

**Manufacturing Systems Technology**  
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**Module - 02**

**Lecture - 07**

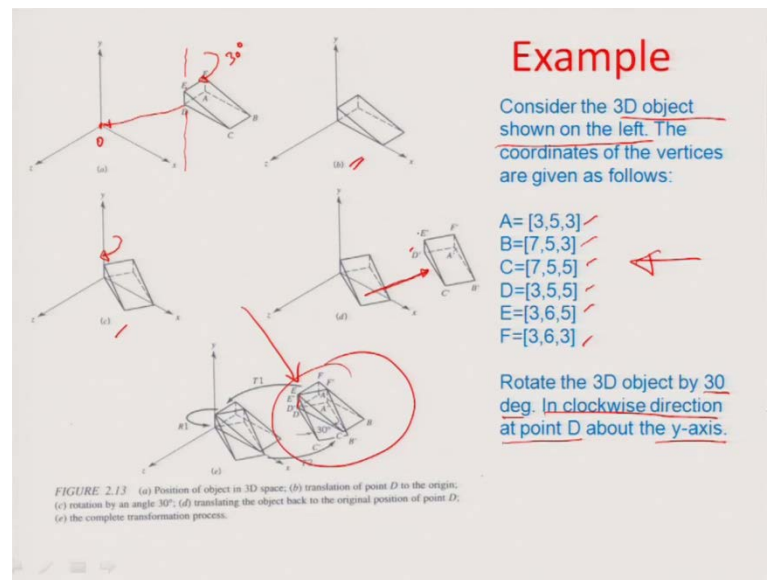
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**Review of previous module**

- Homogeneous transformation.
- Multiple transformation operations on a single object.
- Example problem.

Welcome to this module 7 on Manufacturing Systems Technology, a quick recap of the previous module. We were talking about homogenous transformation in case of computer aided designing and the need for the homogenous transformation, and then we also started exploring that if you have multiple operations done to a single domestic object in space. Then, how do you actually use homogenous transformation for that in a concatenation problem, where there are various matrices, multiplier matrices to the basic matrix which is basic coordinate would actually result in the modified coordinate in that particular case. So, we also were discussing an example problem where we will use this concept of concatenation and homogenized transformation to actually do a transfer of a three-dimensional object in the space.

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In context of that, we have mentioned that there is this block here which is abcdef which actually is at a certain distance from the origin which is at the center of this orthogonal coordinate system, and we wanted to rotate this block in the clockwise direction about the y axis along the line de, ok.

So, the axis actually is along the line de and the rotation was actually to be in the clockwise manner by an angle of about 30 degrees, and we would like to estimate the new coordinate locations which is actually shown right over here a dash, b dash, c dash, d dash, e dash and f dash after performing the 30 degree rotation. So, we have discussed earlier that we have to first translate this because it is not at the origin. So, you have to translate this block back to the origin first. So, d moves to the point o as you can see here in this translation b and then, followed by that you are basically doing a clockwise rotation of the block once it is at the origin about the point o, and then again taking this back in the module d all the way to the new current location where you can ensure that the d dash, e dash does not change with respect to d e. So, because that is the axis about which the rotation is taking place.

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**Solution**

Since we know about how to rotate at the origin the translation of the point of rotation 'd' in this case to origin is very important.

$D = [3, 5, 5]$  Distance matrix for changing the position of D as  $D = [-3 \ -5 \ -5]^T$

$V_{initial} = \begin{bmatrix} A & B & C & D & E & F \\ 3 & 7 & 7 & 3 & 3 & 3 \\ 5 & 5 & 5 & 5 & 6 & 6 \\ 3 & 3 & 5 & 5 & 5 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$   $T_1 = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$V_{final\ at\ origin} = [T_1][V_{initial}]$  D is origin

$= \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 7 & 7 & 3 & 3 & 3 \\ 5 & 5 & 5 & 5 & 6 & 6 \\ 3 & 3 & 5 & 5 & 5 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3-3=0 & & & & & \\ 5-5=0 & & & & & \\ 5-5=0 & & & & & \\ & & & & & 1 \end{bmatrix}$

So, how do we do this problem? So, since we know about how to rotate at the origin, the translation of the point of rotation which is d in this case to origin is very important and we know that the location coordinates of d has been given as 355 and subsequently, we can make the distance matrix for changing the position of d as minus 3 minus 5 minus 5 transpose. So, that is what the distance.

So, we have to really now take care of what is the order of this matrix, the d matrix which is somehow should incorporate minus 3 minus 5 and minus 5 in a manner, so that the whole coordinates of all the points a to e in the initial object should be mapped in a manner, so that d becomes 0 0 0. So, let us look at the basic matrix for housing all the coordinates which we are considering to be the initial vector over which the transformation has to happen, so that vector can be represented as an assembly of all the different points abcde and f which is there in the whole three-dimensional figure. If we look at their coordinates, the coordinates are all you know starting from point a, let us say which is 353. So, we write the matrix or we constitute the initial matrix in the way 353 and we add an extra dummy variable here for the homogenization set, so that we can actually create the addition as a multiplier of matrices and this figure has been detailed in the earlier modules then can be looked at 7531.

Similarly c can be looked at 7551 d is again 3551 and e subsequently is 3651 and f is 3631. So, this is how the initial matrix goes and we have to do this transformation in a

manner,so that this column right here which is the column d becomes 0 0 0 and 1. That is the sense that is the whole figure moves back to the origin. So, we create a translation matrix  $T_1$  here out of these in a manner,so that the multiplication of this with the basic initial matrix would result in a situation where d column becomes all zeroes. So, that  $T_1$  can be constituted as 1 0 0 minus 3 0 1 0 minus 5 0 0 1 in a manner, so that after moving back to the origin the  $v$  final at origin would be equal to  $T_1$  matrix multiplied by  $v$  initial matrix and that would definitely be in that. Performing this particular transformation would result in a situation where this d column right here goes to 0 0 0 1. Let us look at it.

So, if we just multiply the two matrices, just formulate it here 1 0 0 minus 3 0 1 0 minus 5 0 0 1 with the matrix which is  $v$  initial which is this 3 5 3 1 7 5 3 1 7 5 5 1 3 5 5 1 3 6 5 1 3 6 3 1 so on so forth. We will definitely get situation where if we just evaluate this particular column, what would be the state of this column? So, the first element of this would be recorded as 3 minus 3, the second element as 5 minus 5, the third element as again 5 minus 5 and the fourth element would be recorded as 1. So, definitely the whole row gets converted, the whole column gets converted into 0 0 0 which is what we need of moving back d to origin. So, this is definitely a valid transformation which has happened and that way you can actually look at the first multiplier matrix as  $T_1$ .

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**Solution**

Rotation about the y axis of  $30^\circ$  clockwise

$$R_1 = \begin{bmatrix} \cos(-30) & 0 & \sin(-30) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-30) & 0 & \cos(-30) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$v_{\text{final position of rotated object}} = [T_2][R_1][T_1][v_{\text{initial}}]$

In a similar manner we can perform another operation to the same because we have a rotation along the y axis of 30 degree clockwise. Obviously, all the clockwise entities in rotation are treated as negative angles. So, from earlier knowledge the rotation matrix can be used. Here  $r_1$  is given by  $\cos$  of minus 30  $\sin$  of minus 30  $0 \ 0 \ 0 \ 1$ . Similarly, it is  $\sin$  of minus 30  $\cos$  of minus 30 and  $0 \ 0 \ 0 \ 1$ . So, this is how the  $r_1$  matrix would look like and  $t_2$ .

Now, once you have performed the rotation, the question of translating back the rotated object to the initial position, so that the  $d$  dash matches with the  $d$  would have to be performed. So, the next matrix that you have to really do in order to retranslate the rotated object back would be  $1 \ 0 \ 0 \ 3$  in the positive distance matrix  $0 \ 0 \ 1 \ 5$  and  $0 \ 0 \ 0 \ 1$ . So, if you now compute the final position of rotated object, it will really be a concatenation between the different matrices that have been looked at here with respect to the  $v$  initial which you started with at the beginning of the process. So, I will just write down for the sake of remedy what is going to be the final matrix and how it is going to look like.

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**Solution**

$$V_{\text{final}} = \begin{bmatrix} 3 + 2\sin 30, 3 + 4\cos 30 + 2\sin 30, 3 + 4\cos 30, 3, 3(\cos 30 + 2\sin 30) \\ 5, 5, 5, 5, 5, 6, 4 \\ 5 - 2\cos 30, 5 - 2\cos 30 + 4\sin 30, 5 + 4\sin 30, 5, 5 - 2\cos 30 \\ 1, 1, 1, 1, 1, 1, 1 \end{bmatrix}$$

Simplifying further we have the new positions of the rotated 3-D figure

$$V_{\text{final}} = \begin{bmatrix} 4.00, 7.46, 6.46, 3.00, 2.00, 4.00 \\ 5.00, 5.00, 5.00, 5.00, 6.00, 6.00 \\ 3.27, 5.27, 7.00, 5.00, 5.00, 3.27 \\ 1.00, 1.00, 1.00, 1.00, 1.00, 1.00 \end{bmatrix}$$

So, the  $v$  final if you do all these different multiplications of the first translation, the first rotation and then the first translation matrix with respect to the  $v$  initial, the  $v$  final would work out to be something like  $3 + 2 \sin 30, 3 + 4 \cos 30 + 2 \sin 30, 3 + 4 \cos 30, 3 + 2 \sin 30$ . That is what the first element would look like. The

second element would look like  $5.556$  and  $6$ . The third element again would be  $5$  minus twice  $\cos 30.5$  minus twice  $\cos 30$  plus  $4 \sin$  of  $30.5$  plus  $4 \sin 30.555$  minus twice  $\cos 30$  so on so forth and the last element finally would look like  $1 \ 1 \ 1 \ 1$  and  $1$ , all ones.

So, the dummy element comes out of this old transformation. So, simplifying further we have the new positions of the rotated three-dimensional figure as  $v$  finally equals  $4.005.003.271.00$  and you have the other lot of coordinates emerging from the computations shown earlier which would give you the final locations of the rotated object points. The fourth point becomes  $3551$ . Now, this also gives us clarity of our transformation method because you can see that the point  $d$  is not really changing from the initial and the final. So, it still remains as the same coordinate  $3 \ 5 \ 5 \ 1$  and so is to with the next coordinate of  $e$  which again because it is the  $d$  axis along which the rotation is taking place the  $e$  dash and  $e$  and the  $d$  dash and  $d$ . They are all same in the figure and the final coordinate of  $f$  is  $463.27$ .

Also what is interesting here is that the dummy variable one comes out at the end of the matrix as this is the same as we assumed initially. So, these in fact the first three rows of this matrix represent the final transformed coordinates of the point, you know  $abcde$  as you know equivalent to rotation being performed at that particular place. So, it is a really very complex kind of backend computations that are needed for justifying the relocation of an object is really a complex set of concatenated matrices which would actually transform the initial coordinates into the final coordinates.

So, with this I would like to close this module and in the next module, we would like to look at some different aspects of CAD like curve fits etc, particularly when the topology to be mapped is very complex. What you do in those cases, we will have to evaluate them in a proper manner.

Thank you.