Basics of Noise and Its Measurements

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Lecture - 32
Discrete Fourier Transform

Hello, welcome to Basics of Noise and its Measurements. This week we are having the discussions on the Discreet Fourier Transform. In the last class, we had introduced this concept of DFT and its relationships and what we are going to do now is continue with the examples which we discussed in the last class, and actually used the data points which generated in the last class, to compute the discreet fourier transform of the function which was described. So that is what we will do, will we continue the dissension discrete fourier transform.

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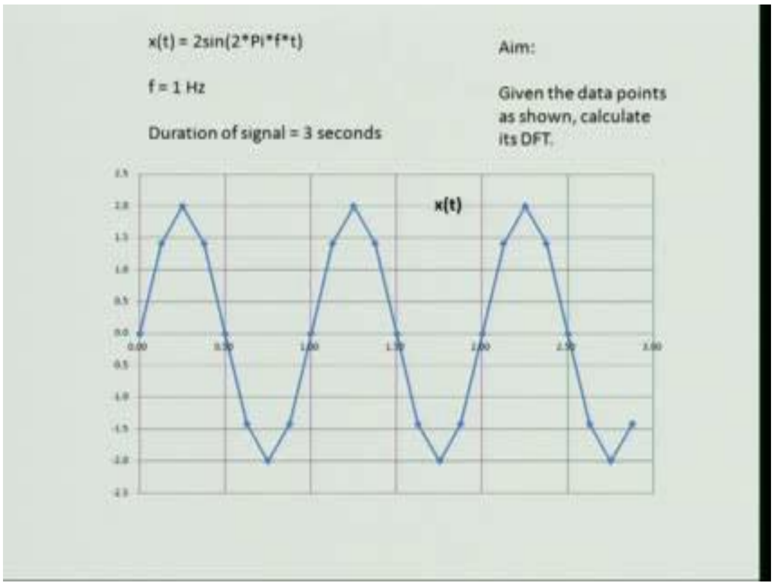

These are the points which we have generated. This is x 0, this is x 1 so these individual points of values of x n. So this x 0, x 1, x 2, x 3, x 4 so on and so forth. As I go on for 3 seconds, this is my x 23, I have a total of 24 points but because from my numbering start from 0, so I end up with x 23. And remember here, that I am not completing the cycle, because I have duration of 3 seconds and I have done 24 points; 8 points per second. The last point if I completed this cycle it would the 25th point, but here I am just doing 24 points. It is important understand that. So this is the thing.

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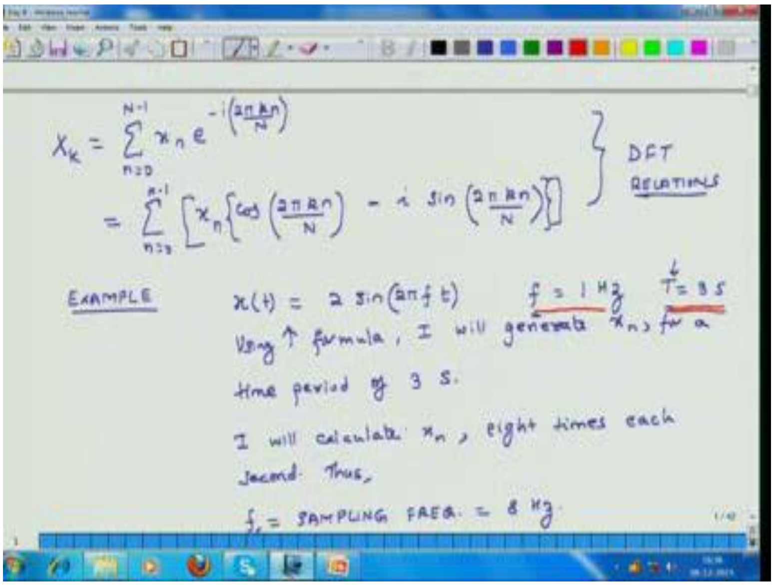

Another thing I wanted to mention is that, in this relation X k equals x n exponent to this entire term, there is no explicit mention of time. The information about time is embedded in the frequency duration of time and sampling frequency, but in this formula there is no explicit mention of time. The values of x are mentioned which is x n, but the values of time are not explicitly mentioned. When we do a fourier transform, all we require is a series of numbers. Then, how they are related to time information is actually dependent on these parameters, f s duration of time and so on and so forth, so this again important to understand.

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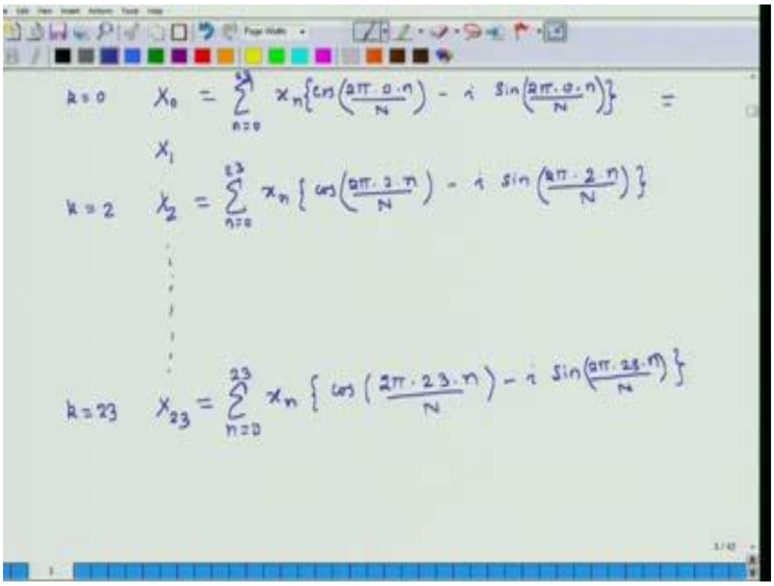

What we will do is, we will compute couple of values. So, for k is equal to 0, X 0. What are we going to compute, this capital X implies that it is the for a domain representation of the signal. So, we will be calculating X 0, x 1, x 2, it will be X 23. This is equal to summation of n equals 0 to 23 x n cosign of 2 pi and the value of k when k is equal to 0 2 k pi k n so 2 pi times 0 times n over N minus i sign 2 pi times 0 times n over N. This is what we are going to sum on. We have put the value of k as 0. Here, let us do k is equal to 2, so that will give us the value of x 2 and that will be equal to summation n equal 0 to 23 x n cosine of 2 pi, and the value of k is 2 n over capital N minus i signed 2 pi 2 times n over capital N. I can calculate all the values of x 0, x 1, x 2 and so on and so forth.

And I will just still write down one more number. This is for k is equal to 23, if I plug in the k is equal to 23 I get x 23 and that is equal to n equal 0 to 23 x n cosine of 2 pi times 23 times n over N minus i sin 2 pi times 23 times n over n. I can calculate these values. I have formula it is ready straight forward, I can do this calculation and what I will show you is the actual values for the function.

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n	t	f(t)	0	0	1	1	2	2	3	3	4	4
0	0.000	0	0	0	0	0	0	0	0	0	0	0
1	0.125	1.414	1.414	0.0000	1.366	-0.3660	1.225	-0.7071	1.000	-1.0000	0.707	-1.2247
2	0.250	2.000	2.000	0.0000	1.732	-1.0000	1.000	-1.7321	0.000	-2.0000	-1.000	-1.7321
3	0.375	1.414	1.414	0.0000	1.000	-1.0000	0.000	-1.4142	-1.000	-1.0000	-1.414	0.0000
4	0.500	0.000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000
5	0.625	-1.414	-1.414	0.0000	-0.366	1.3660	1.225	0.7071	1.000	-1.0000	-0.707	-1.2247
6	0.750	-2.000	-2.000	0.0000	0.000	2.0000	2.000	0.0000	0.000	-2.0000	-2.000	0.0000
7	0.875	-1.414	-1.414	0.0000	0.366	1.3660	1.225	-0.7071	-1.000	-1.0000	-0.707	1.2247
8	1.000	0.000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000
9	1.125	1.414	1.414	0.0000	-1.000	-1.0000	0.000	1.4142	1.000	-1.0000	-1.414	0.0000
10	1.250	2.000	2.000	0.0000	-1.732	-1.0000	1.000	1.7321	0.000	-2.0000	-1.000	1.7321
11	1.375	1.414	1.414	0.0000	-1.366	-0.3660	1.225	0.7071	-1.000	-1.0000	0.707	1.2247
12	1.500	0.000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000
13	1.625	-1.414	-1.414	0.0000	1.366	-0.3660	-1.225	0.7071	1.000	-1.0000	-0.707	1.2247
14	1.750	-2.000	-2.000	0.0000	1.732	-1.0000	-1.000	1.7321	0.000	-2.0000	1.000	1.7321
15	1.875	-1.414	-1.414	0.0000	1.000	-1.0000	0.000	1.4142	-1.000	-1.0000	1.414	0.0000
16	2.000	0.000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000
17	2.125	1.414	1.414	0.0000	-0.366	1.3660	-1.225	-0.7071	1.000	-1.0000	0.707	1.2247
18	2.250	2.000	2.000	0.0000	0.000	2.0000	-2.000	0.0000	0.000	-2.0000	2.000	0.0000
19	2.375	1.414	1.414	0.0000	0.366	1.3660	-1.225	0.7071	-1.000	-1.0000	0.707	-1.2247
20	2.500	0.000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000
21	2.625	-1.414	1.414	0.0000	1.000	-1.0000	0.000	-1.4142	1.000	-1.0000	1.414	0.0000
22	2.750	-2.000	-2.000	0.0000	-1.732	-1.0000	-1.000	-1.7321	0.000	-2.0000	1.000	-1.7321
23	2.875	-1.414	-1.414	0.0000	-1.366	-0.3660	-1.225	-0.7071	-1.000	-1.0000	-0.707	-1.2247
FFT			0	0	0	-7E-16	-4E-15	-3E-15	1E-14	-24	3E-15	5E-15
Mag of FFT			0		7E-16		5E-15		24		6E-15	

This is the value of n. So, n is equal to 0 1 2 and it goes one till 23. Then, my next column is for time, t. So, t is 0 in the beginning, then the next step of t is one over eighth because sampling frequency was 8 hertz, the next value of time is 0.25 and so on and so forth. So, I come up to 2.875. Then I am sampling this for period of 2.85 seconds and, because my sampling frequency is 8 I will get 24 points. This is important to understand.

If I sampled it for 3 second, so there was little error because I said that I sampled it for 3 second it should have been 2.875 seconds. If I have sampled it for 3 second it would be n equals 25, capital N would be 25. My intention was to generate 24 points not 25 points. Then the next column for f t, and this the value of the function which 2 sin times 2 pi f times time. Then these rows correspond to value of different values of k, and here I have included information only till k equals 4 because I could not go on beyond the screen, but this would to continue to 5, 6, 7, till 23.

The first column corresponding k is equal to 0 is specific value far the real portion of that fourier transform. This number corresponds a real portion when n equals 0, this corresponds to the real portion of the component of when k is equal to 0 and n is equal to 1. Here, k is equal to 0 and n is equal to 2, here k is equal to 0 and n is equal to 3 and so on and so forth. These are all, the real component. Similarly, these are all the imaginary components and what I have done here is I have added them up. This FFT, this is sum of real things, sum of all the cosine terms, this is sum of all the sin terms. This is a magnitude, which is this square plus this square and this is a square root of that.

This is a value of magnitude of X k, and if I have to look at his imaginary and real portions X k when k is equal to 0 is 0 plus 0 i. Here, X k so these two columns are when k is equal to 1, then this is the value of X k 0 plus minus 7 into 10 per minus 16 times i. And the overall magnitude is 7 e minus 16 and so on and so forth. Now look at 3, practically all the components are coming out to be 0 right this is e minus 16 is 0, pretty much close to 0. But you come to column 3 and what you will see is that the real component is 0; the imaginary component is minus 24. In this case, the fourier component corresponding to k equal to 3, now 3 is not same as the frequency. Frequency was 1 herds, this is 3. So this is not equal to frequency, but fourier component when k was 3 the value of X k is 0 minus 24 i and the magnitude of his 24.

Then for the fourth component the x 4, it is again 0 plus 0. We can keep on doing this mathematics till we get x is equal to x k, when k is equal to 23 in our case. We get all these values. Then the question is what we do with these numbers, right because this 24 was not the amplitude of the anything and neither 3 was the frequency. So, we have to figure out how to map this information into the real thing. The first thing we will do is, we will learn how to convert the value of X k into specific amplitudes of specific frequencies, because what we saw that amplitude in this case was very much 0, in one case it was 23 and all other places what we saw it was 0. So, we have to convert that into something real.

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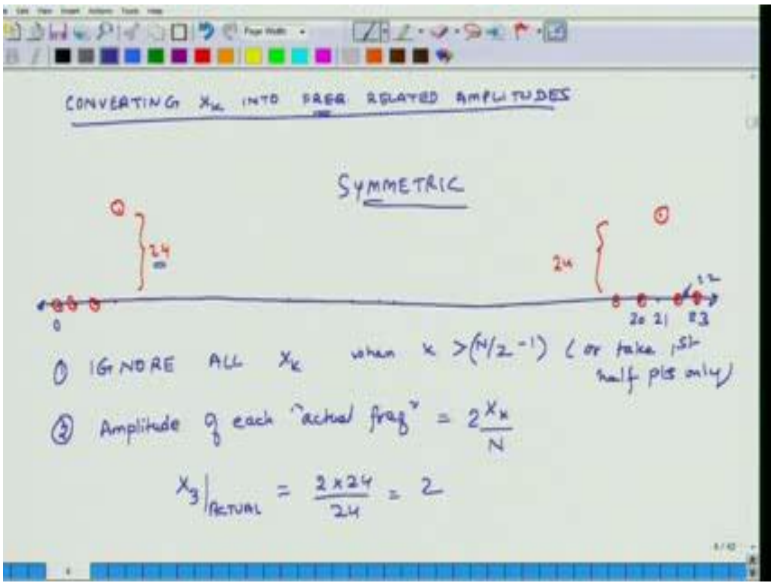

What these frequencies are, we will not talk right now, we will do that later, but right now we will just land how to convert this X k into frequencies related amplitudes. So, we will very quickly draw this spectrum or we will plot the values of X k. We will start from 0 and we will end with 23. So that’s, 22, 21, 20, 19 and here it is 1, 2, 3 and so on and so forth. What we saw was that at k is equal to 0 the value was 0, so it was 0. At k is equal to 1 the value was still 0. Let us see my vertical axis and k is equal to 2 it was still 0. At k is equal to 3 we found that the value was 24, so this was 24. And then you keep on computing you will find that there were lots of zeros, and at all places it was 0, but till you come to 21 and then this is 22 this is 20. So, at 19 also it was 0, it was 0 here, it was 0 here, 0 1 2, and at 3 it was here, it was 0 at 19, it was 0 at 20, but at 21 it was again the magnitude I am talking was 24. Then at 22 at 0, and then also 23 it was 0. If you did the computation this is what you will get.

And you have a total of 24 points. This is how you calculate the frequencies. First thing is ignore all values of X k, when k is greater than N over 2 minus 1. In this case, what does that mean, you take only the first 12 points, N by 2 is what? N is 24, N by 2 is 12, 12 minus 1 is 11. So, you ignore all this k is equal to 12 13, so you have note 12 points. You ignore all those so you take basically the first 12 points, or take first half points only.

This is in plain English; you take only first half points. All others you ignore, you do not worry about those.

Second, amplitude of each actual frequency, we have not figured out how to calculate the actual frequency yet, each actual frequency. See, this case is associated with some frequency number, but we do not know, what is the value of that number? K is equal to 0 means it has associated with some frequency, if k is equal to 1 it has associated with, we do not know the values of these frequency. But, X k is tells us something about the amplitude but not the actual amplitude. So, amplitude of each actual frequency is equal to X k divided by N times 2, because you ignore the other half, you will always find that when you do this analysis, this whole figure will be symmetric. You ignore the second half you take only the first half to account for the other half you multiplied by 2, why is all these happening we will not discus in the particular lecture, but multiply it by 2 and you dived by the total number of points, which was in this case 24.

What does it mean? So, this is the indicated frequency, but actual frequency so x 3 actual is equal to 2 into 24 by 24 is equal to 2. Now, we go back to our original sin function its amplitude was 2, if you remember it was 2 sin times 2 pi f t. And there was no other frequency in the signal, there was only one frequency so all other frequencies are coming to be 0. There is only 1 frequency and that is, its amplitude is 2. What is its frequency? Right now, from this analysis we have not figured that out. So that is something we will discuss in the next lecture.

Thank you.