Basics of Noise and Its Measurements
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Lecture - 11
General Solution for 1-D Wave Equation

Welcome again to this course on Basics of Noise and its Measurements. I am Nachiketa Tiwari; I am the instructor for this course. This is 8 week course and currently, we are in the second week of it. We are covering during this week the wave equation and in the last lecture, what I have discussed was the derivation of the wave equation from three fundamental equations - the momentum equation, the continuity equation and the equation which dictates the process which a gas undergoes that is the gas law.

So, what we are going to do today is actually develop a general solution for the wave equation, and then used that solution to make certain interpretations, and explain the physical meaning of this constant term which emerged in our wave equation, which is C. We had noticed in the last lecture, that the value of c, this physical constant, which was defined as a square root of t times gamma divide by rho naught. The value of this particular physical constant was fairly close to the measured value of speed of sound as it propagates in air at standard room standard pressure and temperature conditions.

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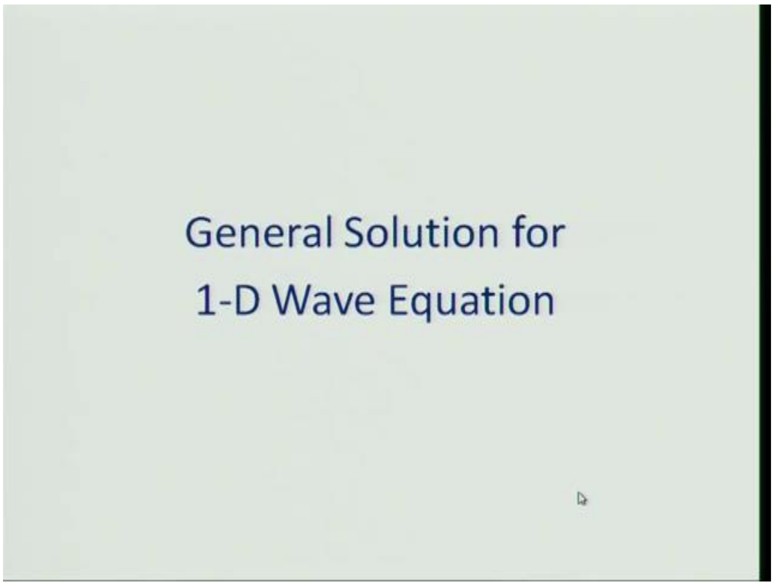

So, again once again, what we are going to cover today is the general solution for 1-D wave equation and then try to understand the meaning of this particular general equation.

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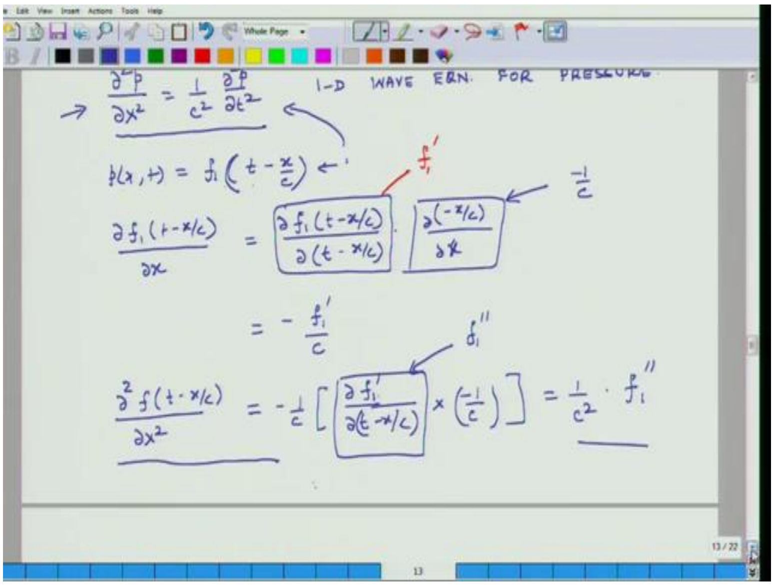

So, I will start by writing the wave equation once again for pressure so that is my 1-D wave equation for pressure. Now, when you look at this equation, you observe that it is a partial difference equation, and it has partial second order partial derivatives for pressure both in time and also in space that is x, so that is the first feature which you observe. And then the second feature, which you observe is that if you take the ratio of second order derivatives with respect to x and with respect to time, what essentially you get is a constant; and in this case constant is 1 over c square. So, it is the constant and the value of the c square is gamma times pressure divided by density. So with this understanding, you try to figure out what could be a possible solution which is as general as possible.

And you do some thinking and you realize that maybe if I have a function p of x t and if it is such that it is function - some function f 1, which depends on time and position in such a way that it does not independently depend on time alone and it does not independently depend on x alone, but rather it depends on a combination of time and position in such a way. Then may be this solution is going to work for this differential equation; and the way to figure that out is that you plug the equation or this relationship this function back in to the pressure wave equation and see whether it satisfies so that is what we are going to do.

So, before we do that we have to compute the derivatives of f with respect to x and t, second order derivatives so with respect to - second order partial derivatives with respect to x and t. So let us do that. So we will first compute the first order derivatives with respect to x is nothing but first order derivative with respect to t minus x over c times right, excuse me. So, what I do is I label this thing as f prime, and then this thing is minus 1 over c. So essentially, I get this minus f 1 prime over c, so that is my first order derivatives of f 1 partial derivatives with respect to x. Similarly, now I can compute its second order derivatives, so it is minus 1 over c with respect, so I partially differentiate f 1 prime with respect to t minus x over c multiplied by the derivative of t minus x over c with respect to x, so I get another 1 over c. So, this is nothing but 1 over c square times f 1 prime prime, because I am calling this as f 1 prime prime. So, this is my left hand side of this particular equation.

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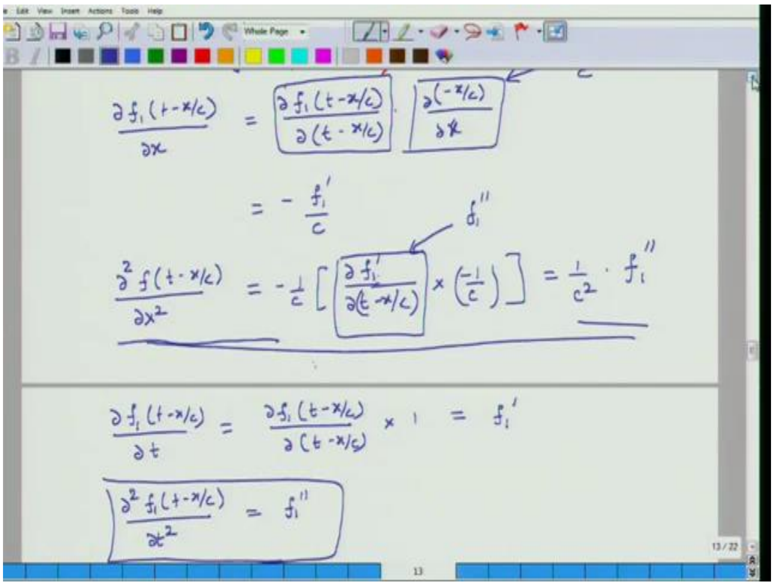

Now, let us calculate the right hand side of the equation. So partial of f 1 with respect to t is nothing but partial of f 1 with respect to t minus x over c and multiplied by partial of t minus x over c with respect to t, so that is 1. So this is nothing but f 1 prime and similarly, the second order derivative in time is del t square is nothing but f 1 prime prime. So, if I plug this relation, and I also plug this relation back into my original wave equation…

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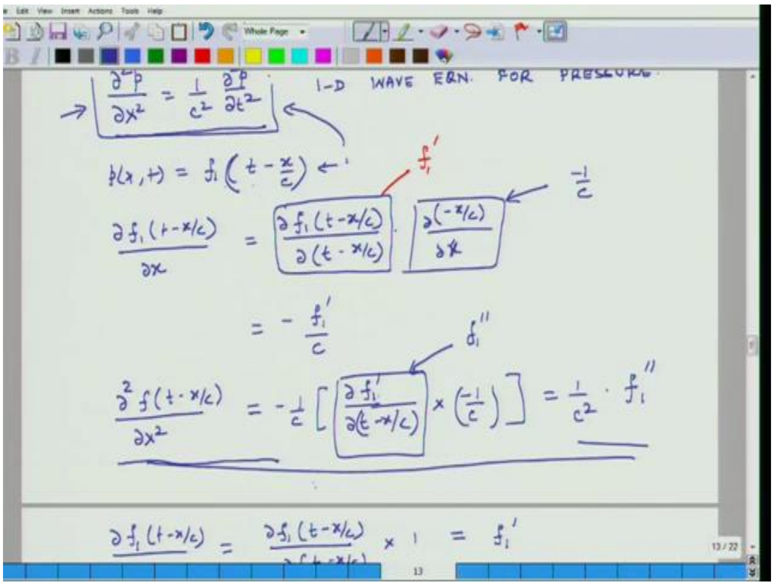

I will see that this equation is identically satisfied, it is identically satisfied.

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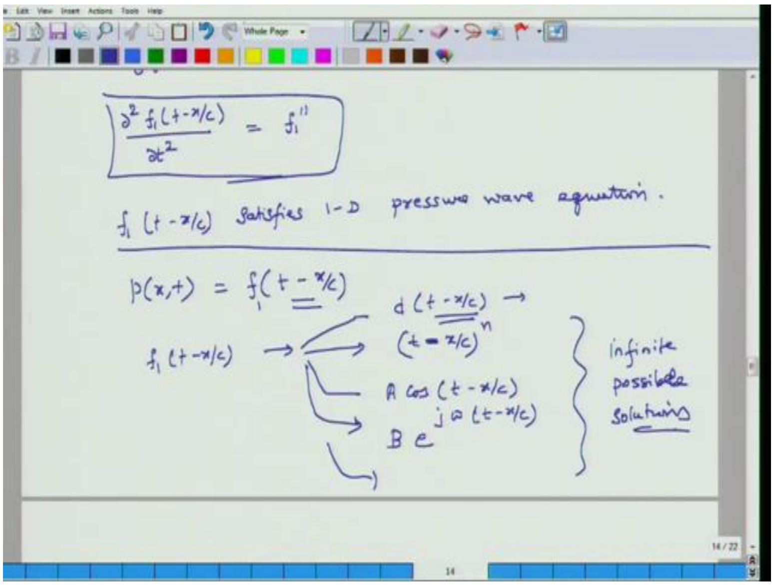

So what that tells me is f 1 of t minus x over c satisfies 1-D pressure wave equation; so this is one thing that we assumed. So we assumed that pressure is a function of t and x such that it is dependent not independently on t or neither independently x, but a combination of t and x in a certain way, and this combination has to be of this way; if it was follow this way, then it will satisfy it.

Now, let us look at some actual solutions. So one actual solution could be, so this is f 1, so one actual solution could be, so we will just write down some possible solutions; one solution could be some constant, I will called it d times t minus x over c, this is one possible value of f 1. There could be an infinite possible solutions, another possible value could be t minus x over c to the power of n, another possible value. As long as we have in parenthesis t minus x over c, any of these solutions is a possible solution. Now whether it is matches reality or not that is a different question, but all these things or all these function are possible solutions of the pressure wave equation. And there solution could be cosine of t minus x over c some amplitude. Another possible solution could be another amplitude e to the power of j omega t minus x over c; this is another possible solution. So, we have infinite possible solutions. Now, the only constant is the t and x has to come in this form t minus x over c.

Now, we will look at the physical significance of this particular class of solution, because it is not just one solution, but it is a class of solution. And in this class, the t minus x over c comes as a good and goods package you cannot separate them out; it comes as one block. So, this is a class of solutions.

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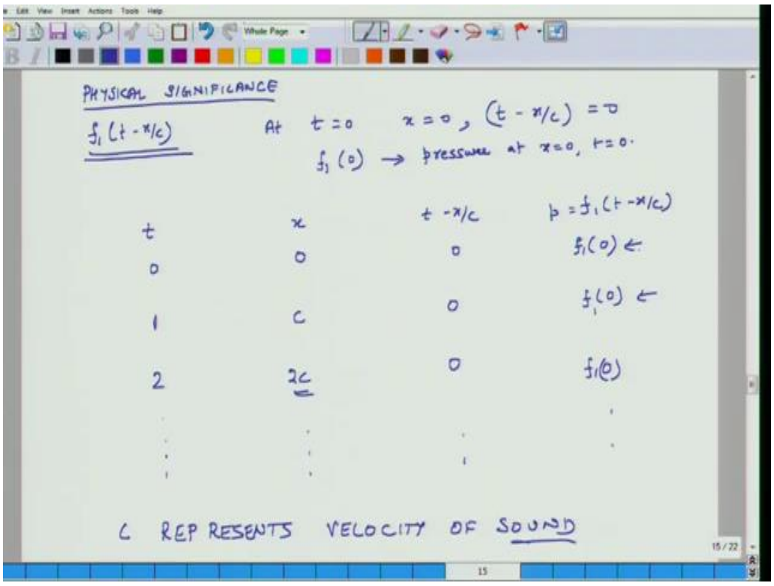

So, we will look at the physical significance. So, this is one class of solutions which satisfies the pressure wave equation. Now t can have any value, and let us say that I start counting t from time t is equal to 0. So let us say, I am counting at t is equal to 0, and let us say at t is equal to 0, x is also 0. So if I am having a reference frame I am looking at something at x is equal to 0 at the beginning of time, then in and that at time t is equal to 0, x is equal to 0; t minus x over c will be 0. So then the value of function will be f 1 of 0. So, this will be the valve of function and this will be the pressure at x is equal to 0 and t is equal to 0. So the value of pressure at origin at the beginning of time will be f 1 of 0.

Now, we will do a matrix t, x, t minus x over c, f 1 t minus x over c. So at the beginning of time and also at the origin; at the beginning of time t 0, and if I am also at that point of time at the origin, then x is 0 then t minus x over c is 0, and the value of pressure which I will record because this is pressure, so this will be f 1 of 0. Now, let us look at so this what is there then I am looking at my watch and then I see that 1 second has elapsed, so then t becomes 1 second. And then I move a distance c away from the origin, if I move away the distance c away from the origin, after 1 second I move a distance c away from the origin, so in that case t minus x over c is 0, because x is equal to c, and then the value of pressure will be f 1 0. So here this is the value of pressure at origin at beginning of time this is the value of pressure at x is equal to c after 1 second.

Then after 2 seconds, let us say I move by distance 2 c; then t minus x over c is still 0; and the value of pressure after 2 seconds, at position 2 c away from the origin is still 0. So I can keep on doing this, but what this chart tells me is that whatever pressure is present at the origin at the beginning of time that pressure moved after 1 second by a distance of c, and the same pressure moved by a distance of 2 c after 2 seconds, which means that the pressure - the value of pressure is moving with a speed of c meters per second. It is moving with the speed of c meters per second, and it is moving in the positive direction, because the value of c is not a negative c, but it is positive c.

So, what this tells me that if the pressure is such that it can be express as a function f 1, and f 1 depends t minus x over c then essentially it means that there is a pressure function which is moving at a velocity of c in the positive x-direction. So that is what so from that we say because of this understanding, we say that c represents velocity or speed of sound; it represents speed of sound.

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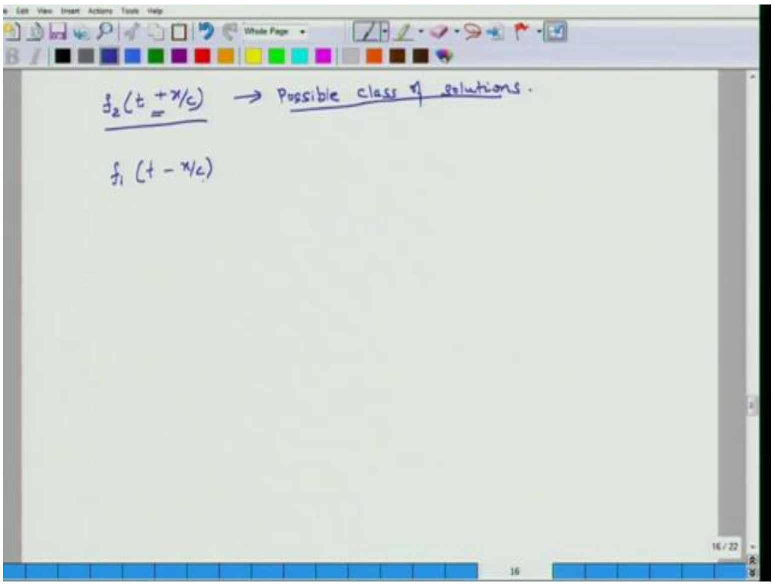

Similarly, we can also have a solution f 2. Here, instead of t minus x over c, we can have a solution t plus x over c, so this is also a possible class of solutions. Again, in this class of solution, there could be infinite numbers of possible solutions, but if the solution belongs to this category, then again c will represent if we go through the analysis which we did earlier, c will in this case represents again the velocity of sound. But, because I have the term positive x over c in rather than negative x over c, the sound or the pressure instead of propagating in the positive x-direction, it will propagate in the negative x-direction. And the propagation velocity of that pressure wave will be c meters per second. So in both the cases, f 1 t minus x over c, but in both the cases f 1 t minus x over c, and f 2 t plus x over c, for both these class of solutions, c represents the velocity of sound, and the negative sign implies that the pressure wave is traveling forward that is in positive x-direction. And the positive sign for the second class of solution implies that the pressure wave is traveling in the negative x-direction.

So, that concludes my module or my lecture today and tomorrow, we will close the journey and move onto actually the next range of topics starting next week. So tomorrow, we will again cover some two or three important concepts. And next week onwards, we will start actually working on details of how to measure sound and all the associated details.

Thank you very much and look forward to seeing you tomorrow.

Thank you.