Manufacturing Systems Technology

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Module - 02

Lecture – 12

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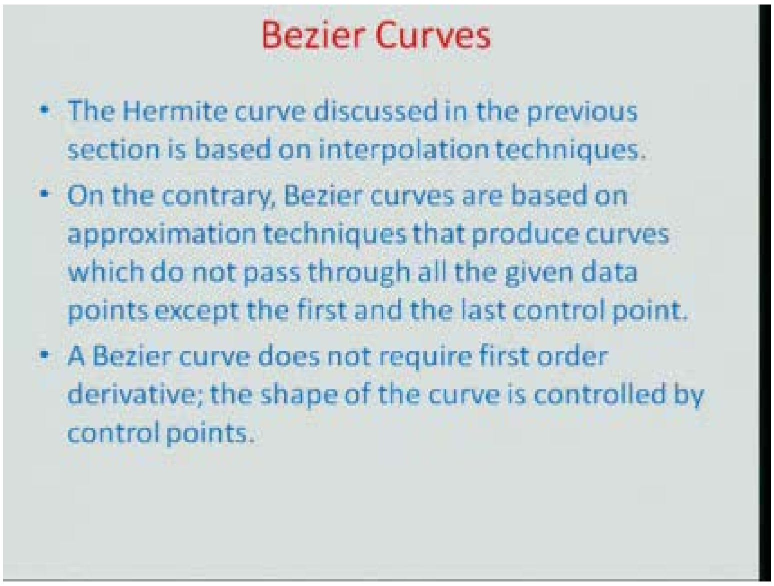

Hello and welcome to this manufacturing systems technology module 12. We had in the last module discuss how to fit the Hermite cubics polynomial to 2 points and to the practical example where we mapped 2 points with their slopes according to variety of family of curves, variety of different curvatures. What involved I mean in fact floating those curves between the 2 end points. So, we had discuss last time that it is very difficult for an unrealistic actually for a mythology to define the slopes of certain coordinate points particularly when the topology becomes very complex. And therefore can we have an equation or a or a fit layer really without considering the slope also would be able to vary and manipulate the curvature of the curve between the 2 points just in the same manner as we did earlier in the Hermite cubics explain.

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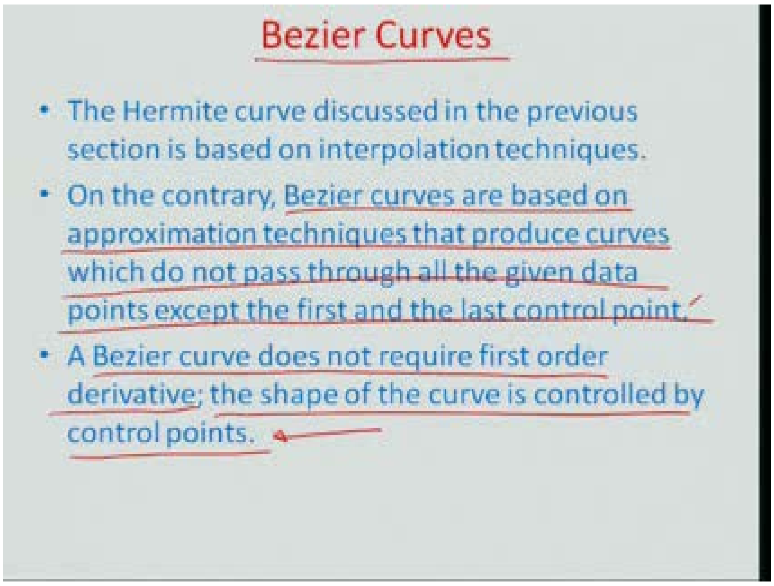

So, answer to that is this new curve fit which is known as the Bezier curve fit, and the Bezier curves are based on approximation techniques that produce curves which do not pass through all the given data points except the first and the last control point. Although there is contribution from the various other data points to the overall curvature of the curves passing through the set of points section. So, the Bezier does not require first order derivative, and therefore we can do away with the slope at a certain point which was major practical problem when we talk about Hermites in earlier. And the shape of the curve is controlled only by varying the control points as illustrated here.

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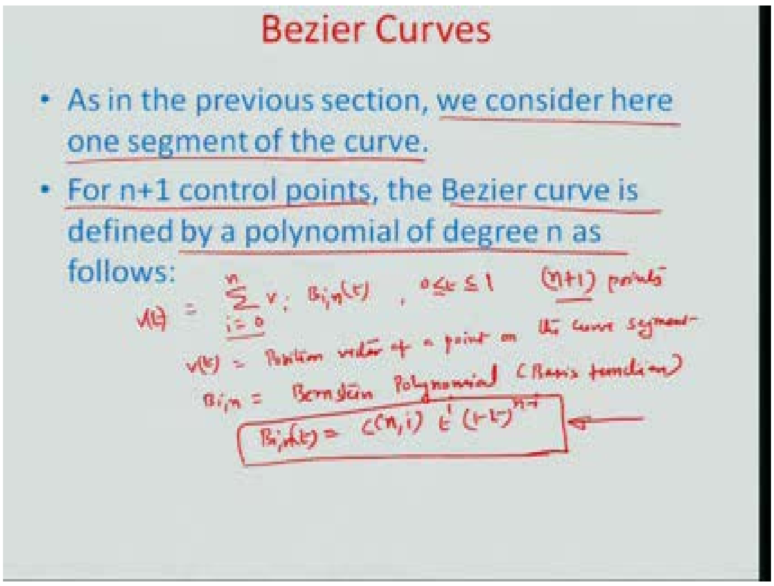

So, let us looks at how such you know controls or how such kind of a curve fits can be obtained. So, we consider here 1 segment of the curve, and for n plus 1 control points the Bezier curve now is defined by a polynomial of degree n, which is written as vt equal sigma, i varying between 0 and n, v i, v i n function of t; t is the parameter where t varies again between 0 and 1 and vt is the position vector really. So, vt is the position vector of a point on the curves segment; obviously, when I am talking about various 0 to n points; it means actually n plus 1 points on the curve segment. And the B i n is a polynomial which we also otherwise known as Bernstein polynomial. And so this is actually the basis function of floating the Bezier fit component and the B i n is a function of t can be mapped as the combination of n i times of t to the power of i 1 minus t to the power of the n minus i, that is how you plodding the or formulating the basis function known as the bunts steam polynomial in this particular case.

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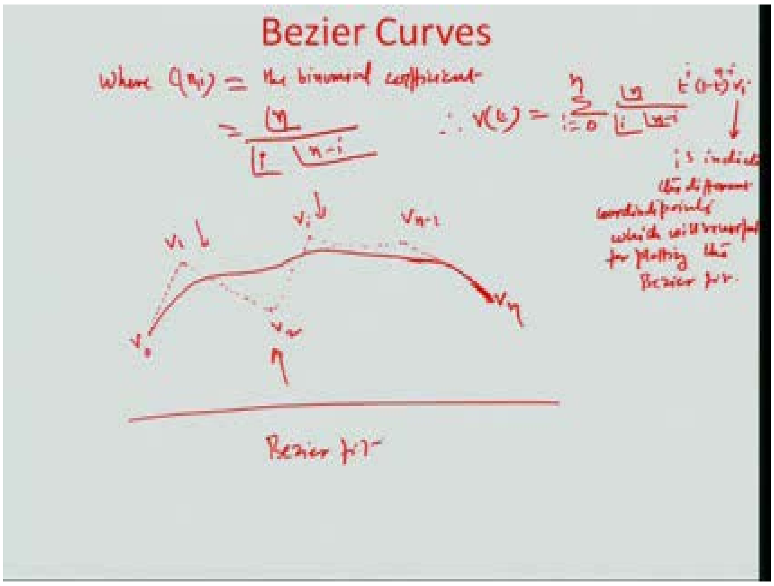

So obviously, now the expansion of this polynomial can be carried out by opening the combinations. So, c i n c, let us say we call it c n i, and is the higher index on which is there. So, c n i use actually the binomial coefficient and it can be expanded as n factorial by i factorial times of n minus i factorial from common mathematics knowledge. So, therefore vt the polynomial can be written down again as a sigma i varying between 0 to n for the n plus 1 control points factorial n by factorial i times of factorial n minus i t to the power of i 1 minus t to the power of n minus i v i, where these i is indicate the different points or you say different coordinate points, which will be useful for plotting the Bezier fit. So, this tuition is something like illustrated here that, lets say we have a set of n plus 1 points starting between v 0, another points somewhere here v 1, another point may be somewhere here v 2, then probably somewhere here which is v some point i may be and then you know all the way to v, let us n minus 1 and finally v n.

So, these are the different n plus 1 points and space, if you just connect them with the broken line, because you know they are not in totality representation is only limited to few points here although the number of points are n plus 1. So, we use a broken line to connect all these. So, the Bezier ultimately which would get formulated on the Bazier fit would looks something like this between the different points and the mayor variation of such points here would have a curvature variation, and local domain which can be used to create again a family of curves which can be fitted to the realistic situation.

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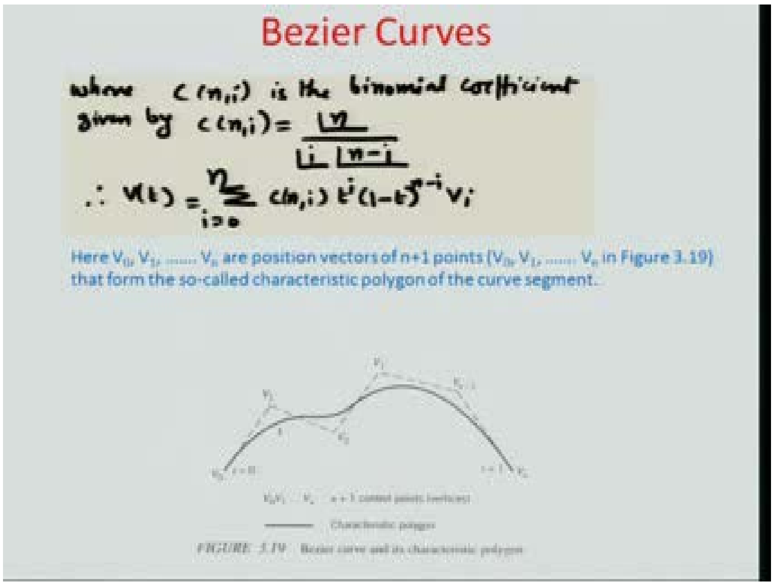

So, that is how the Bezier fit is achieved, which work how day property based the study of what are the different aspects associated with this particular fit.

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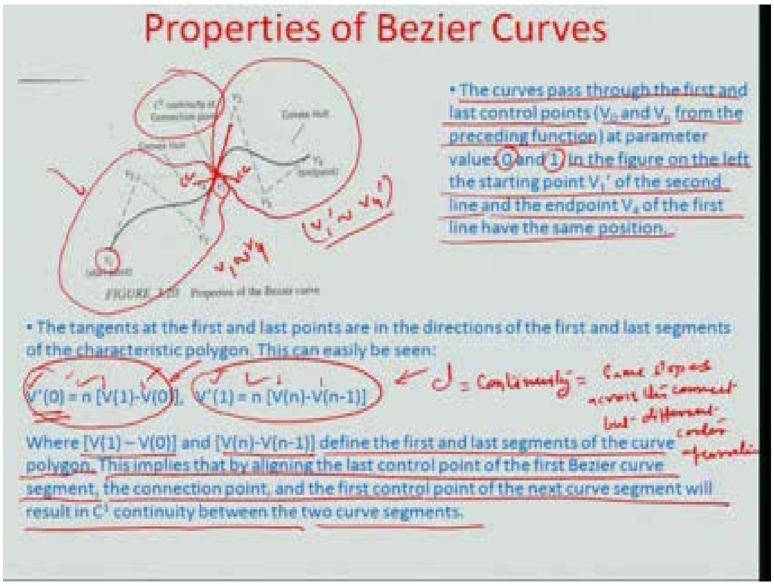

So, let us just first of all try to summarize by saying that the curves would pass through the first, and the last control point you can see here that the curves are passing either through v 1 or this v 4 here incidentally for the first polygon which is this particular. You know polygon the fit is between v 1 to v 4 and for the next polygon, which is let us say this particular polygon; the fits start between v dash 1 to v dash 4, the dash nearly indicates just different notation of representing the different points, and don not confuse it as slope for anything and this particular case.

So, here what the Bezier fit is trying to show it is of continuity between 4 points, which has been used to fit 1 section of the curve, and 4 points which is being used to fit the next section of the curve as sort of a continuous relationship between these 2 for sets. So, the curves pass through the first and the last control point v 0, and in this case n is equal to four. So, in this case it starts from v 1 and v 4, and this again start originates from the preceding functions. So, the preceding function in this particular case was this, and there is a c 1 continuity meaning there by the slopes are same although the center of curvature or both the curves may be different. So, there is some degree of flexibility in that aspect. So, there is c 1 continuity of connection between these 2 different convex cells, and in the figure on the left you know another thing is that you can correspond correspondingly vary the v in the manner.

So, that corresponding to t equal to 0 and t equal to 1 at represent the first, and last point which are essentially to be included with in the curves, it is not necessary that we include the remaining points, and in the figure on the left the starting point v dash one of the second curve the end point v 4 of the first curve have the same position coordinates. So, in such a case, if you the tangents were supposing of supposedly defining c 1 continuity; remember c 1 continuity means that same slopes across the connect, but not necessarily the same center of curvature, but different center of curvature you can see for example, here the center of curvature of this end. For example, of the curve somewhere here, and that on the other end is probably somewhere here c, c are different only what is important is at the slope of both the curves and both the directions same to each other.

So, the slope here add v dash 0, I would say v dash 0 is the first part of the slope at the first part of the curve would should be equal to n times of v 1 minus v 0, I am going to prove this expression in the next subsequent slide, and v dash 1 is actually equal to again n v n minus v n minus 1, where the v 1 minus v 0 and v n minus v n minus 1 defined the first and last segments of the curve polygon. So, this implicates implies that the by lining the last control point of the first Bezier fit the connection point, and the first control point of the next curve fit results in a c 1 continuity between the 2 curve segments. So, slope at the one of the last curve should be equal to the slope at the 0 of the succeeding curve that is what goes less? So, let us now try to arrive at these 2 conditions mathematically, and see by putting the polynomial function and trying to find out what is the v, whether we are able to hit upon this condition here.

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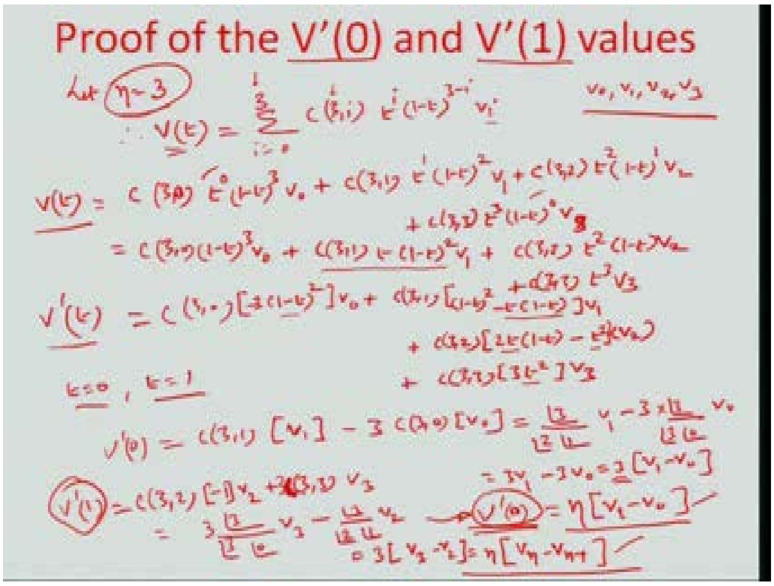

So, let us look at v dash 0 and v dash 1, and how they would be obtained mathematically or numerically. So, let us assume a case equal to 3 different points, 4 different points with n varying between 0, 1, 2 and 3. And we want to write down the vector position the parametric form of the vector position of the fit equation, which is vt equal to sigma i varying between 0 to 3 c 3 1 combination 3 1 n, now minus 3. Therefore, I am substituting the value of any all the cases times of t to the power of i times of 1 minus t to the power of 3 minus i times of v i. So, that is how corresponding to an equal to 3, you can define the vt. So, if I express this function in terms of v 0, v 1, v 2 and v 3 separately, and just we open it up the way we can represent this whole thing becomes equal to vt equals c 3 1, the first term times of; obviously t to the power of, sorry we start with 0.

So, c 3 0 times of t to the power of 0 1 minus t to the power of 3 v 0 plus c 3 1 t to the power of 1 1 minus t to the power of 2 v 1 plus c 3 2 t to the power of 2 1 minus t to the power of 1 v 2 plus c 3 3 t to the power of 3 1 minus t to the power of 0 v 3, and that is how we kind of shape this thing up between all the different points further, if we try to… So, expand this little more carefully would be just eliminating the power zeros, and we have c 3 0 1 minus t cube v 0 plus c 3 1 t times of 1 minus t to the power whole square v 1 plus c 3 2 t square 1 minus t v 2 plus c 3 3 c t q times of v 3, as you can c here. Always we have to also find out what is v dash t. So, the v dash t in this particular case would actual become equal to c 3 0 thrice 1 minus t square to the minus sign v 0 plus c 3 1 times of 1 minus t square plus t times of 1 minus t, this would be a there would be negative sign between. So, minus t times of 1 minus t times of v 1 plus, similarly c 3 2 times of twice t 1 minus t minus of t square times of v 2 plus c 3 3 3 t square v 3.

So, that is how you define the v dash t, and if you correspondingly corresponding the put the value of t equal to 0 and t equal to 1. So, that the end points emerge out of it and if I choose this value here. So, the v dash 0 corresponding to t is equal to 0. Now comes out to be equal to c 3 1 v 1 minus 3 times of c 3 0 v 0, remaining other expressions involve in t and mathematically get the eliminated, because t is equal to 0. And therefore, the this scan also be represented as 3 factorial by 2 factorial 1 factorial v 1 minus thrice of 3 factorial by 3 factorial 0 factorial v 0, and other word it is 3 v 1 minus 3 v 0 or 3 times of v 1 minus of v 0, and we already know that this corresponds to the index n. So, you can have n times of v 1 minus v 0 to be defining the v dash 0 in 1 instance, and in similar manner we can actually suggest or we can actually try to define the other expression where we talk about v dash 1.

And if I just put v dash 1 in this particular expression here. So, all with 1 minus t would get eliminated and the ones which would have the individual t would be retained. So, the only thing which would be available to us is c 3 2 minus 1 times of v 2 plus c 3 3 times of v 3. So, this become is equal to factorial 3 factorial 3 0 factorial v 3 minus and you have 3 here. So, 3 times minus of factorial 3 by factorial 2 times of 1 factorial v 2. So, in this case this comes out to be equal to thrice v 3 minus v 2 or this can be represented as in a generic manner n times of v n, which is the last point minus v n minus 1. So, as you can see here the various v dash 0 and v dash 1 corresponding to the first point, and the last point of a particular curve section in 1 case are equal to n v n minus v n minus 1, and in other case equal to n v 1 minus v 0.

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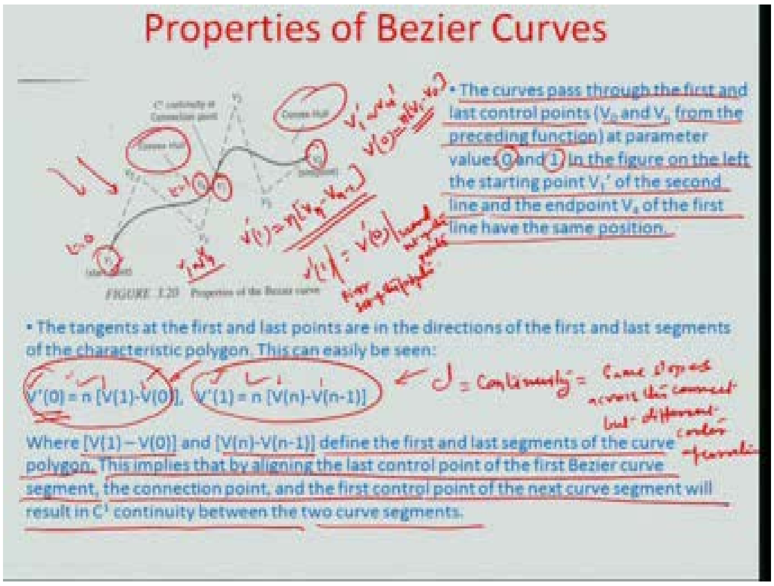

So, in a nutshell is going back to the earlier slide as you see here the for the first convex hull just going to rub off of a few things here for just for the benefit of clarity. So, in this particular case, we are starting to a plot 2 set of points 1 set starting between v 1 all the way to v 4 in other set, which is starting between v 1 dash all the way to v 4 dash. So obviously, in this particular curve segment 1, which is corresponding to this convex hull in indicated here between v 1 to v 4 the last point corresponds to the slope at the last point corresponds to v dash 1; obviously, t is equal to 0 at the first point t equal to 1 at the last point and this happens to be equal to n times of v n minus v n minus 1, as you had just seen in the last derivation and for the second convex hull corresponding to the points v 1 dash to v 4 dash, the slope at v 1 dash is corresponding to the slope at t equal to 0 for this new convex hull, which is actually equal to n times of v 1 minus v 0.

So, in order to certain the c 1 level of continuity; obviously, the v dash 1 has to be equal to v dash 0. So, v dash 1; obviously, is for the first set of the points and v dash 0 is corresponding to the second set of the points. So, this is something that you have to be careful when you try to do a practical example problem.

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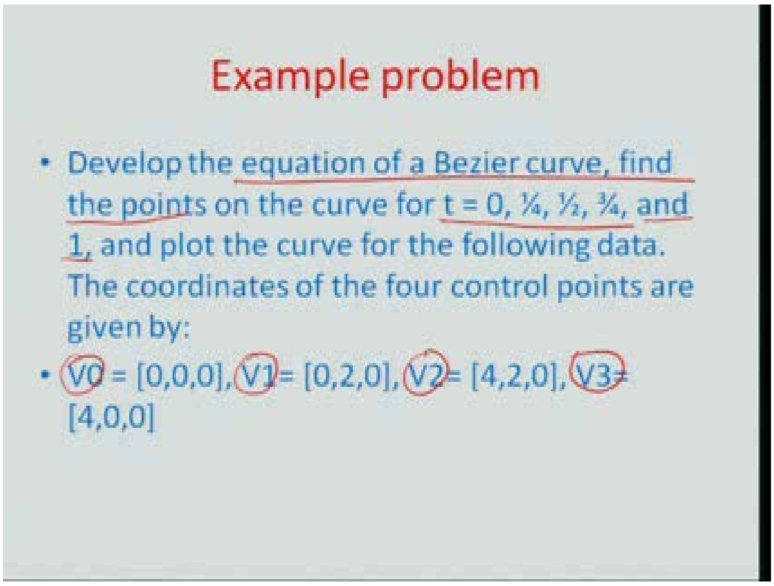

So, we want to just solve a practical example of Bezier fit, where we want to develop the equation of Bezier curve, you know will try to find out intermediary or intermediate points corresponding to t equal to 0 one fourth, half, three by fourth and 1. So, we splitting it up into 0 point 25 increments. And we plot the curve for the following data points coordinates of 4 points are given v 0 to v 3, and these are only a 2 dimensional curve, because you can see the z coordinate to be 0. So, basically the curve is on the x y plane, we want to find out what is the function functional fit between these points, and also map corresponding to the different values of t for the function.