Basics of Noise and Its Measurements
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 44
Short Term Fourier Transforms & Spectrograms

Hello, welcome to Basics of Noise and its Measurements. This is the second day of the last week of this course. Yesterday, we were discussing Short Term Discrete Fourier Transforms and we had discussed as to how we can splice up a long time series signal, and successively FFT to get time related information, time related frequencies spectrum of this, the entire signal. This approach becomes particularly handy and useful, when we have non-stationary signals, that is, signals for which the frequency spectrum keeps on changing, as time marches forward. Now, what we will do today is, we will actually continue that discussion on Short Term Discrete Fourier Transform. We will actually look at some of examples and then we will conclude today's session by discussing this, another very useful tool, known as a spectrogram.

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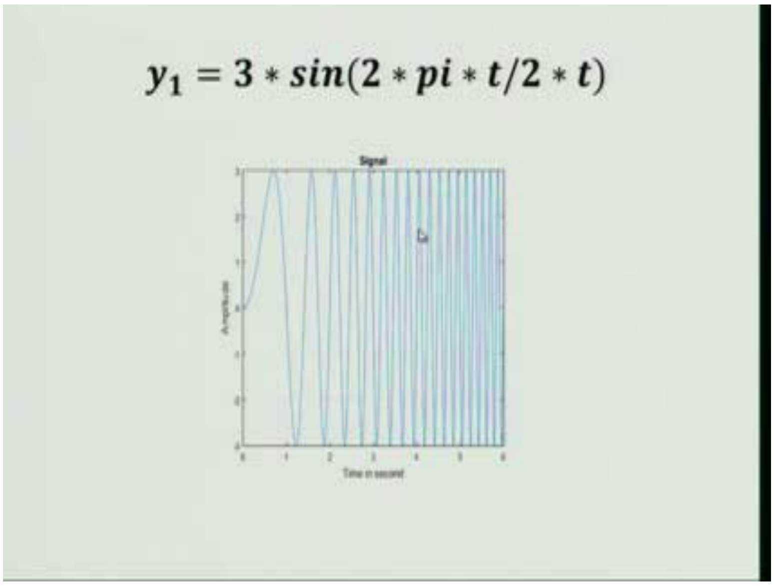

So, that is what we are going to talk about today, Short Term Fourier Transforms and Spectrograms. Now, I wanted you to look at this signal, and this is, y equals 3 times sin of 2 pi times t over 2 times t. So, that is basically pi times t square over 2. Here, my frequency... So, the reason I have put t over 2 separately and I have not merged with the other t is that my frequency which is this thing, t over 2, it is actually increasing, as time is marching forward. So, at t is equal to 0, the frequency is 0 hertz; at t is equal to 1, frequency is half hertz; at t is equal to 2, frequency is 1 hertz, and so on and so forth.

When I plot this signal on x, you know, on the time axis, that is the plot, time in seconds, and I have plotted it for 6 seconds; vertical axis is amplitude, and what you see is that, as time marches forward, the frequency of the signal keeps on going up. You may hear something similar, for instance, if there is a car, and which is rising and it is accelerating, you may have, you may hear some noise, kind of similar to this kind of a signal. So, now, if I take the FFT of this, if I take the FFT in maybe first second, my frequency will come out to be, say, you know, the most dominant frequency will come out to be, maybe something close to half a hertz. But, as I increase, move my window rightwards, the frequencies which are dominant, as the, you know, as my window moves rightwards, the dominant frequencies, they keep on going up. So, this is one example, and in this example, the frequencies themselves are shifting with time, and this is an example of non-stationary signal.

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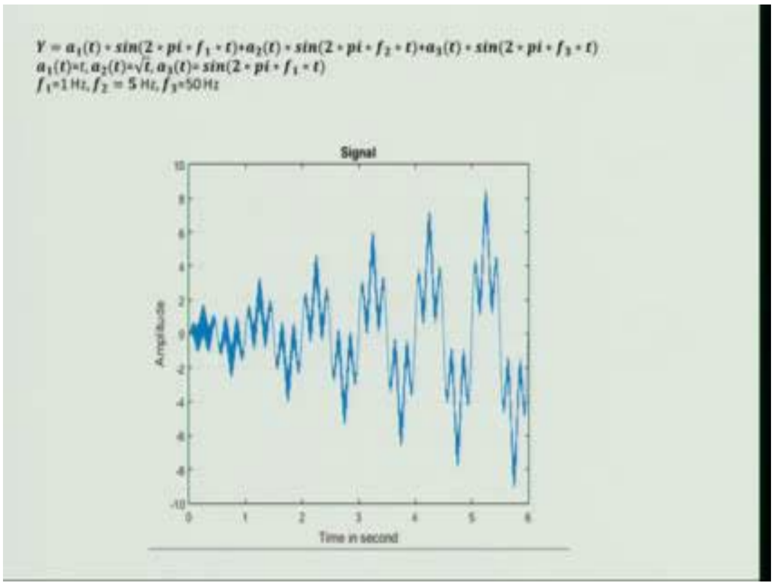

Now, this is another non-stationary signal. So, in this case, the frequencies were shifting with time. They were actually, in this case, increasing with time; this is another example. So, what you have here is, another signal, on the y axis time, x axis time, y axis amplitude, and you have, y equals a 1 and a 1 is the amplitude, and that is a function of time. So, a 1 t times sin 2 pi f 1 t, plus a 2 t sin 2 pi f 2 t, plus a 3 t sin 2 pi f 3 t. Now, what are the values of f 1, f 2, f 3? f 1 is1 hertz; f 2 is 5 hertz; f 3 is 50 hertz. So, the frequencies themselves are same, but the amplitudes are time dependant. So, a1 is linearly changing with time; a 2 is having a square root relationship, and a 3, interestingly, we have chosen that, it is a sinusoidal function of time. So, the amplitude, amplitude of this signal, last component of the signal, which is 2 pi times 50 hertz, it is basically, 2 pi times 1 hertz t. So, the amplitude of this signal, initially, it is 0; then, it goes up, then it comes down; then, it goes up, then it comes down, and so on and so forth. So, this is another thing.

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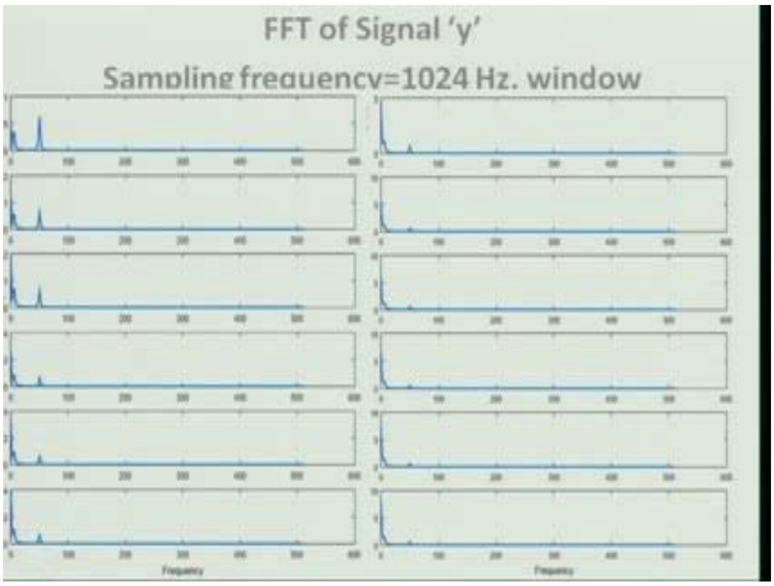

Now, what we will do is, we will use our windowing technique and use the, we did the short time DFT of this second signal, and we will show you some of the plots. So, so, what you are seeing here, are 12 graphs; 1, 2, 3, 4, 5, 6, and then, start again, 7, 8, 9, 10, 11, 12. These are 12 different graphs. As you would look at these graphs, please be aware that, the x axis is same, 0 to 100, or 600 hertz; but the y axis, the range is different. So, the range is, minimum here, it is 0 to 1; here, it becomes 0 to 2; but here, it becomes 0 to 4; here, 0 to 5; and then, finally, it becomes 0 to 10. So, as you are making judgments about these graphs, please be aware that, the vertical axes are changing.

The first graph corresponds to window which is half a second long. And, this is the first half second of data, of that entire 6 second long signal; and, we took its FFT, and the results have been plotted here. What you see here is that, you see a peak at 50 hertz, which was indeed the case because there is a 50 hertz frequency. Then, you also see a small, another peak, at maybe 2 hertz.

So, had we plotted this on the log scale, this would have been clear. So, this is at 2 hertz, because, there is another frequency at, I am sorry, 5 hertz. So, this is 5 hertz, but then, you will see, if you zoom in on this, you see some abstract data at1 hertz; in the, below1 hertz; and the reason for that absurdity is that, our time, duration of time is only half a second. And to capture accurately the FFT of a 1 hertz signal, which is this first component in the signal, I should have signal, which is at least 1 second long; even at 1 second, it is not always guaranteed, but it should be at least 1 second long. So, you see some absurd results; but you get ok results at, for 5 hertz and 50 hertz frequencies.

Then, you see that, the value of a 1, a 2, and a 3, it keeps on increasing with time; I mean, this also increases, and it only actually becomes small, after one cycle is over, when, after 1 second. So, that is what you actually see, that, it has increased; everything is increasing. And then, this increases; everything is increasing, and all these amplitude, the first 2 amplitudes, they go on increasing; same thing here; the first 2 amplitudes keep on increasing, and it is just that. And, this amplitude, actually, if you calibrate yourself, because the vertical axis scale is different, this actually goes up and down, and goes up and down and up and down because here the scale is now 0 to 10.

On the time scale, it is this analysis, this short term DFT is giving us decent results; but on the frequency scale, at least for 1 hertz component, which is this, this particular component, we are not getting good data. So, this is there. So, then, we said ok. So, this is again, in this case, what we are doing is, we are taking FFT of the first half second; then, we move rightwards by another half second, and then, take the second half second, then third half second, fourth half second and so on and so forth. So, that is how we are getting 12 different graphs. While generating these graphs, we were not having any overlapping of the windows.

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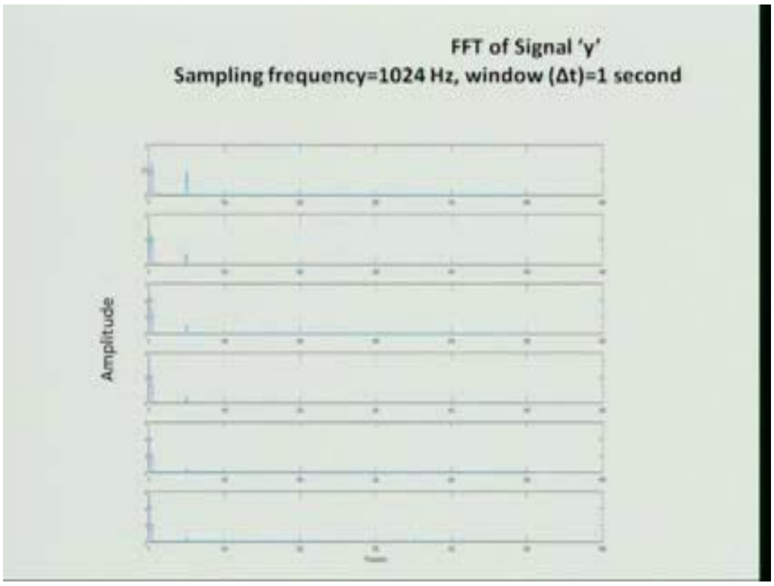

So, but… So, this is there. Then, this is, these are 6 graphs, and these correspond to windows which are 1 second long. And, once again, here also, the windows were not overlapping. So, we took one first second, second second, third second. So, that is why we have 6 graphs. And, what you see is something very interesting here. So, if you zoom in, you get much better resolution, somewhat better resolutions of 1 hertz and 2 hertz, but you something, see something very interesting that, at 50 hertz you are seeing 2 frequencies; you are seeing actually 2 frequencies at 50 hertz; and you wonder, why is this happening? And, the reason for that is this.

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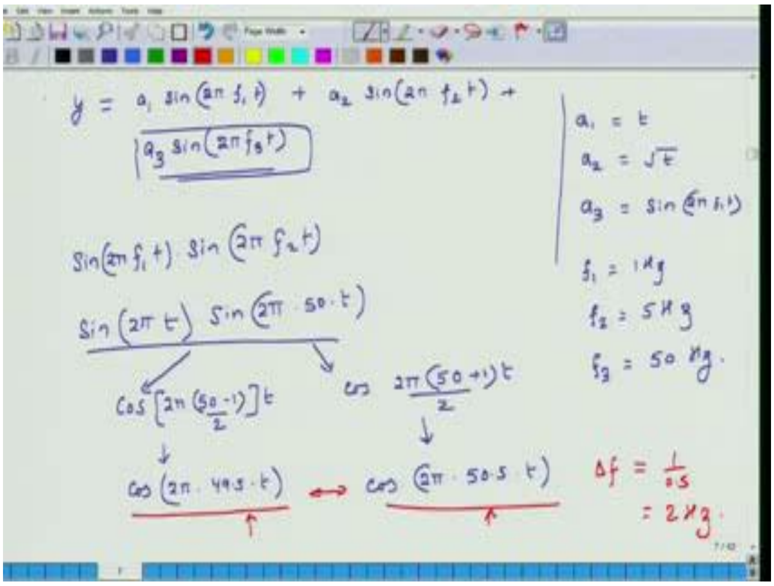

So, actually, I will explain this on the... So, what was our original signal? Our original signal was, y equals a 1 sin 2 pi f 1 t, plus a 2 sin 2 pi f 2 t, plus a 3 sin 2 pi f 3 t, and we had defined that a 1, a 2, a 3, they were not constants, but they were functions of time. So, a 1 was basically t; a 2 was, we had defined it as root of t, and a 3 was sin of 2 pi f 1 t; and f 1 was 1 hertz, f 2 was 5 hertz, and f 3 was 50 hertz.

Now, what is happening is that, at around 50 hertz, in the graph when you see it, you are seeing 2 frequencies; you are not seeing 1 frequency; we will see that again. And, why is this happening? So, we will look at this component in detail. So, a 3, is basically sin 2 pi f 1 t times sin 2 pi f 2 t, right, which is… And, what is f 1, or sin 2 pi t times sin 2 pi times 50 times t; this is what is there. So, this is sin a times sin b; and if you go back to your classes of trigonometry, you will see that, these multiples of 2 sin, sin of a, and sin of b, it can be resolved into the cosine of a minus b and a plus b divided by 2. So, this is nothing, but it will, this gives us 1 component, cosine of 2 pi 50 minus 1, divided by 2, t, and the other component you get is, cosine of 2 pi times 50 plus 1 times t divided by 2, which is cosine of 2 pi times 49.5 times t and this is cosine of 2 pi times 50.5 times t.

Now, these phenomena... So, the actual signal when you do the FFT, when it does the FFT, it is detecting these 2 frequencies, 49.5 and 50.5 hertz. It is detecting these, and it is doing a good job; it is doing; this is the right thing. It is detecting these 2 frequencies, and it is detecting it why. So, in the first graph, when the duration was extremely small, it was not able to detect these 2 frequencies. It was not able resolve these 2 frequencies, because our delta f resolution was what? Our total duration was half hertz. So, our resolution was 1.0, 1 over 0.5 and that is equal to 2 hertz. So, it could not separate those 2 frequencies; it could not separate those 2 frequencies accurately.

When I increased my del, duration of time to 1 second, my resolution went down to 1 hertz; it went down to 1 hertz, and so, it is able to resolve. So, that is why, you are seeing these 2 peaks. These 2 peaks should have been there, because, when you multiply 2 frequencies, you can also express them as sum of 2 different frequencies. And so, these 2 frequencies are actually real, but you do not see, when your duration of time is very less, because, your resolution is not that much. So, that is what we mean that, if my duration becomes very less, one problem I am having is at 1 hertz; one problem I am having is at 1 hertz, because when time is very less. Other problem I am having is also at 50 hertz, because there also, there are 2 frequencies present, but it is not detecting them, because, again, the resolution is not there. So, again, when I reduce my time, I lose frequency resolution; when I increase my time, I lose time related information. So, this is what is happening.

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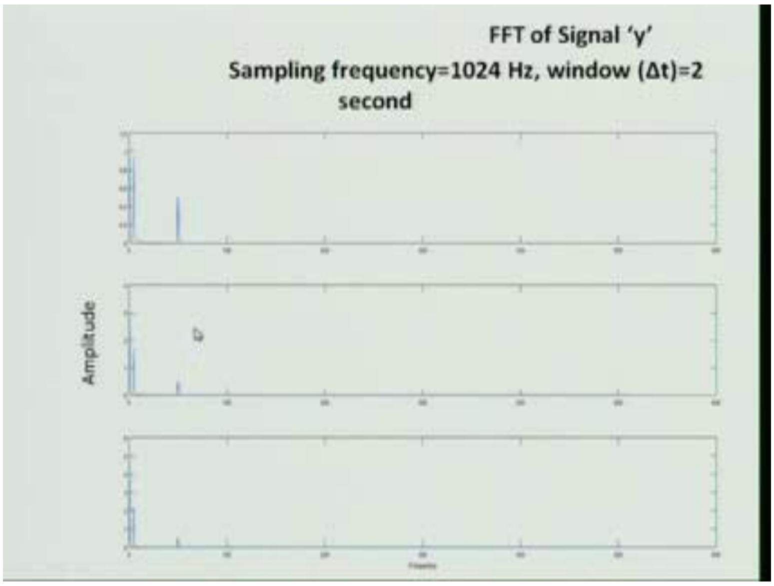

Now, let us look at t is equal to 2 seconds. So, this is t is equal to 0 to 2; t is equal to 0 to 2, 3, 2 to 4, and t is equal to 0 to 4 to 6 seconds. And, what you see here is that, the time information now. So, again, I mean, you have 2 frequencies here. So, frequency information is there. All the frequencies have been correctly identified but the amplitudes, you cannot say, at what time these amplitudes correspond to, because this is for a 2 second long interval. It is for a 2 second long interval. So, is this time, the first, the first graph is for 0 to 2 second; is this amplitude corresponding to t is equal to 0, or is this amplitude corresponding to t is equal to 1, or is this amplitude corresponding to t is equal to 2; you do not know.

The only thing you know is that, for that whole duration, the amplitude is 2 seconds. If you use this to reconstruct, with the inverse FFT reconstruct, you will not get good answers. So, so, this is what I wanted to show in context of this short term digital Fourier transforms. This is very important to understand that, windowing is important, and it helps us. But, you are seeing it here, very clearly that, if I increase the time, I get a better frequency resolution, but I lose time information; and, if I reduce the time, I lose frequency information, but I get better time information. So, I have to balance, and I have a compromise between these two conflicting stands. So, this is the signal we were discussing.

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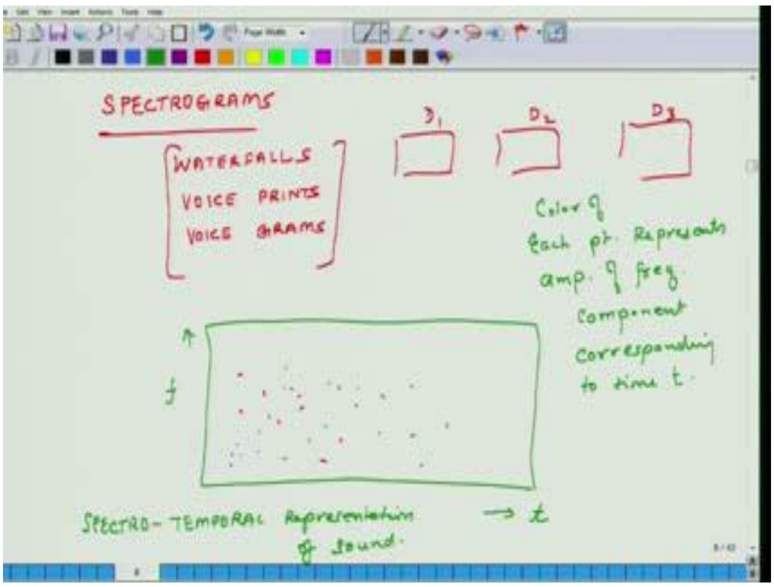

So, the next topic we will cover is spectrograms. So, these spectrograms come by different names. They are also known as waterfall diagrams, or waterfalls. They are also known as voice prints, especially, if you are synthesizing and analyzing voice samples. They are also known as voice grams and the concept here is pretty much identical to what we had discussed in context of short term discrete Fourier transforms, but the nature of representation of these graphs is, or this data, is different. So, in short term what we had done was, we had generated a large number of graphs, you know, for different durations; so, duration 1, duration 2, duration 3, and so on and so forth.

So, you have, you have to make 12 different graphs, and then, you have to look at those graphs carefully, and interpret, and see what sense you make out of them. Spectrograms, you know, you compress all that information into one single picture. So, in the spectrogram, what do you have? You have, on this x axis, you have frequency; or, you can also have time. So, the one which we are going to see will be frequency, time; on the y axis you plot frequency. What do you plot here? Here, at each point, you plot different points. So, you plot all sorts of points; each point, point represents amplitude of frequency component, corresponding to time t, so each point and the color of the each point. So, I should also written, color of each point, each point represents. So, if it is…

So, if your scale is such that, red is extremely large and blue is extremely small, then, red will represent that, the amplitude is maximum; and, blue will represent amplitude is minimum. So, this is like a 3 dimensional plot in 2 dimensional spaces. So, this is what this is, ok. And this, these are also known as, there is some fancy terminologies, Spectro-temporal representation of sound. So, what is a spectro? You are plotting, having the frequency information also and then, tempo will mean, you are also giving time information through this is spectrogram.

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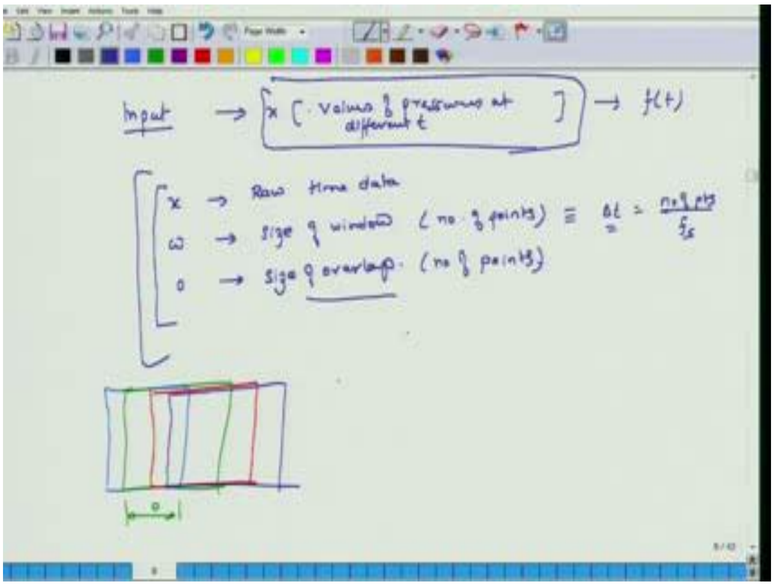

You can generate this spectrogram; your input will be; what is your input going to be? Input is going to be a string of numbers or what are these numbers? They are basically, values of pressures, which you have measured. So, this is basically a function of time. So, these are all the values of pressures at different values of times, different t. And then, using this windowing method, you generate the spectrogram. So, what I am going to show you, I have some, a bunch of spectrograms for 2 different functions, and in… While you are generating this spectrogram, there are 3 or 4 important parameters you should think about. So, the first parameter is x, or... So, that represents your raw time data.

Then, the next important parameter is, let us call it w. So, this represents size of window and typically... So, so how long your window has to be? Is it going to be half a second long? 1 second long, and so on so forth; but you do not typically represent the size of the window in time, but rather, in terms of number of points. So, if you are sampling frequency is, let us say 1000 hertz, then, you say size of window equals 500; that will mean that, the duration over which you are going to do this thing, will be half a second, right. So, this corresponds to, delta t is equal to number of points divided by f s.

So, from this, you can calculate number of points, if you are more interested in delta t.

And then, the third one is o; this is size of overlap, size of overlap. So, what, what is this, and this is, again in terms of number of points. So, you can have window 1; then your second window can be like this. So, whatever is the number of points in this range that is your overlap size, this is your o. Then, your third window could be this; your fourth window could be this; and, so on and so forth. So, this is also, overlap, size of window, all these are specified, at least in context of MATLAB, in terms of number of points.

And then, the spectrogram, you can plot it in all sorts of ways. So, what we will do is, we will actually look at some of the results. So, this is the function; this is the function we have discussed, and what we are going to do is, we are going to look at its spectrogram for different cases. In first case, we will have half a second size of the window, and there will be no overlap.

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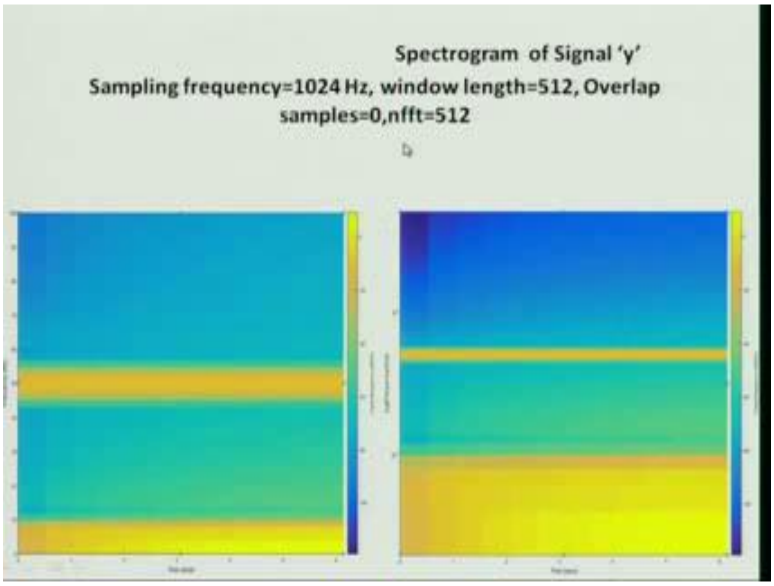

Then, we will look at... So, this is the spectrogram, and we are seeing 2 pictures, and I will explain you; but, both of them are for half a second. See, it says window length is 512; sampling frequency is 1024, which means the window size is half a second long. And, it says overlap samples is 0. So, both of them are overlap samples is 0. The only difference here is that, in this case, my y axis is frequency in a linear scale, and here, I am plotting on a db, here I am plotting on a db; that is the only thing.

So, here, I am actually plotting it on a, you can, do not worry about d b, its basically on a log scale, and x axis is time. So, what, what do you see? So, here, only at low end, I have only 3 frequencies totally. One is t is equal to, f is equal to1 hertz, 5 hertz and 50 hertz. What do you see here? This entire band is yellow. So, ignore this one, because I do not see frequencies, at low frequencies clearly. But, if you look at this log scale plot, then, this is; no, this is 1 hertz, and this is 10 hertz. This tells me that, all the frequencies are present between 0 and 10, which is wrong. It is wrong because, my duration of time was very small. So, I am losing frequency related information. I get a good frequency related information at 50 hertz; this is 50 hertz, because, the time period for 50 hertz is very small, compared to the duration of my window. So, I get decent information here. So, I am losing time information here; and herein, overlap was not present. Now, you…

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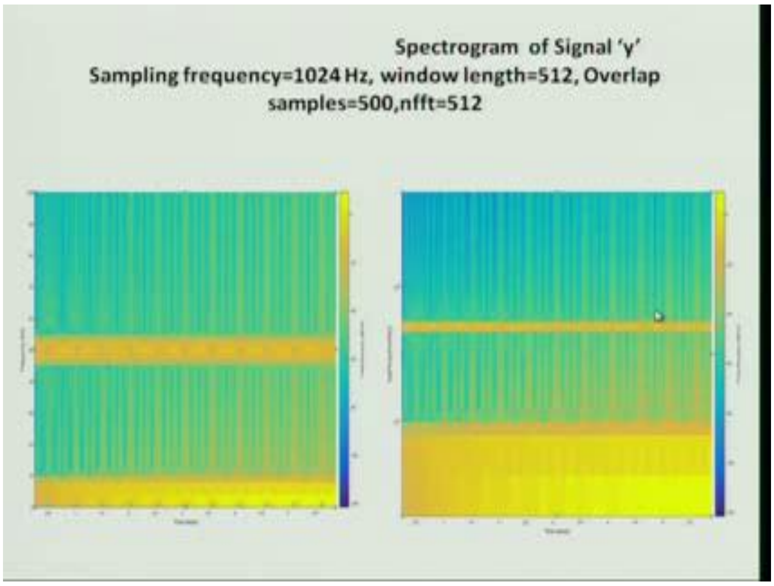

And, this is the same graph, but overlap was present. So, you see some, probably more gradual transitions of... So, I am not going talk too much about that, but here, this is the same plot, but overlap was more. There was overlap, so, it is moving gradually rightwards.

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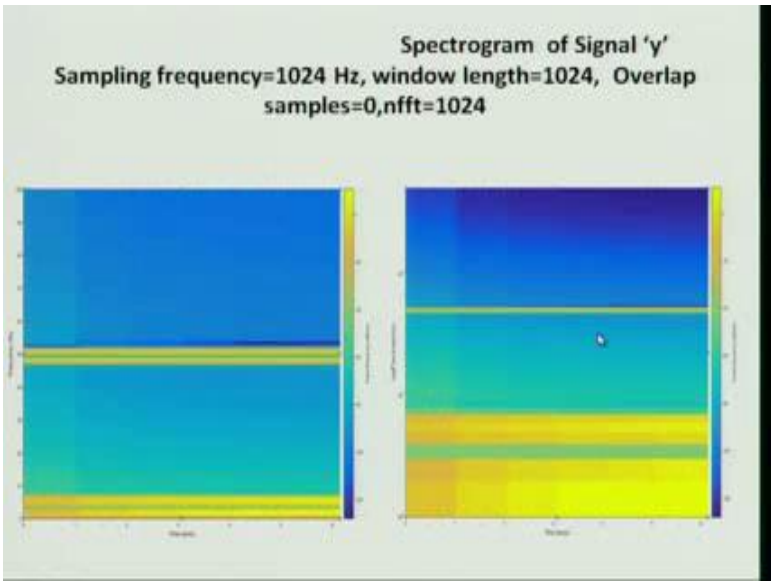

Then, the other one is, sampling frequency was 1000; window length was 1 second, or 1024 points. And then, again, I will look at this; and when the, this thing, becomes1 second, you see first is that, you see 2 lines here; at 50 hertz, you see 2 lines. Yellow means, its intensity is more; this is more, this is less. This yellow, this color is also on a db scale. So, this is 0 db; this is minus 120 db. So, 120 db is extremely small. So, this yellow means more, which means that, there are 2 frequencies present, which is actually the case, which we had seen earlier.

Then, the other thing is that, around 5 hertz, you see one band; then, you have a green, right, which means there is no frequency at this end. And then, you have some more information here. So, you see some, 2 frequencies here, third frequency here, fourth frequency here. But, here again, it is a still pretty wide; actually, this log scale should have been, maybe, we should have made it better. We should have started from 10 to the power of minus 1, then, I could have clearly seen 1 hertz.

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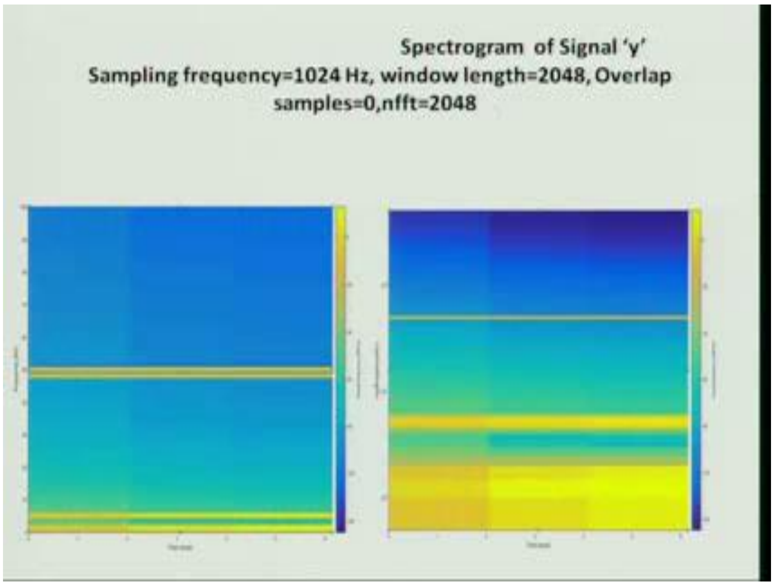

Then we look at the third graph, and here, the window length is 2048 points. So, since our sampling frequency was 1024, this is 2 seconds long. And here, what you see is, there are 2 frequencies here, very close to each other. Then, there is a third frequency at around 5 hertz, and then, this is 10 to the power of 0 which means 1 hertz. So, you see a dark color here; you know, between this point; and then, here it becomes lighter progressively. So, there is some energy at around 1 hertz. So, you get more information for 1 hertz, when you increase the sampling size, which is what we expected. but then, what you see is that, the time related information is getting increasingly blurred, because, I have one, this whole is one window, 0 to 2 seconds. So, nothing is changing in 0 to 2 seconds.

But in reality, the amplitude has grown for the first frequency from 0 to 2; because, its amplitude was t, but that is not shown here. But, if you go back, in the first case, you see that amplitude is growing; you know, this is, this is orangish, which is minus 20 db, and this is yellowish, which is 0 db; so, amplitude is growing. So, time related information is more present in this graph, when your sampling frequency is, when your window size is small; the time related information you are losing. So, you, this entire thing is homogeneous; then, you have another homogeneous color; then you have another homogeneous color.

So, you lose time information, but you get frequency information more. So, this is, again, to show these plots, and then, the second set of graphs which I am going to show you, is for this function; y equals 3 sin 2 pi t over 2. So, here, the amplitude is constant. The only thing which is changing is frequency, as a function of time; initially, frequency is small, and then, frequency keeps on ramping up. And, what we have done is, again, generated the signal for 6 seconds, at a sampling rate of 1024.

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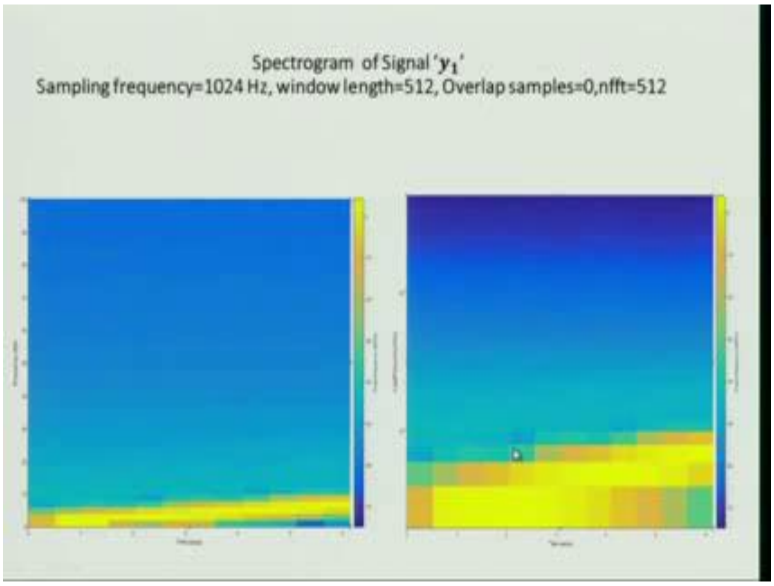

So, this is my window length, when my window is half a second long, or 512 points... So, again, if we look at this graph, not necessarily this graph, but on the decibel scale graph log where we had the frequencies on the log scale, you see that, you have a lot of time related information, but especially at, or for the frequency, you do not see a whole lot of stuff.

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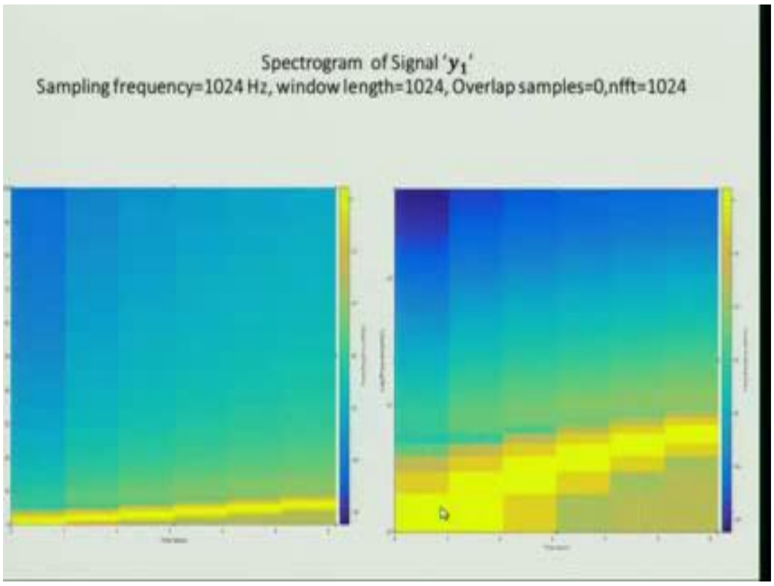

And then as the, as this window length increases then this time related information starts getting blurred, and you get more and more information about frequencies, because the resolution on the frequency… see, see, here on the y axis, resolution here is less, but here, you get more finer bands.

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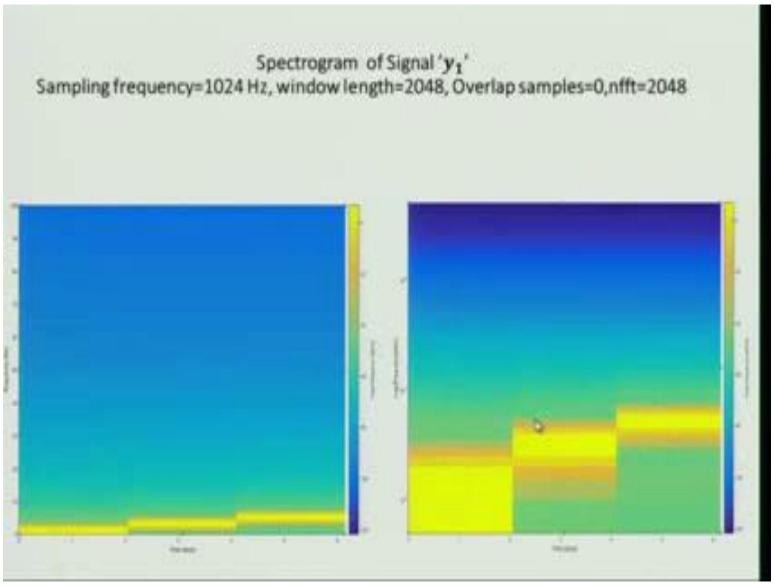

So, you get finer bands. . So, you get more information on resolution; here, the bands are even finer. So, you get more information on frequency. So, these are the 2 things I wanted to share, and I hope that, these concepts and tools, for spectrogram and a short term DFT, they will be useful for your applications. And, what we will do tomorrow... So, this concludes our lecture for today, and what we will do tomorrow, will be something related to reverberation time, and concepts related to reverberation.

So, thanks a lot for patiently listening to this, a little long lecture, and we will meet tomorrow and have great day. Bye.