Basics of Noise and Its Measurements

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Lecture 29
Fourier Transform

Hello, welcome to the Basics of Noise and its Measurements. I am Nachiketa Tiwari. Today is the fifth day of fifth week of this course. This week we have been discussing Fourier Series, Fourier Integral, and today we will be discussing Fourier Transform of various functions which may not necessarily be periodic in nature. The technique of fourier transform is extremely powerful, and it is very widely used across all sorts of domains when we want to represent time signal in frequency domain. This technique helps us several ways and most importantly, it helps us resolve or identify specific frequency components of a signal which contribute to the generation of a signal in time domain.

Suppose, we have a complicated time signal, that is a signal which is a function of time and if we are interested in finding out what are its frequency components which are dominant and if the signal is not necessarily periodic in nature, then the method of fourier transform this technique, it helps us solve this problem. In that sense extremely power full and very widely used. Specifically, in area of noise it is used almost everywhere and that is why it is important to understand it. Now, the other thing I wanted to mention was that, what we will be discussing today is fourier transform form a theoretical stand point.

But, when we use computers to conduct these transforms, to develop these transforms then we use slightly different method so we do not use analog formulations of the method. And what we do is something known as DFT, that is Digital Fourier Transforms. That is something we will cover in the next class or actually next week, but today and tomorrow we will cover more or less pretty much the details of fourier transform from a mathematical and analog stand point.

You had wondered that, you have these a fourier integrals which we have already discussed and they help us represent a signal in frequency domain, even if these signals are not periodic in nature. Then what is the additional advantage we get from fourier transforms. So the answer to that is, that fourier integrals which we discussed in the last class, they are essential valid for real functions. But, if I have function is complex in nature you know acrostics and in noise, lot of times we handled complex entities for instead we have discussed complex amplitudes, complex frequency, complex time signals. Then if we have to resolve this types of signals in the frequency domain and that is fourier transforms comes in handy. So, that is the back drop.

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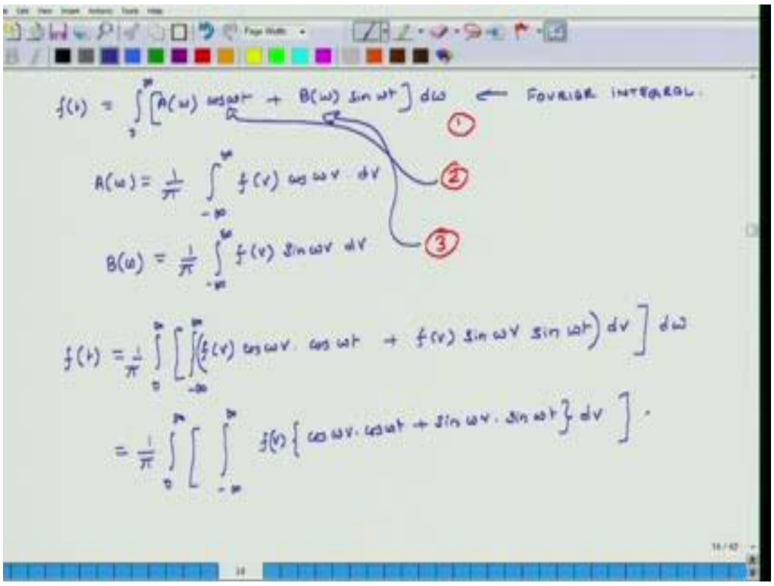

Let us consider a function f t, and we have seen that its fourier integral is 0 to infinity A function of omega cosine omega t plus another function B of omega sin of omega t d omega. This the fourier integral. We had seen that A of omega equals 1 over pi 0 to infinity, excuse me, its minus infinity to positive infinity f v cosine of omega v times dv and B of omega is 1 over pi minus infinity to infinity f v sin of omega v dv. Let us number these equations, this is equation 1, this is equation 2, this is equation 3.

If I do my mathematics, and what I do is I put this here and this equation in place of B omega then my f of t equals 1 over pi 0 to infinity and instead of A omega I m going put this entire expression, so this is f of v cosine of omega v and I have to integrate it for 0 to infinity and then cosine of omega v times cosine of omega t plus instead of B omega I am going to put this expression, so it is f of v sin of omega v sin of omega t. I have to first integrate this whole expression with respect to dv and then I have to integrate this thing with respect to d omega and the other thing, which is erroneous here is that this limit should have been minus infinity to infinity. This is equal to 1 over pi 0 to infinity minus infinity to infinity, I will take f v as common and in the parenthesis, I will have cosine omega v times cosine omega t plus sin omega v times sin omega t, and then I have dv and d omega

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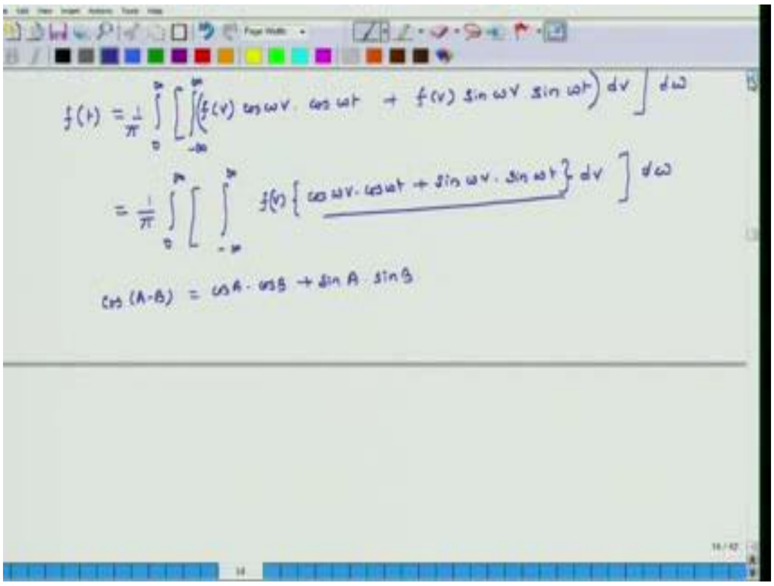

Now, I know that cosine of A minus B equals cosine of A times cosine of B plus sin of A times sin of B. This is of that form this entire underlined term is of this form, which is of cosine A minus B.

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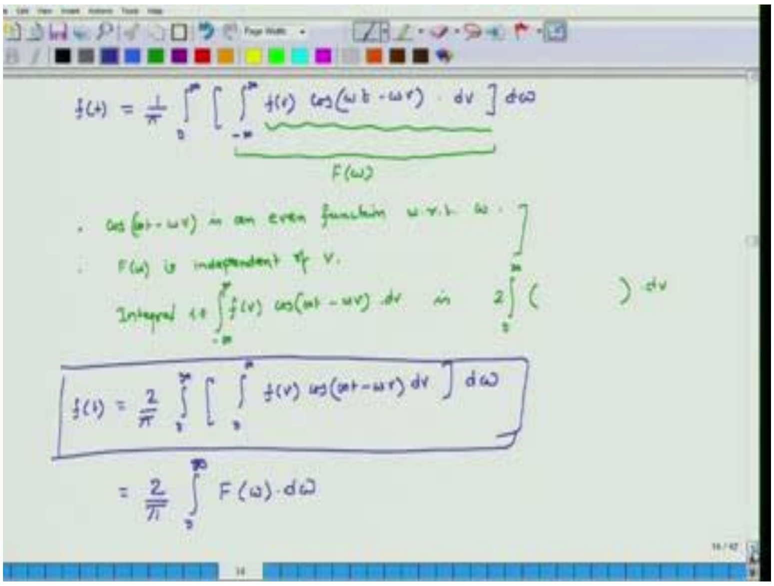

What I do is, I rewrite this expression f of t equals 1 over pi integrated from 0 to infinity minus infinity to infinity f of v and then cosine of omega t minus omega v, and this has to be integrated first with respect to dv and then with respect to d omega. Now we look at this expression and we make some observations. The first thing we note, let us call this whole integral of f v cosine omega t minus omega v times dv, when I integrate it, let me call this as capital F of omega. It is not just the integrand; it is the integral of the entire expression so we make some observations.

The first thing is that, cosine of omega t minus omega v is an even function with respect to omega. Second, when I integrate this whole thing f of v times cosine omega t minus omega v dv, then the integrand of F omega it has v, but once I integrated it F of omega is independent of v, it is independent of v. And from this two, I can say that the integral of this f v cosine omega t minus omega v that is minus infinity to this thing. The integral that is this thing is nothing but 0 to infinity 2 times the same thing. Again make this detection from this.

What I can do is, I can rewrite my f t as 1 over pi 0 to infinity 0 to infinity again, and then because I change the limits so I am going to put 2 so excuse me, its 2 over pi f of v cosine omega t minus omega v dv d omega. This is one part, and this I can write it as 2 over pi 0 to infinity and I can write this integral as F of omega d omega. What I have done in this class still so far is that we started from a function right from a fourier integral we substituted the values of A omega and B omega into the fourier integral, I mean that is all we done. We have in substituted those values, did some mathematical manipulation and we have transform these equation f of t as a function which is nothing but 2 over pi integral of F of omega d omega from 0 to infinity. Where, F of omega is defined as this function, which is f of v times cosine omega t minus omega v times dv integral of 0 to infinity.

So, this is what I wanted to cover in this lecture and I will continue this in the next lecture also, because what we have done till so far it has to continue and then finally we will arrive at the fourier transform of the overall system. So, please stay tune and will continue this discussion and close this in the next lecture.

Thank you.