

Manufacturing System Technology - II
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Lecture - 16

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Alternate Formula for Mean

$$\begin{aligned}
 \sigma_x &= \text{Standard Deviation} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{(n-1)}} = \sqrt{\frac{\sum x_i^2 + n\bar{x}^2 - 2\bar{x}\sum x_i}{(n-1)}} \\
 &= \sqrt{\frac{\sum x_i^2}{(n-1)} + \frac{n\bar{x}^2}{(n-1)} - \frac{2\bar{x}\sum x_i}{(n-1)}} \\
 &= \sqrt{\frac{\sum x_i^2 + n\bar{x}^2 - 2\bar{x}\sum x_i}{(n-1)}} \\
 &= \sqrt{\frac{\sum x_i^2 + n\bar{x}^2 - 2\bar{x}\sum x_i}{(n-1)}} = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{(n-1)}} \quad \left(\frac{\sum x_i}{n}\right)^2 \\
 &= \sqrt{\frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{(n-1)}} \quad \text{This formula is superior to calculation} \\
 &= \sqrt{\frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{(n-1)}}
 \end{aligned}$$

Hello and welcome to this manufacturing system technology part 2 module 16. We were taking about the how to plot the x part the mean from a distribution, so let us and the standard deviation as x. So, lets us actually look added the s x or the standard deviation of a distribution can be represented as sigma x i minus x bar square divided by n minus 1 whole under the root and this further be represented as sigma x i square plus x bar square minus twice x i x bar divided by n minus 1. You can further simplified this by looking at this a sigma x i square by n minus 1 plus x bar square again sigma n minus 1, and the sigma as I had mentioned earlier is for samples from i 1 to n, so exactly in samples. So, that we have twice sigma x i x bar by n minus 1.

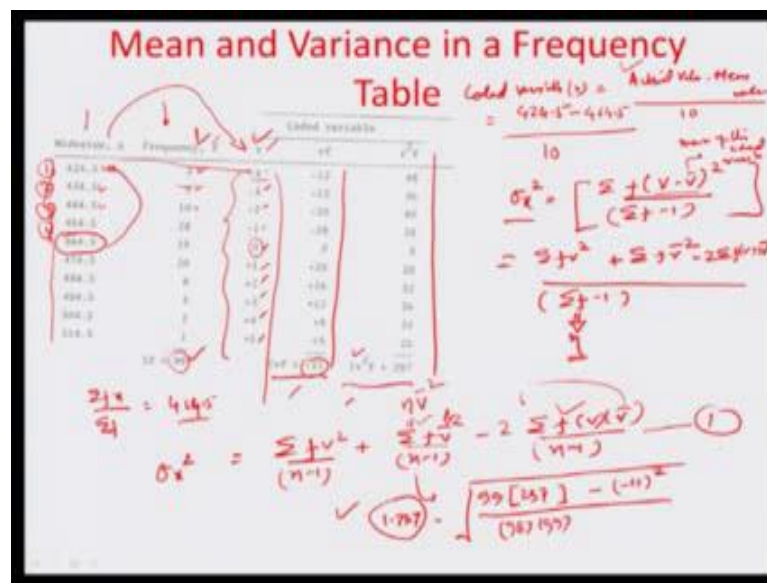
So, have in said that we can also try to further you know simplify it by a substituting this as n x square, because you are repeating the x bar square exactly and time here divided by n minus 1 minus twice of if we multiply and divide by n, we get this as x bar sigma a x i by n. And this can further be represented as root of sigma x i square by n minus 1 plus n x dash square by n minus 1 minus twice n n minus 1 and this is actually x bar square.

So, eventually we can represent this whole thing as sigma x i square by n minus 1 plus n x bar square, I am sorry it has to be minus, because you have a plus n and minus 2 n. So,

minus $n \times \bar{x}^2$ $n - 1$ and this can further be return down as $\sum x_i^2$ by n whole square. So, the final formulation can be a thing as $\sum x_i^2$ by $n - 1$ minus of n times of \bar{x}^2 whole square divided by square of $n - 1$, and this goes away and we have finally, a $\sum x_i^2$ minus $n \times \bar{x}^2$ whole square divided by $n - 1$, so a times of n .

So, this formula actually simplifies the calculation. Any kind of the frequency table it is important that you should be able to pull out a variable, which can be useful and which can be very easily a used for the calculation etcetera.

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So, let us say for example, we want to describe a frequency table as given here, you have several different mid values of the interval x , and there is a frequency also which is distributed or which is actually shown here. So, basically the way that the mid value x equal to 24 to 424.5 would be operating is for the 3 times in this actually comes from the histogram analysis, 434.5 is 4 times, 444.5 is 10 times so and so forth So, have in said that now playing around with these different values, you know they are fight I would say a combers ion and can we do something. So, that is simplified form of this values can emerge which can be the bases of calculating our distributional, and then later on whatever simplifier value has whatever formulation has been done to obtain that simplified value can be a used to reconvert this back into the original values. So, that main value etcetera and all standard deviation etcetera can be required.

So, here we want to coded this variable, and make it into a coded variable v in a manner.

So, that the coded variable v is basically the main value. So, in this particular case, let us say for the observation number 1 or serial number 1, the main value is 424.5. So, we are actually trying to see how much different it is from the mean and in this whole distribution it is so happen that if we go the $\sum f x$ by $\sum f$, you would get the main value to be around 464.5 in particular case which is actually this particular value here. So, we are trying to take of this value every time, so 424.5 minus 464.5 and then divide the whole thing by 10.

So, this is the coded variable, coded variable basically means you know the v variable means basically the actual value minus the mean value divided by the factor of 10. So, now you have converted all these different values 1, 2, 3, 4 into their respected coded variable as you can see v_1, v_2, v_3, v_4 , so on so far. So, t as you can see the coded variable is much simpler, and it is very easy to calculate on the basis of the coded variable. So, you can use of frequency in the coded variable and do all the calculation as per the coded variable, and then whatever transformation you had used for obtaining these variable actual variable minus mean value by 10, we just that should be reward that back, so that you can get the original value that is the whole idea.

So, here the standard deviation for example, can be represented as a $\sum f v^2 - v^2$ being the mean of the coded variable of the coded variable square divided by $\sum f - 1$. And if I just word to simplified this value equal to resulting $\sum f v^2$ plus $\sum f v^2$ square minus twice $\sum f v$ times of v^2 divided by $\sum f - 1$. And in fact if I further simplified this would resulting a value of $\sum x^2$ equals to, let us just put the value of $\sum f$ here as the total number of observation or the total number of the a sub roof size this is n , let us say there are n observation as equal to 99 as you can see in this particular case. So, you can represent this by $\sum f v^2$ by $n - 1$ plus $\sum f v^2$ square by again $n - 1$ minus twice $\sum f$ times v times of v^2 divided by $n - 1$.

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$$\begin{aligned} \sigma_x^2 &= \frac{\sum f v^2}{(n-1)} + \frac{n v^2}{(n-1)} - \frac{2 \sqrt{v} \sum f v}{(n-1)} \\ &= \frac{\sum f v^2}{(n-1)} + \frac{n v^2}{(n-1)} - \frac{2 \sqrt{v} \sum f v}{(n-1)} \\ &= \frac{\sum f v^2 + n v^2}{(n-1)} - \frac{2 n v}{(n-1)} \\ &= \frac{\sum f v^2}{(n-1)} - \frac{n v^2}{(n-1)} = \boxed{\frac{\sum f v^2}{(n-1)} - \frac{n (\sum f v)^2}{n^2 (n-1)}} \\ \sigma_x^2 &= \frac{n \sum f v^2 - (\sum f v)^2}{n(n-1)} = \sqrt{\frac{n \sum f v^2 - (\sum f v)^2}{n(n-1)}} \end{aligned}$$

And further I would just like to illustrate here that the sigma x again can be more simplified as... Now, I just convert whatever we have in this expression write about here into a more a simpler form for as to be able to calculated through this table or this variable approach. And let us see what are the basic minimum values, which are needed in terms of the frequency v, a frequency f and the coded variable v in order to able to calculate the mean as standard deviation of the distribution. So, I just write this down as sigma f v square by n minus 1 plus, and this becomes equal to as you know a this is sigma f v dash square, we are v dash a basically is a the mean value of the coded variable which is unique value. So, I can just simply write that you merely repeating this n times right.

So, sigma f v dash square is basically meaning to say, you know v dash square times of ninety nine which is the sigma f value v that being uniquely common in particular distribution that we are considered. So, this can be represented as n v dash square by n minus 1 minus of twice v dash times of a you can write the value of a sigma f v here by pulling out v dash outside the summation. So, we are left with if we just a allow this to come out we are having sigma twice v dash times of sigma f v by n minus 1 and infact what I am going to do here it is sort of multiply, and divide the value by n where n can be represented as sigma f. So, that I have a final expression sigma f v square by n minus 1 plus n v dash square by n minus 1 minus twice v dash times of n times of sigma f v by sigma f which is actually equal to the variable v dash as you may recall earlier. So, this is actually the mean value of the coded variable distribution. So, this can be replaced by v

dash and this you divide by $n - 1$.

So, then you have again a $\sum f v^2$ by $n - 1$ plus $n \bar{v}^2$ by $n - 1$ minus twice and you have n times of \bar{v}^2 divided by $n - 1$, and you can further represent this by you know you can actually assemble them together as $\sum f v^2$ minus of $n \bar{v}^2$ by $n - 1$ or other words $\sum f v^2$ or $f v^2$ square $n - 1$ minus of n times of this \bar{v}^2 value which is actually $\sum f v^2$ by square of n times of $n - 1$ this goes away in a similar manner and this the square of standard deviation σ_x^2 , I am sorry. So, σ_x^2 then becomes equals to simply speaking the you know lots of simplified this, more you can have $n \sum f v^2$ minus $\sum f v$ whole square divided by n times $n - 1$ or in other words the σ_x can simply be the under root of this particular term and $\sum f v^2$ minus $\sum f v$ whole square divided by n times of $n - 1$.

So, all you need to do is to sort of now calculate on the coded variable what is the $\sum f v^2$, and you also have to calculate what is the $\sum f v$ whole square. And that can be very easily calculated by constructing to simpler columns, and 1 column just multiplying frequency with respect to the coded variable in other square of the frequency with respective the coded variable, remember these coded variable are very easily handle computationally and you can easily you know in expensively calculate and this is probably only showing few variable, but in actual cases there may be thousands and thousands expect variables. So, you now have the capability to quickly calculate the standard deviations σ_x from distributional impact standard deviation, and simply we calculated by using formula obtained in this the last step here which is $n \sum f v^2$ minus $\sum f v$ whole square divided by $n - 1$ n times of $n - 1$.

So, you can simply substitute these different values 99 being n times of 297 minus of the minus 11 $\sum f v^2$ divided by n times of $n - 1$ whole on to the root. So, this comes out to be equal to... So, the standard deviation of the distribution actually comes out to be 1.737. So, have in said that you know in very simplify manner by using less computation you are able to calculate what is standard deviation of the this particular distribution, you know mean for calculating that mean of the distribution just need to reward back the transformation that had been use for doing the coded variable and with the standard deviation will be remain same.

So, that is the promise that is used all these circumstances. So, having said that I think we have a very good idea now of how to start floating the external charts, and close probably module in the interest of time, but in the next module will actually start floating the charts and learning the more about the charts a particularly when we are talking about quantitative description of parameter or quality parameter like X spar or R chats. So, till then and till than good bye.

Thank you.