**Basics of Noise and Its Measurements**
**Prof. Nachiketa Tiwari**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Kanpur**

**Lecture - 14**
**Planer Waves in Closed Tubes**

Hello, welcome again to Basics of Noise and Its Measurements. The last class what we have discussed was transmission line equations for pressure and velocity wave equations and we will continue that discussion today, and what we will do today is actually use those equations to solve for propagation of planar waves, planar sound waves in closed tubes, so that is again one-dimensional system. And we will actually learn how to use these equations for different situations and in this particular case; we will learn that how these equations can be used for closed tubes.

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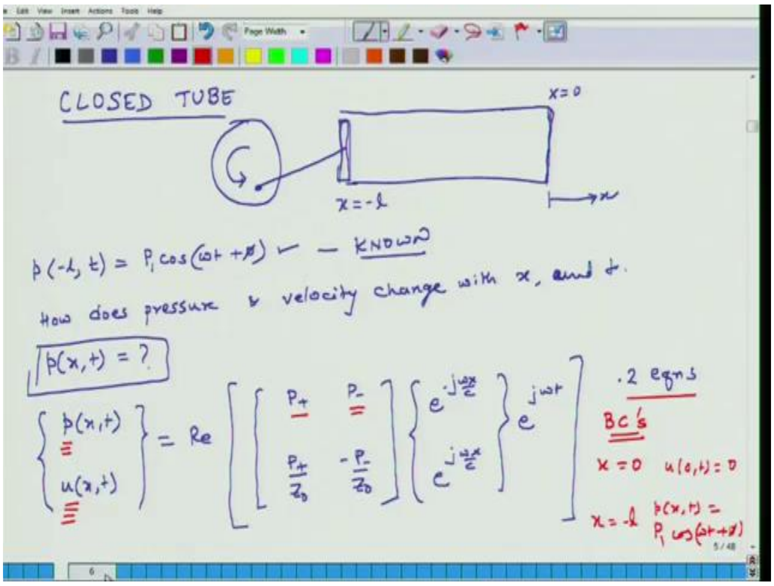


So, with that introduction, I will now move on and we will start actually developing solutions for a closed tube, so closed tube. So consider a tube, and it is at one end, it is closed; and at the other end, let us assume that I have a tight piston, and this piston is moving back and forth using some mechanism. The coordinate system is position such that the closed and of the tube is located at x is equal to 0, the direction of positive x is this thing. And so in that case the value of x and the open end of the tube is x is equal to minus l, here l represents the length of the tube. Now, when this piston moves back and forth, it generates some pressure at x equals minus l. So, the pressure being generated at x equals minus l, p at x equals minus l and time, and that is known because we know how much the piston is moving, and that is defined as some real number p 1 times cosine of omega t plus some phase value which is phase value of phi.

So what is it that we are interested in, what we want to know is how does pressure and velocity change with x and time. This is what we want to know or mathematically we want to know that what is this function x p of x and t. So we know this, we know the physical a nature of the system, we know that it is l long, it is closed at x is equal to 0, it is being excited by a piston at its open end and the pressure at the open end is P 1 cosine omega t plus phi, this is known and with all these information, we are suppose to predict the value of p, not the value the function p of x and t.

So, what we will do is that, we will use our transmission line equations to solve this. And we will do that by actually start we will to write this transmission line equations. So, we have seen and the proved it in the last class that for one-dimensional systems, pressure and velocity can be expressed as this particular relation. Now in this relation, so these are two equations, and the number of unknowns, let us see what are the things, which we do not know? We do not know P plus, we do not know P minus, and we also do not know this function p x over t, and we also do not know u.

Everything else in these two equations are known; we know how time changes, we know at different values what is the value of x, we know Z naught because we know that it is air, we know its density, we know velocity as sound in air. So we have four unknowns, and we have two equations. So, we cannot get all these four unknowns using these two equations. So, we have to get two extra conditions to solve for four these unknowns. And those two extra conditions are actually the boundary conditions of the system or which I will write it as BC's.

What is the boundary condition? It is the condition which exists at a particular boundary of the system. So, in this particular case, the system has two boundaries. The first boundary is x is equal to 0, and the second boundary is x is equal to minus l. We have to look at this system physically and then decide is they are something special about this system at x is equal to 0. Then we look at this picture, we see that, yes, at x is equal to 0, the tube is closed. And what that means is that if this tube is closed at x is equal to 0 that is there is a wall at this point then air cannot move across this wall, which means that the value of velocity u of 0 t is 0 because air cannot move across this wall, so that is the first condition - extra condition.

So, now we have three equations and still four unknowns. So, then we look at the other boundary and then, there we have been told while we were formulating the problem that the pressure at x is equal to minus l is p cosine omega t plus phi so that is the other boundary conditions. So p of x and t equals P 1 cos omega t plus phi. So now we have these four equations, four unknowns, and we can solve for all these variables P plus, P minus, function p and function u, so that is how we are going to proceed.

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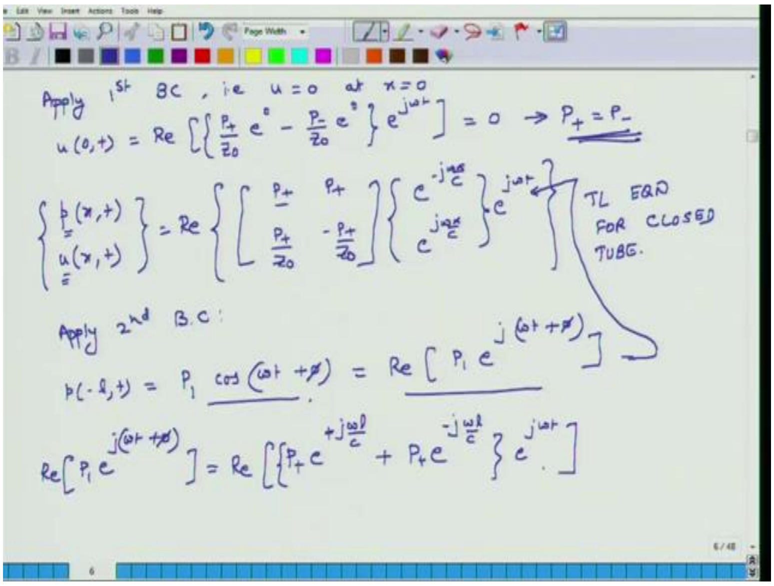


Now, if we apply first boundary condition that is u is equal to 0 at x is equal to 0, so then u of 0 and t is equal to real of P plus by Z naught e to the power of 0 plus, so I am just basically applying this boundary condition on the transmission line equation minus P minus over Z naught e to the power 0 e j omega t. And this value is we know that it is 0; so this equation will be satisfied only if P plus equals P minus. So with this, we now rewrite our boundary transmission line equations, so my transmission line equation for closed tube will be real of P plus and instead of P minus I will again write P plus because for a closed tube P plus equals P minus P plus over Z naught and minus P plus over Z naught. So, this is the transmission line equation for closed tube. This is not a general equation; it is valid only for closed tube.

So, now, we have again one unknown, two unknown, and three unknown, and we have two equations. So, now, what we will do is we will apply the second boundary conditions. So, now we apply second boundary conditions. And what is the second boundary condition; it is that p of minus l and t equals, what is it, P 1 cosine omega t plus phi. I can express this entire expression in also this format real of P 1 e j omega t plus phi. Why is that, because e to the power of j omega t equals cosine of omega t plus phi plus j times sine of omega t plus phi. And if I take the real portion of that I end of with just the cosine term.

Now, this is my boundary condition and I plug this boundary condition in the first equation. So the left hand side is p of x and t which is this thing, so I write real of P 1, and please remember P 1 is real, we have defined P 1 is real. So this is the left hand side of this transmission line equation at x is equal to minus l. And this equals real of P plus e minus j omega l over c, and because x was minus l, so this minus and minus becomes plus, plus P plus e j omega x, and here x is again minus l, so it becomes minus j omega l over c, and this thing I have to multiply by each omega t and close the brackets.

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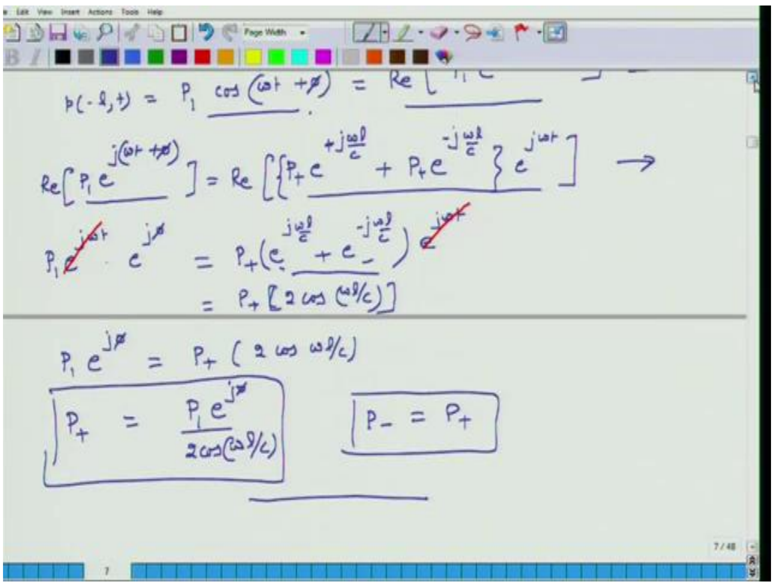


So, this equation will be valid only if the term inside the bracket equals the term inside the bracket on the left side. So what does that mean that P 1 e j omega t, and I am going to expand e j omega t plus phi as e j omega t plus times e j phi, and this equals P plus e j omega l over c plus e minus j omega l over c e j omega t. So, e j omega t and e j omega t get cancel out from both sides. Also this term is nothing but P plus excuse me times two cosine of omega l over c, because when you expand e j omega l as cosine omega l over c plus j times sine omega l over c, the sine terms from this and this term may cancel out. So this is what you get.

So, my equation becomes P 1 e j phi equals P plus times 2 cosine omega l over c or P plus is equal to P 1 e j phi divided by 2 cosine omega l over c. So, now, I have also calculated the value of P plus; and we know that P minus was equal to P plus. So, now, I can use both of these equations in my transmission line equations for the closed tube, and I can get the value of functions p and u so that is what we are going to do.

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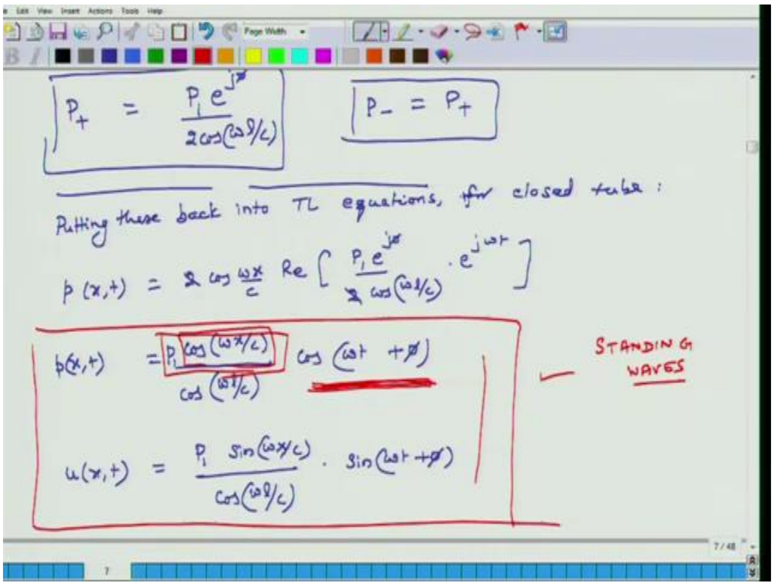


Now, so putting these back into transmission line equations for closed tube. What do we get, so what I am going to do is I am going to write the results directly, because the mathematics is pretty straightforward. So what we get is p of x and t equals 2 cosine omega x over c times real of P 1 e j phi over 2 omega l over c, I am sorry, 2 cosine omega l over c times e j omega t, and this 2 and this 2 cancel. So, essentially what I get is cosine omega x over c divided by cosine omega l over c. And if I take the real component of the term in the brackets, P 1 is any way real, so I just take it out and here I get cosine of omega t plus c.

If I do the similar math, what I get is that u of x and t equals P 1 sine of omega x over c divided by cosine of omega l over c times sine of omega t plus phi. So this is the expression for pressure. So, these are the final equations representing pressure and velocity function as they exist in a closed tube where that closed end of the tube is at x is equal to 0, and the open end is where the pressure is prescribed as some cosine function. So, once we have these two boundary conditions, this is the solution which we get. This brings us to the closer of this particular lecture, but before I formally close this, I wanted to make one or two important observations about this solution.

And the observation, the first thing is that these two equations, they actually both of them are equations for standing waves. So, these both of both of these equations represent though that of standing waves; also if you put in the value of x is equal to 0 in this equation, you will see that u comes to be indeed 0 at x is equal to 0, which is consistent with our boundary condition. And also if you put in the value of x is equal to minus l, and you do this whole computation, you will find that the boundary condition for known pressure at x is equal to minus l is also satisfied, so that is the second comment I make.

And the last comment is that because these are a standing waves what it means is that the amplitude of the motion at a given value of x is not P 1, because at a given value of x, so what do what do I mean, see the amplitude of this cosine term let us look at that pressure equation, for the amplitude of the cosine term is 1, because time could vary anywhere from positive infinity to negative infinity, so the amplitude of this cosine function is one. But, if I am looking at this standing wave at a particular point, the value of x is at that point fixed.

So, the amplitude of this thing at a given point is fixed, and for instance if omega x over c is the such if x is such that omega x over c is pi over 4 then the value of cosine of pi over 4 will always be 0.707. So what that means is that at x is equal to at that value of x where omega x over c equals pi over 4, the amplitude of this portion, this entire portion will be P 1 times 0.707. And this amplitude will vary from one value of x to other value of x. This is very particular for a standing waves because and that is why we know them as a standing waves, because at every point the amplitude is overall amplitude of the motion is fixed. And this amplitude varies from one point to other point, so that is the implication.

And with this, I want to close this lecture, and thanks to you, and look forward to seeing you tomorrow.