

**Manufacturing Systems Technology**  
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**Module – 04**

**Lecture – 20**

Welcome to this module 20 of Manufacturing Systems Technology.

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**Numerical Problem**

- Suppose 500 units of a shaft are to be manufactured within  $1 \pm 0.003$  in. Suppose there are three alternative machine tools with the information given in Table below. Use the models developed earlier to perform the turning operation.

The other data are:

- Unit raw material cost = \$10.00
- Unit salvage value = \$2.00
- Process average = 1.0015 in.

**TABLE 5.1 Basic Data on Types of Machine Tools**

Types of Machine Tools	Standard Deviation, $\sigma$ (in.)	Processing Cost per Unit, $\$$ (\$/unit)	Processing Time per Unit, $t$ (min/unit)	Setup time, $S$ (minutes)
Turret lathe	0.007	7.00	1.00	15
Engine lathe	0.001	10.00	0.90	30
Automatic screw machine	0.0005	15.00	0.70	60

In the last module, we had discussed about numerical technique or numerical way to address machine selection problem, today we are going to realistic look into a case study and try to determine, how to select the machine. So, let say there is a manufacturing unit, which has this requirement of making about 500 units of a shaft and is manufactured within 1 plus minus 0.003 inches tolerances. Therefore, there is a range, which the upper specification limit and the lower specification limit allows for the particular design as far the manufacturing process goes.

And supposing there are three different alternative machine tools you know, with information, which is given here right in this table, which you need to consider for doing selection between different processes. Now, the information given by the manufacturer for the different tools one of them is turret lathe, another is engine lathe and the third one

is automatic screw machine is in terms of a processor standard deviation, that is recorded in inches meaning there by that this sigma for the turret lathe here you know sigma  $\sigma_1$  is represented as 0.007 inches. There is also a processing cost per unit, which has been given and this cost may include many things including consumables and some of the things needed for running the machine.

So, this is about 7 dollars per units that is, what the processing cost at current level of production 500 level of production has been mentioned, what is also given interestingly is the processing time per unit, which is 1 minute per unit and the setup time, you know the setup time is very important particularly when you do batch production processes. Because, the moment you change the design or the moment you change the particular manufacturing lot, you need to set up the machine again for the new design to be rolled out.

So, the setup time is given to be a 15 minutes for the turret lathe, similar data points have been given by the manufacturer for the engine lathe and the automatic screw machine. And given this capability of the process, we need to somehow see which of these tools are fit for doing, what we are doing here in terms of the design that we are manufacturing, which is 1 plus minus 0.003 inches diameter of the particular shaft.

There are some other data, which are important and which are there the part of the production process. One is the unit raw material cost it is; obviously, market defined, so that is about 10 dollar per piece, the raw material comes into the process. Remember this is the process there is an input side, which again the cost comes from the market on this particular side. And then; obviously, the output cost, which would be defined by the degree of rejects of the particular process you know.

And; obviously, the rejects also get sold at the price, so in this particular case this salvage value for the rejects is about 2 dollars per piece, that is how the process defines and the process average comes out to be about 1.0015 inches. So, that is how the  $\mu_j$  or the process average for all the  $j$ th machines have been defined as. So, this data is also a part of the manufacturer data that has been supplied by the machine manufacturer.

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**Solution**

Normal Variates

$$Z_{j|k}^L = \frac{k_{j|k}^L - \mu_j}{\sigma_j}$$

$$Z_{j|k}^U = \frac{k_{j|k}^U - \mu_j}{\sigma_j}$$

$\mu_j = 1.0015 \text{ in}$   
 $\sigma_j = 0.007$

$k_{j|k}^L = 0.997 \text{ in}$   
 $k_{j|k}^U = 1.003 \text{ in}$

$$Z_{j|k}^U = \frac{1.003 - 1.0015}{0.007} = 0.21$$

$$Z_{j|k}^L = \frac{0.997 - 1.0015}{0.007} = -0.64$$

$$S.C_{j|k} = \phi(Z_{j|k}^L) + 1 - \phi(Z_{j|k}^U)$$

$$= 0.24109 + (1 - 0.58317) = 0.26109 + 0.41683$$

$$S.C_{j|k} = 0.67792$$

$$k_{j|k}^g = \frac{S.C_{j|k}}{1 - S.C_{j|k}} = \frac{0.67792}{1 - 0.67792} = 2.1048$$

So, let us now, check for the different situations for all the three cases, what are going to be the different costs and the time parameter associated with this particular problem. So, let us first follow the method described in the earlier module, where we talk about calculating the normal variates. So, let us first determine the normal variates. Obviously, you had earlier seen the normal variates, which exist are actually towards the lower tolerance side and the upper tolerance side and these variates are defined in terms of the tolerance system or the tolerance specification; that is provided, which is represented here as  $t_{l, j k}$  minus the process mean represented as  $\mu_j$ .

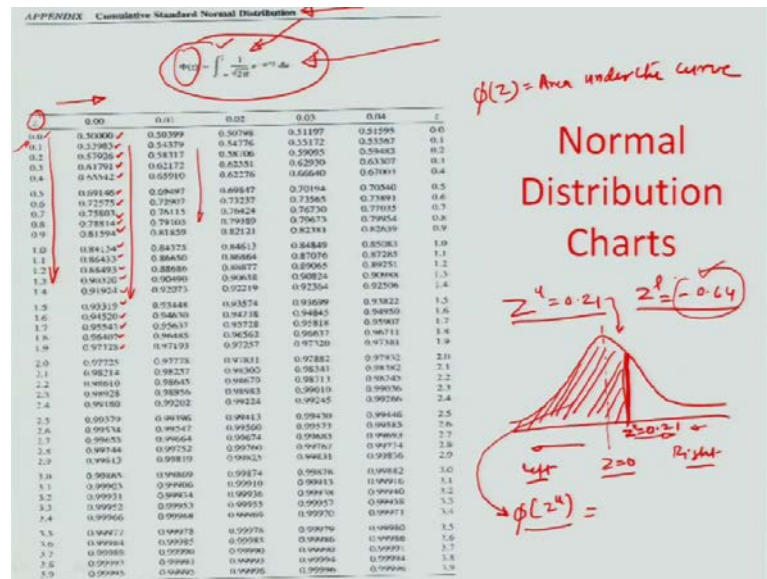
And how; that has spread in terms of the standard deviation of the particular process; that is in question and this values are been given by the machine manufacturer. Similarly on the upper side, it is the tolerance plus the basic dimension given on the upper side, so upper specification limit minus the process mean by the process of standard deviation given by the machine manufacturer. So, the various values here as recorded in the question are that if you look at the range of variation of the diameter, the value that has been given is 1.1 plus minus 0.003 inches.

Meaning there by, the upper tolerance and the lower tolerance specification limits are recorded as 1.003 inches and 0.997 inches. One is 1 minus 0.003, 1 plus 0.003 as the upper tolerance and the lower tolerance limit. So, obviously, we need to calculate the standard deviation recorded for the different systems that are in the questions are given in this particular table. And, so we can actually calculate the Z variate values.

Given all these information and make a Z upper, j k as the upper tolerance 1.003 inches. You already know that the process mean has been provided in the question as 1.0015 inches that is how you have defined the process mean. And, here then the upper variate comes out to be 1.003 minus 1.0015 divide by this standard deviation for the j th process and the standard deviation for the first case that is turret lathe cases recorded as 0.007. So, these come out to be equal to 0.21.

In the similar manner, we also calculate the lower variate as the lower specification, which is 0.997 minus 1.0015 the mu j value divide by the standard deviation in this particular case and this is actually recorded as minus 0.64. So, very important preposition that needs to be understood here is that, how do you really find out the cumulative distribution functions given these two Z variates. So, the question is how you now find the area under the curve and as I think, earlier illustrated if you really look at the way that normal distribution goes, which is actually defined by an integral as you can see right away here.

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So, this is actually a sort of area under the curve and it is numerical integral problems. So, you have defined values of Z on the basis of, which you can calculate corresponding to certain value of Z with the first decimal place represented in a columnar manner and the second decimal place represented in the row wise manner. The entries here are basically those cumulative distribution functions of this 5 Z value, you know area under the curve.

So, it is numerical integral, it has been already calculated, we just want to use the help from this numerical integral table. So, these are also known as standard normal distribution table, it is available everywhere in any statistics book commonly found. So, we want to now calculate that corresponding to our Z values, which was represented and we had calculated as on the upper side 0.21 and on the lower side minus of 0.64, how do we really find out the normal distribution function.

So, further I would just like to channelize a little more as to, how this minus 0.64 can be obtained. You see the normal distribution has been calculated only from the 0 value onwards and there are no negative values in this table particularly, which we are considering and one of the reasons is that, it is a symmetric function. If you look at the normal curve, it is represented by this bell shaped inverted bell shaped curve and let us say about the mean and particularly in a case, where mean equal to 0, the curve is exactly a replica the symmetric replica of each other if we divide into a right quadrant and another left quadrant.

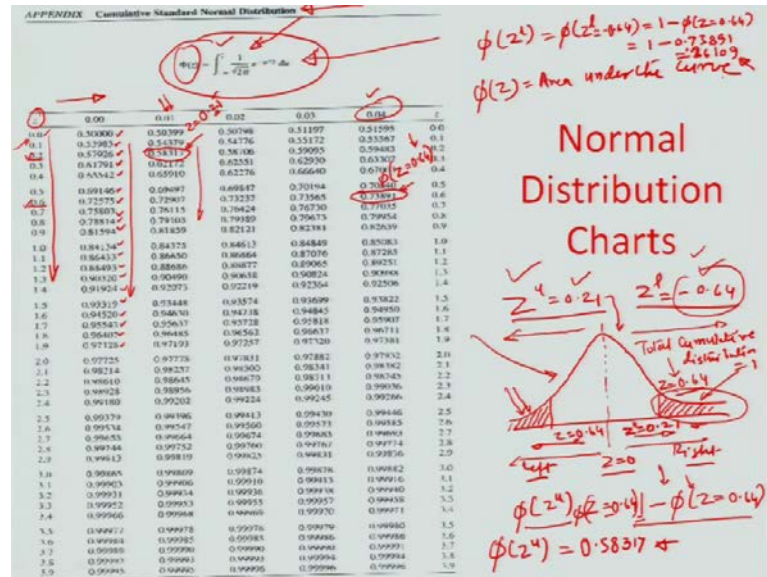
Meaning thereby, the area under the curve exactly starting from Z equal to 0 onwards towards the right is exactly similar to that towards the left that is why we call it as symmetric function. And therefore, you know just mentioning all the positive Z values would be sufficient, self sufficient to define the cumulative distribution function as represented here in this particular table was the negative values can be calculated identically.

So, one of the things which are important again is that  $\int Z$  is area under the curve. There are few important points, how this area is been mapped in this particular situation. Whenever we say a particular  $\int Z$  value, the way the numerical integral has been calculated ((Refer Time: 11:57)) represents the area up to that particular value. So, let us say there is some value here corresponding to Z equal to 0.21 as in the case of the upper tolerances limit that we have obtained is that Z variables in the last line.

So, the area under the curve or the five the cumulative distribution function of the Z u is represented as the area towards the left of this particular line here at Z equal to 0.21. So, the area of the area under the curve up two Z equal to 0.21 is the area that we are considering or calculating as the cumulative distribution the integral functions that you have seen in this particular domain here of Z u and which is subsequently calculated as this numerical values recorded in the particular table. So, obviously, now the question

arrives that how we can calculate the 0.64 minus value and how we can arrive at the cumulative distribution for Z lower equal to 0.64.

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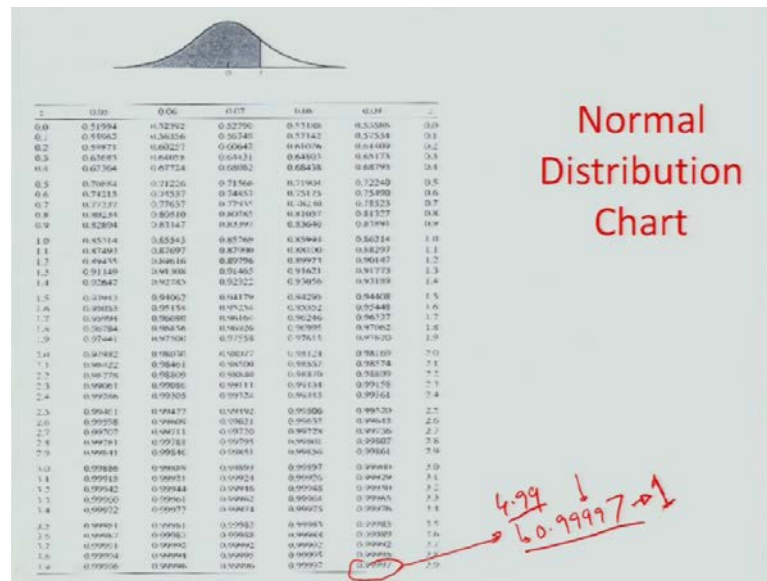


So, again if we use the same, similar logic you know for calculating the 5 for Z values somewhere here equal to 0.64 negative minus. So, the area under to the curve should be represented by the small area towards the left of the line corresponding to 0 equal to Z equal to minus 0.64. So; obviously, if the area as I suggested towards the right here is same as the idea towards the left because of the symmetricity of the function, so this area, which we have hatched here can also be recorded as the area towards the right of the line Z equal to positive 0.64.

So, they are exactly the same this area and this area right here there exactly the same, because of the symmetric nature of the problem or symmetric nature of the function. So, having said that then the question is, how do you define the Z equal to 0 point minus 0.64 we can say that this area right about here is 1 minus and the cumulative distribution function of Z corresponding to positive 0.64.

Because, we know that area under this complete curve whether its towards the left of the line Z equal to 0 or right of the line Z equal to 0 should be equal to 1. So, the total cumulative distribution function is unity.

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In fact, if you look at this particular table after like the value of Z equal to, let us say in the range of 4.99 beyond this particular range already the value achieved is 0.99997 you know. So, it is typically considered to be equal to 1 and beyond that almost the normal curve has achieved the full radius. So, we now, quite comfortably placed in terms of the area under the curve corresponding to the upper tolerances, where the upper tolerances variate and the lower tolerances variate.

Let us now go back in to our problem and try to provide a solution here. So, therefore, as I think we had already seen in details earlier this scrap coefficient in this particular case for j k value can be recorded as the cumulative distribution of the lower variate plus 1 minus the cumulative distribution function of the upper variate and from the normal tables as we had seen before corresponding to 0.21 if I try to see 0.2 and this is at the value corresponding to second place of decimal 0.01; that means, this value correspondence to Z value equal to 0.21 this is the first place of decimal and the columnar manner this is the second place of decimal on the row wise manner.

So, the value comes out to be equal to 0.58317 the other value corresponding to 0.64 this is a the first place of the decimal 0.6 and we read the fourth column corresponding to 0.04. So, the value here 0.73891 corresponds to the cumulative distribution function for the Z equals 0.64, 0.6 the first place of decimal 0.04 second place of decimal. So, that way it is you know the cumulative distribution value the value of the numerical integral 0.73891.



So, that is recorded for  $Z$  equal to the positive 0.64 and I think the issue here was to really determine the cumulative distribution function corresponding to  $Z$  equal to minus 0.64 and I had earlier illustrated in detailed manner, how this is recorded as one minus the cumulative distribution corresponding to  $Z$  equal to 0.64. So, I would say that let's write it here the lower variate, which is minus 0.64 corresponding to minus of 0.64, let me just write it in a proper manner.

So, that it is not confusing, can be 1 minus the cumulative distribution of  $Z$  equal to positive 0.64, 1 minus of 0.73891, which records as 0.26109. So, you have now two values 1 is the lower value corresponding to minus 0.64 and the scrap fraction here is defined as 0.26109 plus 1 minus of the upper value, which was again recorded here as 0.58317 from the normal tables.

So, this correspond to 0.26109 plus 1 minus this whole 0.58317 becomes 0.41683 and therefore, the total scrap fraction in this particular case comes out to be 0.67792. So, that is how you have derived you arrive that the scrap fraction for in this particular case; obviously, the technological coefficient for this scrap can be defined as the scrap fraction in the  $j$   $k$  case divided by 1 minus this scrap fraction in the  $j$   $k$  case it comes out to be 0.67792 by 1 minus 0.67792 and this is equal to 2.1048.

So, that is how the technological coefficient of the scrap may be defined determined you can accordingly map the technological coefficient of input the number of input units required the number of scrap units which are been sold. So, probably we are at the end of time today in this module, but I am going to actually discuss starting from here that we have defined already scrap quotient and the scrap fraction for the  $j$ th process for the  $k$ th alternative tolerances system, now having said that can we now derive everything in terms of cost equation. So, we will do that in the next modules.

Thank you.