Manufacturing Systems Technology

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Module - 03
Lecture – 14

Hello and welcome to this manufacturing system technology module 14. So, quick recap of the last module, we were actually talking about how to do a surface patch, and we wanted to find out a mortality where algebraically we could determine what is the quantum of calculations involved in pointing out to a surface patch; this patch would be used for any complex topology which is not a regular geometry, and it can be varied according to some parameters in the manners that it fits the actual surface; and that is how you actually represent a surface by mapping it topologically between various patches and interconnecting those patches with respect to 1 another. Just as you do the curve based fitment of a geometry, you are doing the surface based fitment in this particular case.

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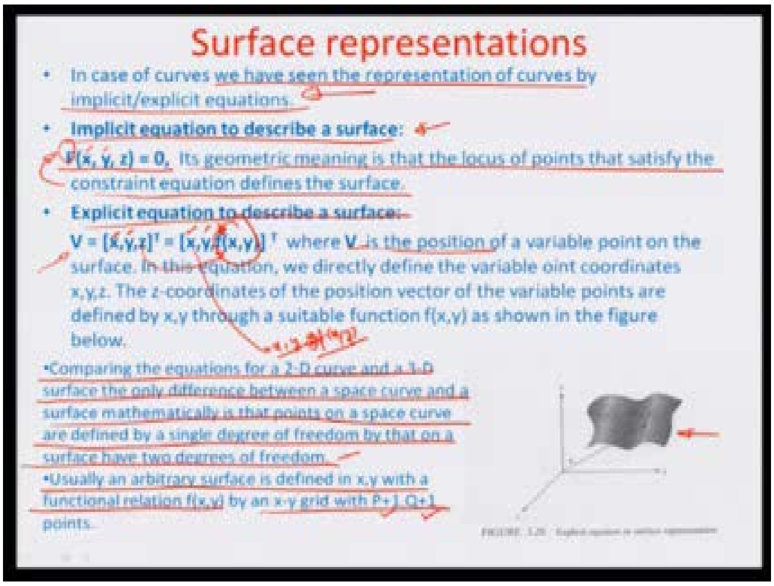

So, let us look at the surface representation, just need to extrapolate the curves by other dimension. So, in case of curve for example, we had seen that the representation of curve were done by implicit and explicit equations, same thing choose rules 2 for this also. So, we can have a implicit equation to describe a surface, just as you have done earlier for the 2 dimensional curve or even a 3 dimensional curve. So, let us say you can have a functional relationship fxyz equal to 0, its geometric meaning really is locus of points that satisfies the constraint equation, and that defines the surface, and they are all sort of downed by this functional relationship of xyz equal to 0. So, this the implicit form of the equation, and is not very clear as to how there is a relationship happening between x or y or y or z. Although there is overall governing relationship between all the 3 variables, which happens for an explicit equation to describe a surface the same thing can be represented in a more direct manner, and looking at the independent coordinates x and y and varying them, and studying the function relationship of z with respect to x and y. So, the z varies as a function of the various xs and y coordinates on a surface.

So, that is how you explicitly represent the position vector in this particular case on a 3 dimensional plane. So, 2 variables are completely random and then this functional relationship is a governing relationship between the z, and the way it goes with the different values of x and y. So, comparing the equation for a 2 d curve and a 3 d surface the only difference between space curve and a surface mathematically is that points on a space curve are defined by a single degree of freedom and that on a surface by 2 degrees of freedom. So obviously, the x and y here are 2 independently changing variables with no relationship what. So, ever between x and y the only other relationship which holds to here is that. z equal to f x y; that means, z is dependent both on x and y is independent variation, but in a way z is constrained by a certain value of x and certain value of y.

So, therefore, you have a 2 degree of the freedom system here that we are talking about and usually an arbitrary surface is defined in xy with a function relationship xy by an xy grade with p 1 p plus 1 q plus 1 control points something like this is what a surface patch would expected to be what it would look like to be…

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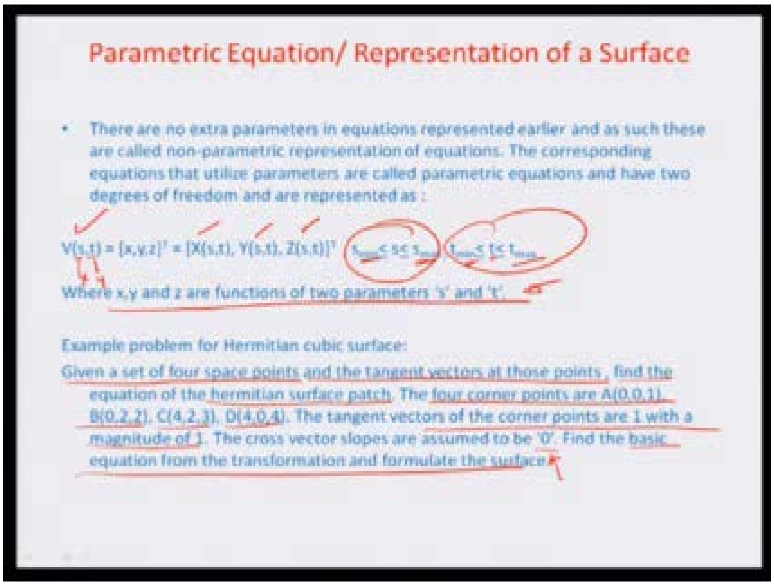

So obviously, we can also just as we made parameterization parameterization of the explicit functions of the earlier kind in case of curve, we added a variable t. So, that the local variation of t would lead to a sort of local domain in which you could more closely study a curve of a complex nature, this case also surface can be very closely studied for a given a global domain of that surface, you know by just having instead of 1 parameter, because there are 2 degrees of freedom 2 different parameters, just as in the curve case 2 d curve case 1 was independent variable. And then was 1 degree of freedom system, we were using 1 parameter t to define the x values, and y would automatically get defined by functional relationship as the x varies in this case, because x and y are independent.

So, we have to consider 1, x 1 parameter t probably in one of the directions may be y another parameter x in the directions of s in the in the directions of x. So, that both independently can vary to have that independent local domain variation given a global domain definition of the particular curve. So, in this case again vst again, we call these 2 parameters smt 1, and actually x another representing y can be defined as xst, yst, and zst. So, in case of s, the s varies between sum minimum s and maximum s. So, does the t the t varies between minimum t and maximum t and xy and z are functions of 2 parameters smt in this particular case. Let us look at a problem to have a more appropriate definition. So, we have been given a set of 4 space points, and the tangent vectors of those points are also given.

So, find the equation of the Hermitian surface patch. So, we want to construct a cubic polynomial based fit, but now it is the surface fit rather than a 4 corner points have been given as 0 0 1, b 0 2 2, c 4 2 3, and d 4 0 4. The tangent vectors at the corner points are 1 with magnitude of one. So, the cross vector slopes are assumed to be 0, there is requirement of a cross vector slope meaning there by the variation of parameter with respect to s, and then with respect to t. So, it is a double slope that we are considering there. So, we call it cross slope. So, as you see as the equation evolves this will automatically come into picture. So, find we have to find the basic equation from transformation, and formulate the surface or see the level of complexity, which is involved in this particular calculation.

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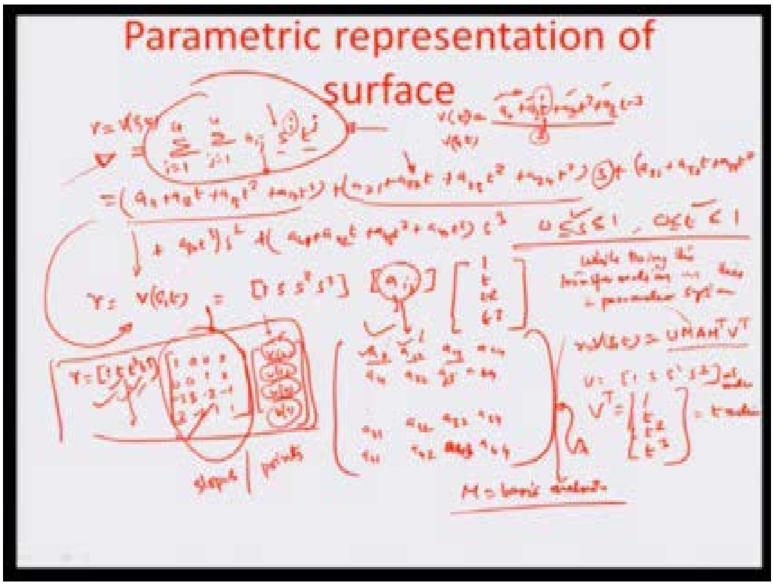

Once we start doing it, so obviously if I look at this equation, I would be able to write the vector vst as 2 different independent parameters. So, I say there are 2 parameters; one is i and one is j, and one is actually on 1 variable s, another parameter t. So, I call i equal to 1, 2, 4, sigma j equal to 1 2 4 aij s to the power of i t to the power of j. So, if I compare it with what we had done before we were talking about a case when we just represented 1 parameter. So, basically we talked about let us say the point v, you know as a function of t and we define the cubic polynomial a 0 plus a 1 t plus a 2 t square plus a 3 t cube.

So, this was a 2 dimensional curves case that we were defining. Now we have the surface case; obviously, this t is having another parameter introduced s, and then there has to be just as you had a sort of index order i equal to 0, i equal to 1, i equal to 2 and 3. In this particular case you will have to have 1 index order for t, and the other for s, because now it is a sort of a multiplication between s and t which would be able to. So, we are talking about vst, so obviously if I spread this out, I would have formulation a 1 1 plus a 1 2 t plus a 1 3 t square plus a 1 4 t cube, and plus you know a 2 1 plus a 2 2 t plus a 2 3 t square plus a 2 4 t cube into s; that is how I would like to vary the ss and the ts here just expanding this particular equation. So, I will combine all the s subscribes corresponding to s equal to 1, and all the other values which are there and put it together in a bracket like this similarly s to the power of 0, I put here.

Similarly I will do s to the power 2 and 3. So, I have a 3 1 plus a 3 2 t plus a 3 3 t square plus a 3 4 t cube times of a square plus a 4 1 plus a 4 2 t plus a 4 3 t square plus a 4 4 t cube times of s cube, where s varies between 0 and 1 and t varies between 0 and 1 again. So, there is a local variation in s as well as t both the variables. So, this is how the cubic polynomial is constructed in this particular case only difference here is, there is an additional degree of freedom, because of which an additional subscript is also coming into picture for the various is and various js along the parameter s and t. So obviously, in a similar kind of a manner, you know you can actually represent this in terms of a matrix equation, where r you know can be vst. And this can be represented as 1 s s square s cube 1 matrix times of sum a matrix which is corresponding to aij, and obviously, in the other side you have a t matrix. So, 1 t t square and t cube. So, this how you can write the whole expression here, and this is the 4 by 4 matrix which can vary between a 1 1 a 1 2 1 3 1 4 and so on, 2 1 2 2 2 3 2 4 and this goes all the way to 3 1 3 2 3 3 3 4. So, 4 1 4 2 4 3 4 4.

So, this is how this matrix has been realized aij matrix has been realized. So, while doing the transformation, you can have a similar kind of a transformation equation is the 2 parameter system, and the equation in this case can be in the similar manner as we had done before for a single parameter, if you may remember. So, in this particular case the 1 dimensional case, if you just look at it, you know the final transform matrix was something like a t matrix 1 t t square t cube times of some kind of a basis matrix or n matrix, which was formulated this we had 1 0 0 0 0 0 1 0 and minus 3 3 minus 2 minus 1 and 2 minus 2 1 and 1. And then on the other side we had v 0 v 1 v dash 0 and v dash 1. So, in the similar manner in this particular case because now the parameter has changed from only t to 2 parameter system t and s, the equation would get slightly modified you know in this particular case, and here the equation turns out to be a basis matrix n and there is matrix u here, which corresponds to the s matrix times of a basis matrix m which is the conversion matrix really in terms of numbers, times of the a matrix which is again represented here very beautifully as a 1 1 2 so on so forth, times of the transpose of the basis matrix m times of v to the power of or v matrix to the power of t actually or transpose of v matrix.

So, all these let you know what all these transformations mean just as you had done here, there was a t matrix. So, here this u is basically the s matrix, I will have this as 1 s s square s cube. Similarly just like this t matrix here in the 1 dimensional case and there would be a v matrix here, which is again you know the transposed basically it is the t matrix that we are considering in this particular case which is something like 1 t t square and t cube. So, this is the v transpose. So, it is really the t matrix this is s matrix, and obviously a matrix has already been defined here what are the a matrix is, and m is the basis matrix which is really the list of numbers, you know you remember there is a list of a numbers here, which relates the t to the different points v 0 v 1 v dash 0 1 v dash 1. In a similar manner this m is the basis matrix for this particular problem in question, and that actually happens to be very well recognized.

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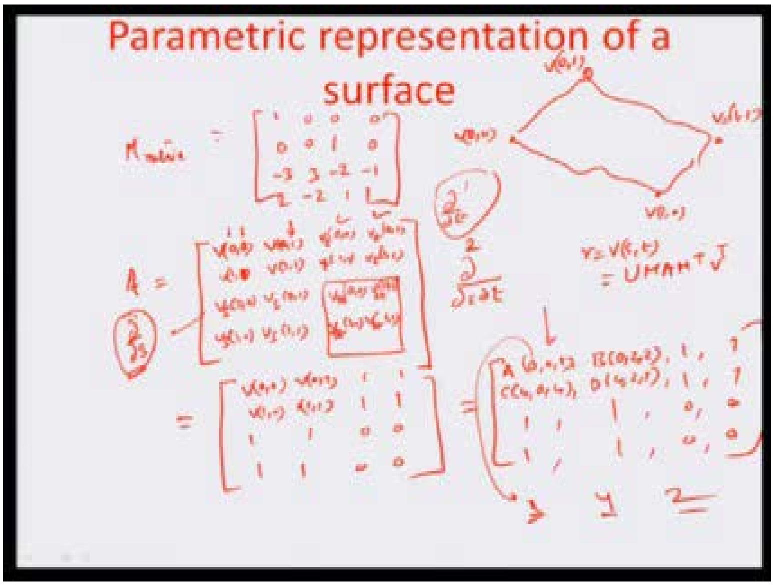

I will just give you the value of the m matrix in this particular case. So, the basis matrix m matrix comes out to be 1 0 0 0 0 0 1 0 minus 3 3 minus 1 minus 1 twice minus 2 1 and 1. Remember we have not found out what is A matrix in this particular case, and the A matrix in this particular case comes out to be equal to in terms of… So, the a matrix was really if you know recall the A 1, A 2, A 3 and A 0; they were really in terms of these points and cross slopes, you know how that is how we incorporated the A matrix here. So obviously, this A 1 1, A 1 2; they should be solved for this particular equation that has been illustrated in this 2 dimensional case problem this particular equation, and in terms of the available slopes and the available points, and if I calculate what is this A1 1 A 1 2

A 1 3 and so on so forth from the calculations it emerges to be the following.

So, the a matrix finally, in terms of the points and the slopes comes out to be equal to the point v 0 0; that means, this is the beginning point probably 1 corner of the particular surface patch that is an question v 0 1, which is the other corner the other extent where only 1 parameter has varied the other has not. So, therefore, it is corresponding to x equals to 0 probably and on to the parametric variations of t from 0 to 1 similarly v 1 0 that is the forth the third corner and v 1 1, which is the forth corner. So, if I am talking about a some kind of a surface patch like this. So, this 4 corners are really the v 0 0, v 0 1 and here, it is v 1 0 and this is v 1 1.

So, these are the 4 corners; obviously, these points have been given in the arguments of the questions, and then there are the various slopes with respect to the t directions. So, you have slopes at the point 0 0 slope at the point 0 1 slope at the point 1 0 and slope at the point 1 1, that is how the a matrix have been formulated and the slope with respect to s. Now this was with respect to t which meaning there by d by dt of that particular function, now this is the d by ds of that particular function; obviously, there are 2 parameters. So, there has to be a derivative in tn derivative in x both the parameter should have derivatives or slopes along the around directions. So, this can be represented as vs 0 0 vs 0 1, similarly vs 1 0 and vs 1 1. And finally, these 4 parameters happen to be very interesting, these are the cross slopes meaning there by these are the derivatives with respect to t with respect to s at the point 0 0 the derivative with respect to t with respect to s.

So, basically this is nothing but d by d 2 by ds dt, that is how you are basically derivating this particular taking finding out this slope with respect to the function there, and this is at the point 0 1. And similarly the cross slopes vst at the point 1 0 the cross slopes vst at the point 1 1, that is how it has been indicated here, and obviously from the question itself you can find out the 4.

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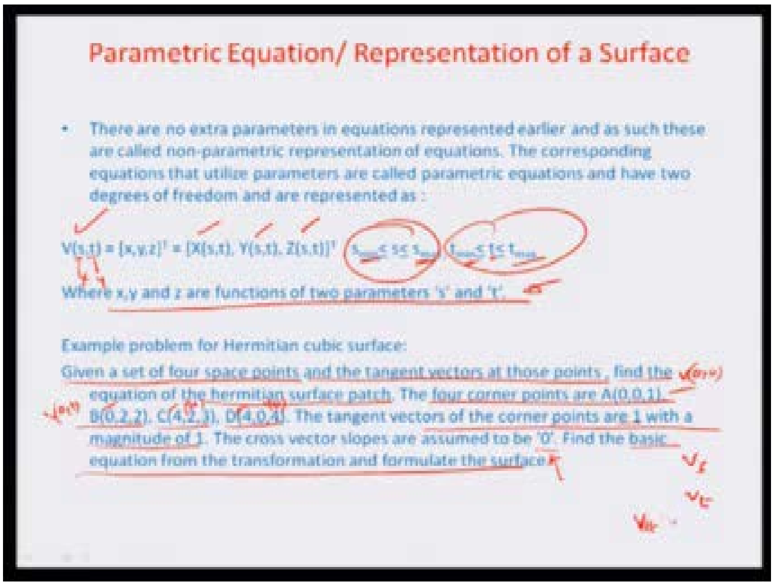

If i just go back to the question, it has briefly mentioned that the 4 points are given here probably corresponding to v 0 0 v 0 1 v 1 0 and v 1 1, and the tangents of the corner points are all one. So, with respect to vs or t they are all 1, and that with the respect to the cross slopes meaning there by with the respect to s with respect to t the cross slopes they are all 0. So, I am going to now change this equation into something which is more realizable and. So, that the particular case that we are considering this actually becomes now the points, let us say v 0 0 v 0 1 v 1 0 v 1 one and all these quantities are unit because they are the single slopes, and all these 4 quantities are zeros, because they are the cross slopes further this can be represented as the point a with 0 0 1 v which is 0 2 2.

Obviously, 1 and 1 would remain there and v 1 0 is corresponding to the points t 4 0 4 and d is corresponding to the point 4 2 3 1 1, and here you have both the single slopes and the cross slopes that is how the matrix base calculation has been done just in an identical manner as we did the 1 dimensional problem. So, now very interestingly if you put these different values. So, there are actually 3 matrices emerging out of it corresponding to the x values of the points, the y values of the points, and z values of the points, you will have a computational basis with which you can quickly compute the by putting the transformation equation which was represented here very clearly as vst equals una m transpose v transpose.

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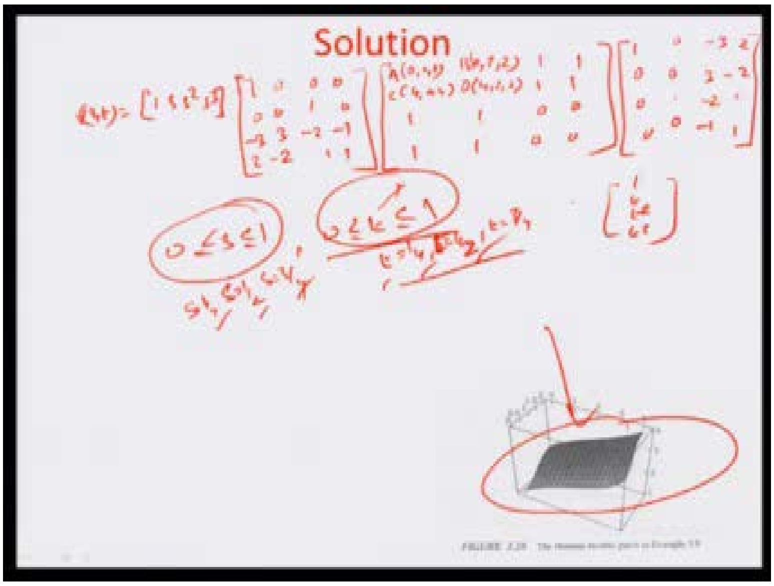

So, I am going to now do this particular calculation in the next slide, and try to plot it and see what happens. So, you have the you know formulation represented as the matrix u which is 1 s s square s cube times of the basis matrix, which has been already calculated as 1 0 0 0 0 0 1 0 minus 3 3 minus 2 minus 1 twice minus 2 1 1 times of the a matrix which is now assumed to be in 3 different forms. So, I just consider the integrated representation here a 0 0 1 b 0 2 2 1 and 1. Similarly c 4 0 4 d 4 2 2 1 and 1, and then you have 1 1 1 1 and 0 0 0 0 times of a matrix which is again. So, this is the, I am sorry a matrix times of the m transpose. So, then transpose happens to be 1 0 0 0, this goes as 0 minus 3 2.

Similarly this happens as 3 minus 2 and 1 and 0 comes here, and similarly minus 2 minus 1 1 and 1, this is how the m transposes obtained times of the matrix 1 t t square tq . So, the whole question here is, now if you vary the value of s between 0 and 1 and. So, do you vary the value of t between the same 2 domains may be in this particular case the s is equal to half in 1 case 1 forth in other case 3 forth in other case. So, is the values of t here, and you can find out by varying the different parameters in a similar manner, you remember we plotted the base here any other curve before we obtain this particular surface based on the various values of ss and ts, and that is how the patch can be formulated mind you here also in a similar manner.

If you would have varied the slope etcetera of the patch, then it would be very worthwhile to have variety of topologies emerging in the surface based on the n slopes or you know cross slopes variations, and that would be giving us a flexibility to fit again the surface patch on the actual surface to that we are actually doing question. So, I think we have done more and less all what is needed to understand from a curve to a surface in terms of the coordinate processing are the computational exhaustiveness of the whole process. The other idea is how to translate all these information into a cam package and all. So, I am going to now slowly talk about issues related to that following this module. So, this module is now coming to end.

Thank you very much.