Basics of Noise and Its Measurements
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Lecture – 08
The Momentum Equation

Hello again, welcome to Basics of Noise and its Measurements. I am Nachiketa Tiwari, this is the second week of our course. Today, what I am going to talk to you about is The Momentum Equation. As discussed and explained in the last class, what this equation is all about is mathematical expression, but differential form expression of newtons second law. And what we will do is, that we will apply basically the same law to a small piece of gas, as it experiences differential pressure over space, and develop a relationship between it is motion, and the forces it experiences on it is boundary. So, that is what we planned to do today as I mentioned the topic or the theme of today's module is the momentum equation.

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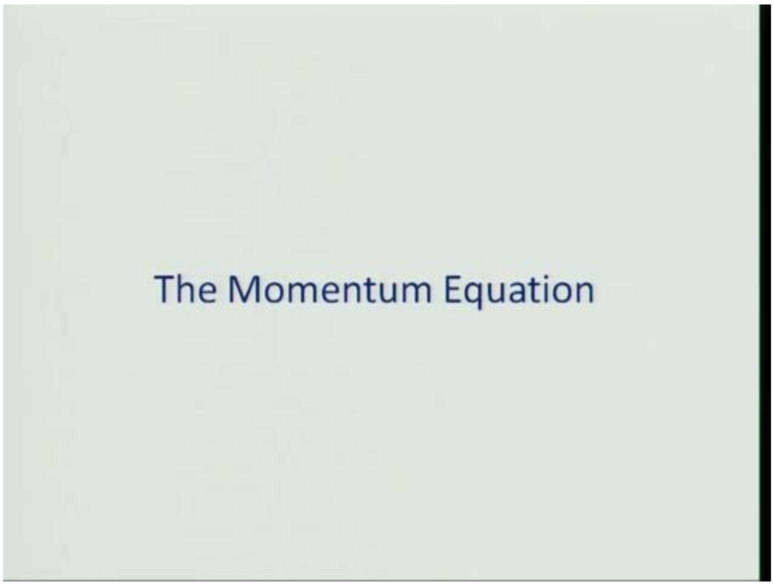

So, what I am going to do is actually, draw a material volume.

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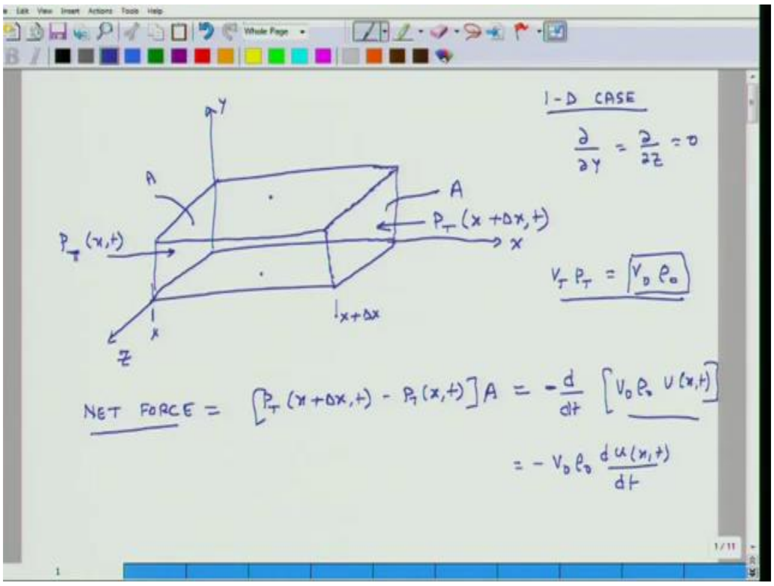

These are my 3 axis; this is my x axis, this is my y axis, and that is the z axis. Let say that, this volume is; let say this coordinate of this particular point is x, the coordinate of this particular point is x plus delta x, and this cross sectional area is a same thing here cross sectional area is a and what we are the way we are conceiving this experiment is that imagine there is a coordinate frame, and with respect to that coordinate frame you have a small prism; rectangular prism of air or some fluid which as rectangular cross section and you are looking at this material volume at a particular time.

Let say, time is t naught and then at that particular time you are taking a lecture camera, use camera and you are look taking a picture of it. And you are seeing using that camera or some sensor you are detecting what kind of forces, are being applied to this material volume. So, because this is 1 decade there is no variation on this surface and this surface at least in context of the pressure which is being applied. So, the only variation of pressure which this material volume experiences is in the x direction, because we have already assumed that this is 1D case, one dimensional flow.

So, because of this partial of pressure with respect to y and partial of pressure with respect to z equals 0. So, on this surface, let say and again because this is a fluid and we are also assuming in this case, that the fluid is inviscid. So, it cannot take any shear forces. So, because of that the viscosity is not playing any role. So, the only forces it can bear are pressures it cannot be a shear forces. So, you have an external pressure, PT on it is left phase and that pressure is a function of x and t. So, it should be actually capital p and then on the right phase you have another pressure, and that is also PT, but p is here as I have explained it is a function of x.

So, the coordinate x coordinate here is x plus delta x and because the time is frozen. So, it is time is t. So, this is the overall situation on my left phase I am having a pressure of PT, which is a function of x and on my right phase I am having a pressure PT, which is different because the value of x is not x, but x plus delta x. Now, using this picture I can apply newtons second law, and what I find try to find is what is the net external force on this material volume and then I equated with rate of change of angular momentum. So, net force equals PT x plus delta x minus PT at position x. So, that is the difference in pressure and since the cross sectional area, I am assuming that it same.

So, if I multiply this difference by area, which is a, then that is the net external force and this force as to equal rate of change of momentum. So, I do a d over dt and the mass of this fluid is basically volume VT times density. So, that is my volume time density is mass, and then if I multiply it by velocity U(x, t). So, that is the momentum the other thing I should mention here is, that because the pressure is acting on the right phase in the negative direction, and in the left phase it is acting on the positive direction.

So, I have to put a negative sign here to a count for this direction to account for the fact that we are taking about pressures and not which act on the invert size. So, with this, this is my first version of ah the momentum equation and now I am going to process it further and bring it to ah something more tangible. So, the first simplification what I will do is, that we had assumed that, this particle is having a constant mass. So, VT rho T is same as v naught rho naught because of the constant mass particle mass assumption. So, as it moves it does not lose any mass. So, this thing it becomes instead of I can replace it by V naught and rho naught.

So, this is the first change I have done the other thing is. So, once and then we should also note that V naught, rho naught is constant. So, I can take it out minus V naught, rho naught d u (x, t) over d t. Next what I do is, I expand d u over d t.

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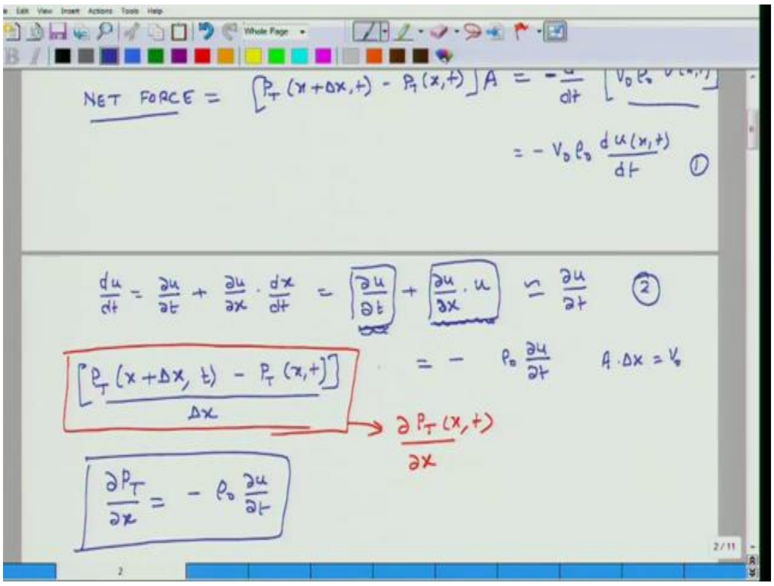

That, I can express because u is the function of x as well as time. So, I can express it as partial of u with respect to time, plus partial of u with respect to x times d x over d t. And please note that x is actually the position of this particle. So, the rate of change of this position with respect to time is nothing, but velocity. So, this I can again express it as, del u over del t times u.

So, I will number this equation as equation 1, and now at this stage we know that partial of u with respect to time is a small entity because we have this smallness assumption. And then over I think I made a mistake partial of u with respect to time is a small entity, partial of u with respect to x is also a small entity, and u itself is also is a small entity. Because in this course we are assuming that all use and pressures and their derivatives all of them are relatively small.

So, if that is the case then this term because it is a product of 2 small entities, is extremely small compared to this term. So, then this approximates to del u over del t. So, this is my equation 2, and I plug this equation 2 back into one. So, what I get is PT x plus delta x t minus p t x t a is equal to minus V naught rho naught del u over del t. Now the next thing what I am going to do is, I am going to multiply this and divide it by delta x. So, once I do that a times delta x is nothing a times delta x is V naught.

So, I am going to erase this and I am going to replace it by V naught, and this V naught and this V naught can cancel each other. So, I am going to erase both of these guys and what I am left with is, and I will move this thing from the denominator here. So, this is my modified equation and if I take if I take the limit of this entire term which is in the red box, as delta x approaches 0 then this thing becomes partial of PT, which is a function of x and t with respect to x. So, essentially I can rewrite equation to as del PT over del x equals minus rho naught del u over del t. Continuing further, now I had explained earlier that pressure which is total pressure which is a function of position and time is nothing but ambient pressure.

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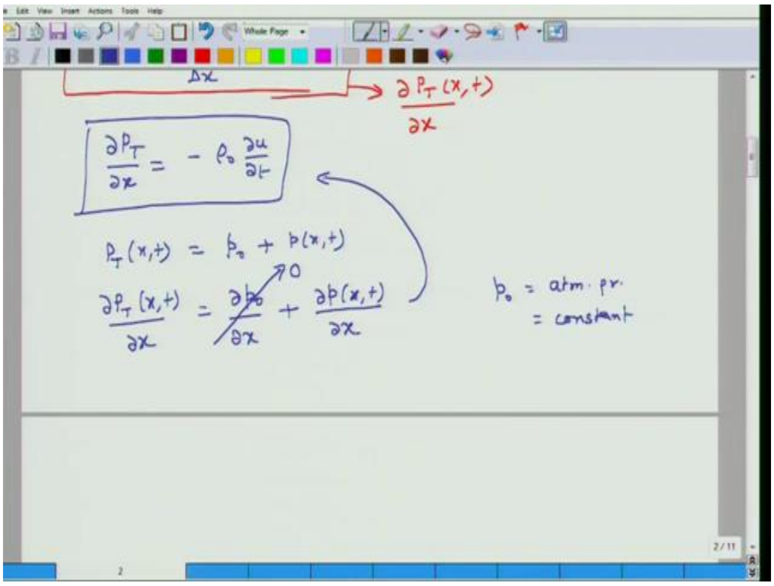

When there was no sound plus some small pressure fluctuation. So, essentially what that means, is that partial of PT, with respect to (x, t) over x is nothing, but partial of p naught with respect to x plus partial of p with respect to x. And because p naught is atmospheric pressure and this was a constant, that is our assumption. So, this term it goes to 0. So, I can use this equation back end to this and what I get finally, is del p over del x is equal to minus rho naught.

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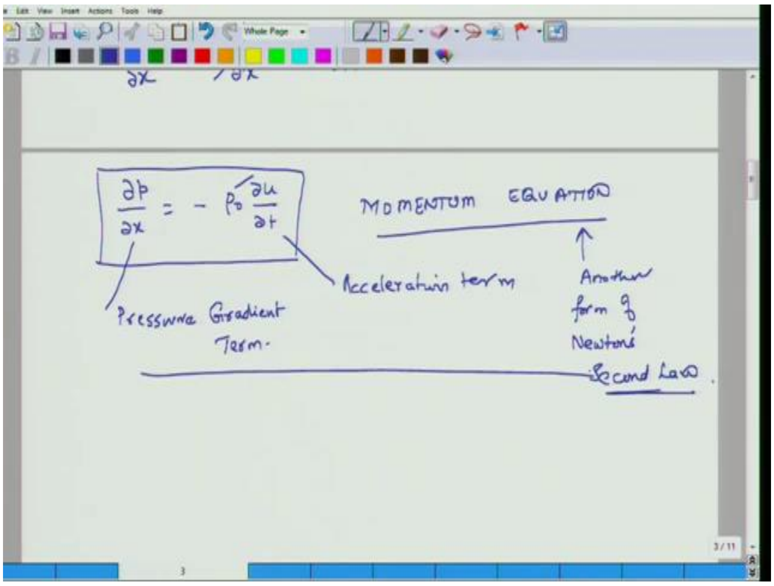

I am just copying this equation back here. So, minus rho naught del u over del t. So, this is the final form of my momentum equation. Essentially what it tells me is, this is the acceleration term and this is the pressure gradient term. So, essentially what this equation tells us is that the acceleration of a fluid particle directly depends on the gradient of the pressure across the fluid particle, the small fluid particle.

Let us that what it means and of course, there is a constant factor which is the density if density is higher the acceleration will be less if density is more and so on and so forth. So, that is what it implies and this is nothing, but another form, another form of newtons second law. So, that is the momentum equation and that is what we had and we were to capture in this particular module and in the next class, what we will discuss is the other 2 equations the gas law and also the continuity equation.

If you have any questions please let us know, please contact us through all the emails and other contact links which have been provided to you and we will be more than happy to address your questions if any. Thank you very much and look forward to meeting you tomorrow.

Thanks.