Basics of Noise and Its Measurements
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Lecture – 33
Discrete Fourier Transform
Calculating Frequencies and Padding

Welcome to Basics of Noise and its Measurements. Over last two lectures, we have been discussing this Discrete Fourier Transform and what we have learnt till so far is how to discretize a function or if we have a discrete set of points how do we use these points to compute the amplitudes of specific fourier components. What we will do in this lecture is continue this whole topic of DFT, and we will also figure out how to extract the specific values of frequencies which corresponds to different value of X k. So that is one important thing which we will discuss in this lecture.

The other thing we will discuss is something which we know as Padding, and I will explain that as we come to that relation. So that is what we are going to do Calculating Frequencies and Padding. The aim of today's lectures is that, once we have generated values of X k, we already figured out how to compute specific amplitudes of frequencies associated with each X k and the way we do it is we multiply that each X k with two and divided by the total number of points, which is capital N and that is the specific amplitude of corresponding to each X k. Now, each X k is also associated with a specific frequency, so that is what we learn how to figure it out.

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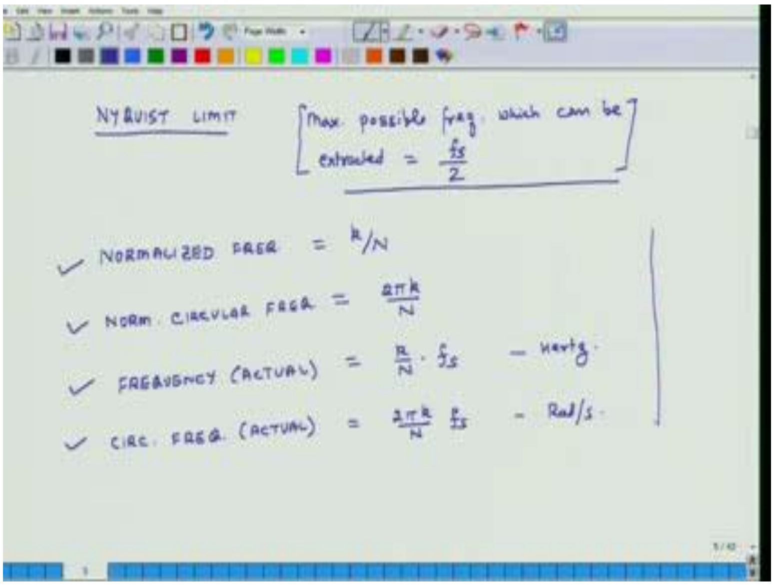

Before we discuss that, I wanted to talk about something called Nyquist Limit. In plain English, this gentleman Nyquist, in plain words what he said is, that if your sampling frequency is 100 hertz, then the maximum possible frequency which you can extract by doing this fourier transform, DFT, discrete fourier transform will be 50 hertz. In mathematical sense, what it means is, maximum possible frequency which can be extracted and that is equal to sampling frequency fs divided by 2.

What does that mean? Our sampling frequency was 8 hertz. So the maximum possible frequency which we can extract out of it was 4 hertz. So that is one thing and associated with this is also this thing, because in last class I had said that you only consider the first 12 points or first half points and ignore the other half points, and the reason for that is. Again, in a very broad, loose mathematical senses that this approach it gives you solutions for a direct problem and also a conjugate problem or as complex conjugate problem. The first half gives you is represent the solution of the direct problem and the second half represent the solution for a complex conjugate problem and somehow the energy over all it gets divided over the entire thing.

When you plot X k, if you plot on the y axis you have the values on X k on x axis you have different values of k, you get a symmetric figure and the energy gets distributed. So that is why you have to multiply it by 2 and get the original amplitude back. This is an important thing that the maximum possible limit which you can extract is fs over 2. Now, let us see what this means.

Now, I am going to introduce 4 different frequencies. First one is, Normalized Frequency, and what is normalized frequency? You can calculate it as k over N hertz. So, if k is 0 normalized frequency is 0, if k is 1 and capital N was in this case 24 the normalized frequency will be 1 over 24. Then, there is Normalized Circular Frequency, and that is 2 pi k over N. Each X k is associated with a normalized frequency, which is k over N. And remember, this is not the actual frequency this is normalized frequency. And if you what to do it in angular velocity terms or circular frequency terms you multiply it by 2. So, this will be hertz, this will be radians per second. Actually, I will not call it hertz but because this is unit less entity, so I will not call it hertz or radians per second.

Then there is the actual Frequency, which corresponds to X k and that is k over N times fs which is sampling frequency. This is actually its units are in hertz or 1 over t units. Then there is Circular Frequency, and this is also actual and this is 2 pi k over N fs. So this will be what, radians per second. You have different values of X k and for each value of X k there is an indices which is k the moment you know that k you can calculate normalized frequency, normalized circular frequency, frequency and circular frequency. You know the amplitude of that frequency and you also know the value of that frequency using this relationship. Now, let us look at this further.

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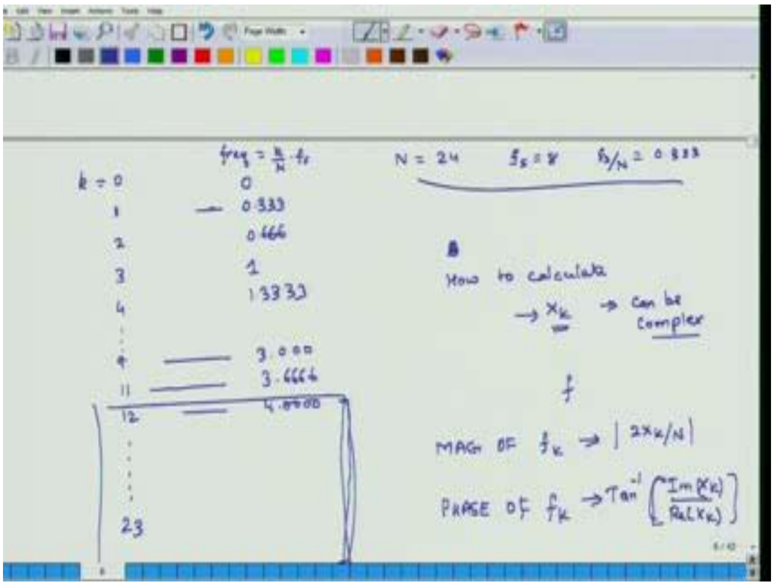

So, k was 0, 1, 2, 3, 4, 11, 12, and it went till 23. If you compute the value of a frequency that was k over capital N times fs. So this one was 0, and what is the value of N? N was 24 fs was 8. So, fs over N equal 0.333. So, this is 0.333 hertz, this is 0.666 hertz, this is 1, this is 1.333. In case 9, then 9 would correspond to 3 hertz, 10 would correspond to 3.333 hertz, 11 would correspond to 3.666 hertz, and 10 would correspond to 4.000 hertz. And, what does Shannon say? That I cannot have any answer in this range, these Nyquist criteria, it says that.

Again, because of this I have to ignore this. I take only first 12 points. Now, you get a little better feel as to why we are ignoring these things, so I take only first 12 points and I get the answer here. I know using this how to calculate. So what we are figured out is that how to calculate X k. Now, this X k can be complex. It can be complex there is no reason and I also know how to calculate frequency. This is X k by itself if I multiply this by 2 and divide it by 24 I get the complex amplitude of that frequencies. Do you understand this?

So, I get the complex amplitude of that particular frequency. That particular frequency will have a magnitude and a phase. The magnitude of f k, k th frequency will be 2 X k over N. The complex amplitude was 2 X k over N, the absolute magnitude because X k can be complex, so the absolute magnitude will be modulus of 2 X k over N. And the phase of k th frequency will be tan inverse imaginary of X k divided by real of X k. This is clear. We will recap.

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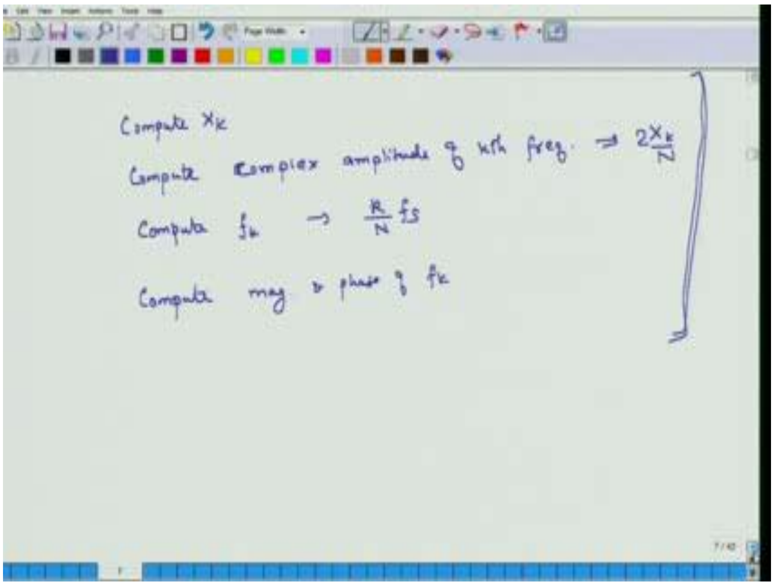

What you do? If you have a data set of data points compute X k. Then compute complex amplitude of k th frequency and this is what X k divided by N times 2. Compute f k and how do you compute f k, it is k over N times fs. And then what you do? Compute magnitude and phase of f k using the relations which we have discussed earlier. So this is the overall summary. If you know this, then you should not be having any problems in doing DFT or Discrete Fourier Transform of any signal. The signal can have complex numbers it can have real numbers, it does not matter you can do this.

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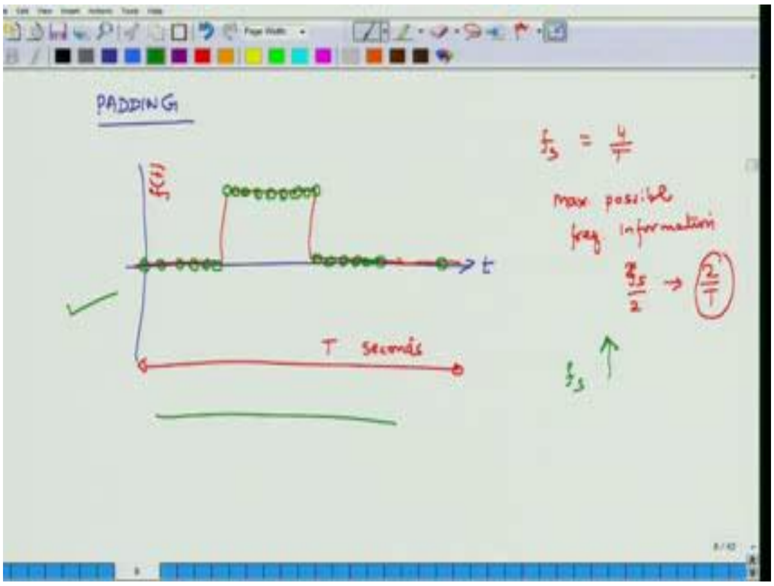

Last thing and this is what we are going to discuss in Padding. Suppose, you have a signal in time and this is not a repeating signal because we can use fourier transform for non repeating signal. Now, I can discretize it here, here, here, here, let us say these are all equal intervals, this is the amplitude, let us say, this is f of t. Now what does this means? Suppose, this is T seconds, then how many samples I have, 1, 2, 3, 4. My sampling frequency is, how much, 4 samples in T seconds. Suppose, this overall is T seconds then it is 4 samples in T seconds.

Then maximum possible frequency which I will get will be what? No maximum possible frequency information it will not be 2, it will be fs over 2 that is 2 over T. Suppose, I want more points, suppose I want more information on frequency, suppose I want this number to be very high, then what do I do? I put more points. That is called Padding. So I can either interpolate between adjacent points, or if two points are close by and if I do not thing it is going to change between two adjacent points I just take a straight things so that is called padding.

So, I basically pad up all the information between two points. Then what happens? This sampling frequency it went up significantly so the maximum possible frequency components which I will get from this will be much higher. So, this is called Padding.

This closes our discussion on Discrete Fourier Transform from the standpoint of having some introduction to it. In the next class, what we will do is, we will have one more lecture on DFT and we will learn a little bit more about it, and then we will close and move on to other topics.

Thank you very much.