

**Manufacturing System Technology - II**  
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**Lecture – 03**

(Refer Slide Time: 00:20)



**Designing quality into products and processes**

- Product design is the prime activity in the process of realizing a product. Therefore it has the greatest impact on the product quality. Loss of quality occurs when there is a deviation of functional characteristics of the products from the target values.
- Taguchi has proposed a philosophy and methodology for designing quality into products and processes. He postulates that the process of designing a product or a process should be viewed as three phases.
- System Design, Parameter Design and Tolerance Design

Hello and welcome to this Manufacturing Systems Technology Part two – module 3. We were talking about how to introduced quality at the product design and the process design stages itself. And as we know that you know so far what we have discussed is that the product design is kind of prime activity in the process of realizing a product and therefore, it has a great impact on product quality in fact it has greatest impact you know the design is such that it is a quite robust then it can be able to allow a very less non conformance at the process and the product level. So, the loss of quality really occurs when there is a deviation of the functional characteristics of the product from the target values. And the design issues about choosing these functional characteristics how much allowance, a tolerance can be given in the functional characteristics, given the aspiration or need of the customer and this has to be than translated in terms of the design, the design of the particular product.

So, you know the philosophy of product design kind of changed by Taguchi, who relished who proposed a philosophy and methodology for designing quality into products and processes. So, then you Genichi Taguchi was actually a tail engineer by a training,

but he work for the you know electrical corporation ECL in Japan and you know does tried to he was a you to actually converted statistician and he tried to develop some methodologies for doing quality for analysis etcetera. Later on his method is very popular in the western and you know in fact, he could give he has in adviser role for several company for long time including in Bell abs, Boeing, Zerax and so on and so forth. So, Taguchi basically was the first proponent of that can design be a made in a qualitative manner or you can trust design product design made into qualitative manner. So, he postulates that the process of designing product or a process should be viewed as three phases one is the design at the system level, then the design of the parameter design level a or design of the parameters level and then design finally, year tolerance to the parameters that has to be. So, he basically organize the thought process of how do incorporated to the design change in a particular design.

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And there when he means you know when he talks about system design it is the process of applying scientific knowledge to produce a basic functional prototype design, now in this phase new concepts ideas and methods are synthesized to provide new or improved products of customer that means, that the basic design concept is established during this phase including the selection of parts, materials and subassemblies and so on and so forth. For example, let say the following question related to an automotive assembly can be addressed very well at system of design level for example, one can say that should the internal combustion engine be made out of the aluminum or cast iron should the breaks of having this additional safety of anti lock breaking kind of system you know, so this

kind of basic questions related to the basic materials to be used, or the sub assemblies used in the design has to be addressed at this system design level. And the relationship between the inputs and the outputs are established this way, and also the functions of the parts and subsystems are determined during the phase once the system design level questions are addressed in the quality you know a planning.

So, the next stage; obviously, is the parameters design. So obviously, as the name suggests the level for product process design parameters are set to make the system performance less sensitive to the causes of variation thus minimizing quality loss. So, choose the parameters the basic parameters for example, in the case of engines a BHP or engine rpm can be a very good parameter. So choose the parameters way so that there are minimalistic variations in the parameter and the capacity of the particular engine is always kept on to deliver of parametric a property or a may be a performance of parametric should always be kept on the over side just by a little bit so that always there is a compliance to the performance metrics. There should not be a situation when the metrics at the border line and the product is not able to achieve the metrics in most of the cases. So, plan the performance metrics in a manner, so that within the capacity or range of the product the metrics lies somewhere.

And in the parameters design wide tolerances on noise factors are assumed to allow manufacturing cost as it is costly to control noise factors. So obviously, the parameters design stage should account for these factors where there is no control on the responsible factor contributing to the non conformance and these are also called so called the noise factor. And you have to design in the process in the way, so that these noise factors are addressed and very tight control is kept and that particular level. And obviously, if you one hand allow low manufacturing cost, it means that you do not have much control on the noise factor at that the particular levels. So on one hand you have to address that issue and other hand you have to address the control of the noise factor so that there is performance compliance every time that issue there is a short of a trade of you know at the parameters design stage itself which as to be obtain by a quality engineer at that stage to necessitate that a certain standard of quality met by the particular product. So, during the parameter design phase, the quality is improved without controlling or removing the causes of the variation. And during you know so then you can do design of experiments simulation and optimization, and you know various other techniques at this particular parameters fixing stage so that you have the right choice of the parameters in the

particular design.

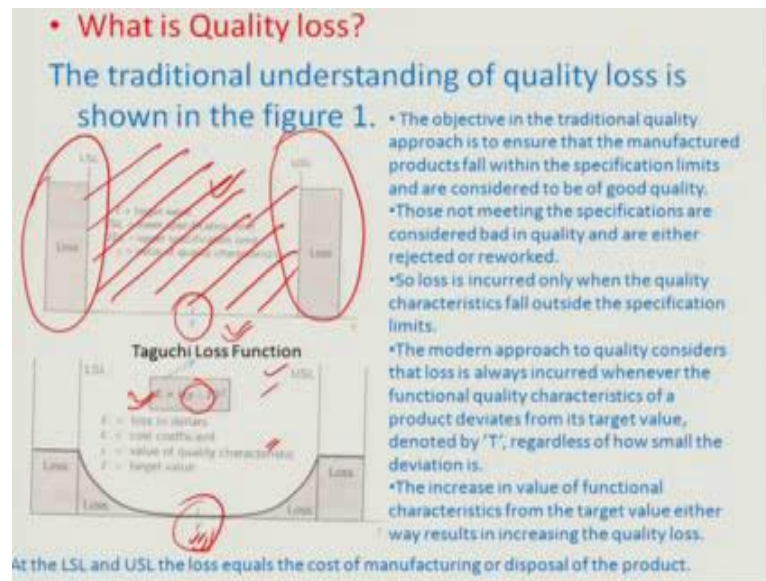
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- **Tolerance Design:**  
The tolerance design phase usually follows the parameter design phase. Quality improvement is achieved by tight tolerances around the chosen target values of the control factors so as to reduce performance variations. However, with quality improvement- that is, reduction in quality loss- there may be an increase in manufacturing cost.

Then we talk about tolerance design, which is one step ahead of the parameters design and it usually follows the parameters design phase. So, obviously, if you introduce tight tolerances regime, there would be an overall quality improvement because in the process selection will also have to be done in a manner so that tight tolerances of the particular specification has to ((Refer Time: 06:56)) the process level. So, you have to obviously, choose the tolerances around the target values very close to the target values, so that there is marginal difference in the output from the parameter which has been design in the earlier stage which can be treated as the allowable for the particular quality system. And; however, with the quality improvement reduction in quality loss which is come in to this quality improvement, there may be an increasing manufacturing cost because of the additional imposition on the process to maintained tight standard of the process. So, tolerance design is basically followed at the level three of the system design, the parameters design and then finally, tolerances design stage of a quality design process.

(Refer Slide Time: 07:47)



So, having said that let us now mathematically try to find out what Taguchi really means by the quality loss. So, if you look at two concepts whichever rather traditional and then after the introduction of the Taguchi concept of design, you can find out that additional system quality monitoring, you had something called a lower specification limit in upper specification limit and the target value would be somewhere between the LSL and USL. And any product which was line in this particular range between the two tolerances limit would be accepted as the product. For the first time, Taguchi proposed that you know this is something like waiting up till non-conformances happened and taken a corrective action. So, this is something like a sort of a breakdown maintenance kind of a situation where there is equipment which has broken down already and then you have to figure out what is the cause you know. So, you cannot wait up till the process goes out of hand or out of the USL or LSL specification, so therefore, any deviation from the target has to be somehow met to be some kind of indicator which in this particular case is known as quality loss.

And this indicator would be so stiff that there is any variation from the target, you should be recording as the square of the variation the loss factor which comes into picture, so that at a very early stage of the process going out of control, you should be able to before the process goes out of the control, you should be able report that yes there is a possibility for the process go out of control by looking at the loss figures, looking at the variations from the target and that is how the whole philosophy is incorporated into the particular matrix or particular quality frame work. So, here for example, if I was talking

about a shaft diameter having a value of one plus minus 0.03 meaning there by the design aspect says that the shaft can go between 0.997 to 1.03. The question is why to allow the process to continuously produce something at 1.01 level or even at 0.998 level which is still within the upper and the lower specification limit, but it should definitely contribute to an indicator which merely a virtual indicator of a loss where this loss then can be philosophically managed with some counter measure, some decision making on the part of administration, so that is comes back to zero now, so that is what Taguchi loss function talks about.

So, obviously, T is the target, y is the value of the quality characteristics which their y minus T is the really the deviation from the target you know basically mapping the losses of the square of deviation from the target. And loss typically mentioned in currency units, so that every design specification noncompliance now mapped as a loss indicating what did they way from and how much away from it is going. Therefore, if can set up way separate tolerances range you know for process control from the design specification range that has already has been available you should not been allow the process to go out to the process tolerances which much, much within the design specification means. So, therefore, in almost hundred percent of the situation, there would be complete conformance to the particular attribute parameters design attribute with tolerances attribute which has been proposed that the design stage, so that is how you mentioned the quality loss.

(Refer Slide Time: 11:21)

### Average Quality Loss

- There is always a variation in the quality characteristic due to noise factors from unit to unit from time to time during the usage of the product.
- If  $y_i$  ( $i = 1, 2, 3, \dots, n$ ) is the  $i^{\text{th}}$  representative measurement of quality characteristics, then the average quality loss can be computed as follows:

$$\begin{aligned} \text{Average quality loss} &= \frac{1}{n} [L(y_1) + L(y_2) + L(y_3) + \dots + L(y_n)] \\ &= \frac{k}{n} [(y_1 - T)^2 + (y_2 - T)^2 + (y_3 - T)^2 + \dots + (y_n - T)^2] \\ &= \frac{k}{n} [y_1^2 + y_2^2 + y_3^2 + \dots + y_n^2 - 2T(y_1 + y_2 + y_3 + \dots + y_n) + nT^2] \end{aligned}$$

(10) (5)

So obviously, do some statistical analysis here to see what this loss can be really



computed as or how to control this loss and then tried to some to realistic examples where we will use this loss factor to identify things related to the process. So obviously, lets a there are there is a process where there is  $y_i$  represents the  $i$ th measurement of the quality characteristics. So, average quality loss can be computed as obviously, this summation of all the different losses divided by  $n$  number of loss lets a there are  $n$  such representative measurement of the quality characteristics  $y$  which has been obtained. So, the overall average loss would be the loss because of  $y_1$  away from the target loss because of  $y_2$  away from the target  $y_3$  away from the target and so on and so forth,  $y_n$  characteristics away from target and this can be plotted as  $k$  by  $n$  times of  $y_1$  minus  $T$  square plus  $y_2$  minus  $T$  square plus  $y_3$  minus  $T$  square plus and so on and so forth up to  $y_n$  minus  $T$  square.

And we can actually in very convenient manner represent this because you know we can probably think of doing some manipulation here. So, let say if we gather together values  $y_1$  square plus  $y_2$  square plus  $y_3$  square plus and so on and so forth up to  $y_n$  square. And then we have minus of  $n$   $T$  square there exactly I am sorry plus of  $n$   $T$  square values which are emanated here and then you have minus of twice  $T$  times of  $y_1$  plus  $y_2$  plus  $y_3$  plus and so on up to  $y_n$ . And so in away what do you are trying to calculate here is related to you know can just manipulated this whole equation that has been formulated and to have realistic in values in terms of the signification of parameters for any such statistical you know distribution like  $\mu$  and the sigma the mean in the standard deviation.

(Refer Slide Time: 13:52)

**Average Quality Loss**

$$\begin{aligned}
 AQL &= K \left[ \frac{y_1^2 + y_2^2 + \dots + y_n^2}{n} + T^2 - 2T \frac{(y_1 + y_2 + \dots + y_n)}{n} \right] \\
 AQL &= K \left[ \frac{y_1^2 + y_2^2 + \dots + y_n^2 - 2Ty_1 - 2Ty_2 - \dots - 2Ty_n}{n} + T^2 - 2T \frac{(y_1 + y_2 + \dots + y_n)}{n} \right] \\
 &= K \left[ \frac{(y_1 - T)^2 + (y_2 - T)^2 + \dots + (y_n - T)^2}{n} + T^2 - 2T \frac{(y_1 + y_2 + \dots + y_n)}{n} \right] \\
 &= K \left[ \frac{(y_1 - T)^2 + (y_2 - T)^2 + \dots + (y_n - T)^2}{n} + T^2 - 2TK \right]
 \end{aligned}$$

So let see how we are now sort of adding, so we supposing the take the n value in the side we have the first term represented as  $y_1^2$  do  $y_n^2$  by n plus n t square by n meaning by t square minus of you have twice of T divided by  $y_1 + y_2 + \dots$  and so on and so forth up to  $y_n$  divided by n. And as we can probably see from here, you have possibility of introducing the mean characteristics quality characteristics  $\mu$  which is actually  $y_1 + y_2 + \dots + y_n$  divided by n in this particular way. So, what I would rather preferred do here is sort of you know calculate by adding and subtracting the values minus  $\mu^2$  and plus twice  $\mu$  times of  $y_1 + y_2 + y_3 + \dots + y_n$  divided by n here, so that I can actually write this down is  $y_1 - \mu^2 + y_2 - \mu^2 + \dots + y_n - \mu^2$  divided by n plus square of T minus twice T  $y_1 + y_2 + y_3 + \dots + y_n$  divided by n, so that is how I can write the value of the average quality loss equal has been computing form before.

So, how this system comes in a simply that if you open this as square bracket you will be left with  $y_1^2 + y_2^2 + y_3^2 + \dots + y_n^2$  minus of sorry plus of  $\mu^2$  because there are n such  $\mu^2$  together and divided by n it  $\mu^2$  and minus of twice  $\mu$  times of  $y_1 + y_2 + y_3 + \dots + y_n$  divided by n. So, I am essentially doing is subtracting and adding the same terms here to this, so that effectively this all can result in only this term  $y_1^2 + y_2^2 + \dots + y_n^2$  by n. So, having said that I can write this down as in full expression  $y_1 - \mu^2 + y_2 - \mu^2 + \dots + y_n - \mu^2$  divided by n minus of  $\mu^2$  plus twice  $\mu$  times of  $y_1 + y_2 + y_3 + \dots + y_n$  divided by n, obviously, there is  $T^2$  plus twice T times of this  $y_1 + y_2 + \dots + y_n$  divided by n.

So, I am left with finally, a situation where this actually  $\mu$  right and so is this and I would be enable with k divided by k times of  $y_1 - \mu^2 + y_2 - \mu^2 + \dots + y_n - \mu^2$  divided by n minus of  $\mu^2$  plus twice  $\mu$  square makes it plus  $\mu^2$  plus  $T^2$  minus twice T  $\mu$ . So, I can say that this is equal to k times of  $y_1 - \mu^2 + \dots + y_n - \mu^2$  plus I can write this as  $\mu - T$  whole square, and this is divided by n. So, that is how solve I can write the average quality loss as this sort of you know pull out all the values together, so that I can write the AQL in a more appropriate manner.



(Refer Slide Time: 18:27)

The image shows a handwritten derivation of the Average Quality Loss (AQL) formula. At the top, the title "Average Quality Loss" is written in red. Below it, the variance formula is shown:  $\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \mu)^2$ . A box on the left contains the sum  $\sum_{i=1}^n (y_i - \mu)^2$ . An arrow points from this box to the variance formula. Another arrow points from the variance formula to a bracketed term  $\left\{ \frac{(n-1)}{n} \sigma^2 \right\}$ . A final arrow points from this term to a box containing the AQL formula:  $AQL = \left[ \frac{(n-1)}{n} \sigma^2 + (\mu - T)^2 \right]$ . Below the AQL formula, the text "Average Quality Loss" is written.

I can say that you know if I look at this particular value here, this is nothing but  $y_i$  minus  $\mu$  square  $\sigma^2$  again probably  $\sigma^2$  varies between  $i$  equal to 1 to  $n$  divided by  $n$ . So, I can make this look like  $\sigma^2$ , so if I suppose you know we already know that  $\sigma^2$  here or may be just the square of the standard deviation is represented statistically. So, this you know  $\sigma^2$  can be represented  $\frac{1}{n-1} \sum_{i=1}^n (y_i - \mu)^2$  varying between 1 to  $n$   $y_i$  minus  $\mu$  square. So, I can write down this whole term as  $\frac{n-1}{n} \sigma^2$ , so that is how I can write this whole term. So obviously, then the AQL becomes equal to  $\frac{n-1}{n} \sigma^2 + (\mu - T)^2$ , so that is how you can record your average quality loss. Now the question is whether you know really this is the average and you know there are some quality parameters which are desirable there are some quality parameters which are undesirable.

For example, let us say if you are talking about vibration and noise in an equipment that is something which is a sort of undesirable parameter, ((Refer Time: 20:10)) you know parameters where you are talking about let say the dimensions of a shaft. So, when we are talking about quantitative parameters like the dimension of a shaft, and how much away it is from the target, the average quality loss probably makes sense. But when we are talking about undesirable characteristics like say friction and wear, where the minimum possible better it is in that kind of the circumstance, it becomes a little difficult to understand this as the deviation from the target. So, will close this lecture here in the interest of time, but in the next lecture, we will probably try and see, if we can do

something with this AQL, so that it change contextually to what is the kind of the parameter that we are discussing.

Thank you.