pair before the other pair this way or that way comes into engagement.

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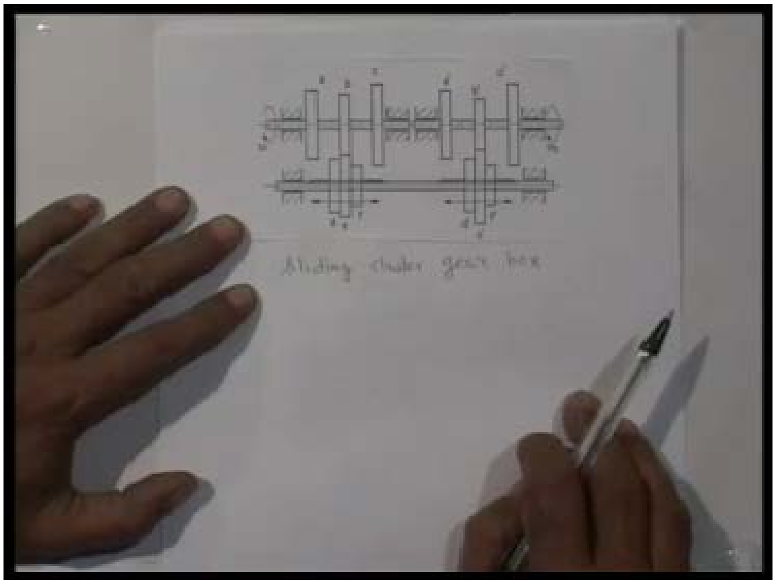

Using the same thing I can get a nine speed gear box. If this is the input shaft and this is the output shaft, and this intermediate shaft I make it splined and here I keep two sliding clusters. Depending on where the connection is taking place, I can get from the same input speed nine different speeds. For example, right now this and this is in engagement and this and this is in engagement so depending on the number of teeth on these 4 gears I get a particular output speed. If I change, slide it this comes out of engagement and this becomes in engagement and here it is the same so depending on the number of teeth on these two gears I get a different speed. I can get three here and three here and by combining them three into three I can get nine different output speeds from the same input speed. This is what is called sliding clusters gear box which are used in machine tools. This is a nine speed gear box from the same input speed I can get nine different output speeds.

The thing to note that because splined shaft are much more costly we have put both the sliding clusters on the same shaft, input shaft, intermediate shaft which is a splined shaft and this is the output shaft, input and output shaft have fixed gears. Whereas, this intermediate shaft which is a splined shaft has sliding gears so this is a case of a nine speed sliding cluster gear box, but as I said for when the gear changes to take place while the whole system is running that is much more difficult to design and we need what I call synchromesh and that is beyond the scope of this course.