Basics of Noise and its Measurements
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Lecture – 09
The Continuity Equation & The Gas Law

Hello, welcome again to Basics of Noise and its Measurements. In today's module we will be discussing two different equations.

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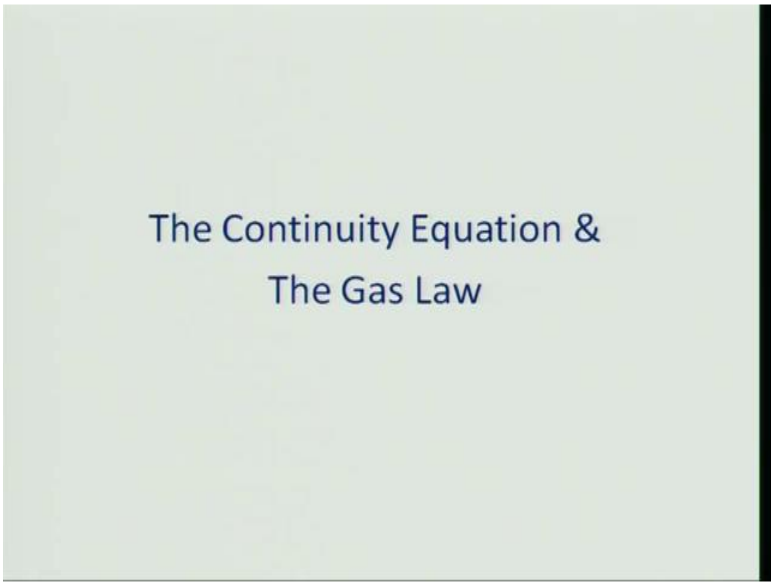

The first one is the continuity equation and the second one is the gas law. For both of these relations, we will be developing their, excuse me differential forms. The continuity equation will essentially capture very basic aspect of nature's reality that is law of conservation of mass. Through gas law, we will essentially try to capture the, behaviour of the material, in this case it is gas and how it behaves as it expands and contracts and undergoes a specific thermodynamic process. So, we will start with the continuity equation and we are going to develop it.

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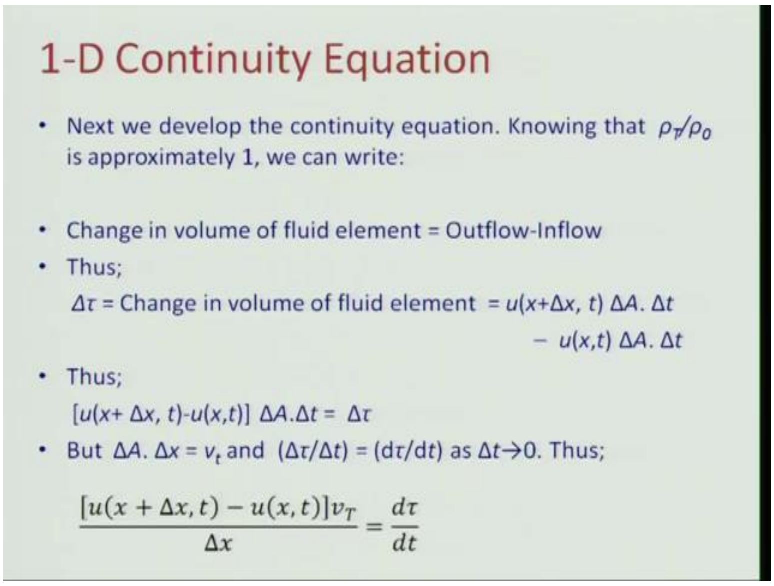

There is a lot of supplementary material in form of power point presentations. So, if you have any questions or need help you can always refer to these power point presentations which are there for your help as supplementary material.

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So, let us start developing the continuity equation.

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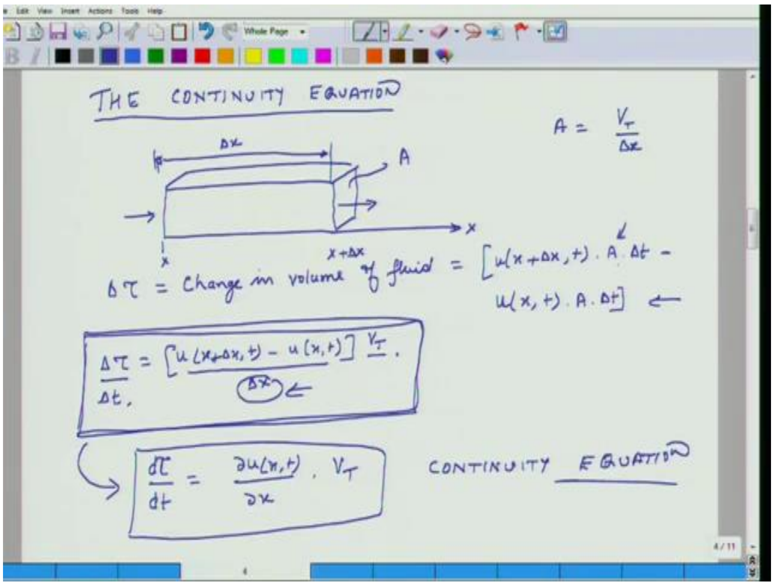

If I have, a 1D tube and some fluid is coming in and it is also going out. Then what essentially this continuity equation or law of conservation of mass says that whatever, what it says is, that whatever mass is coming in whatever mass is going out, if I take the difference of these two masses per unit time then that is the difference if it is not 0, then it will be same as the amount of mass of fluid which is get going to get accumulated in this volume. That is what it means.

Now, we have assumed that this is a constant mass particle. So, in that case, change in volume delta t; delta tau is basically changes in volume of fluid and that equals velocity u which is a function of x plus delta x n time. So, if this is position is x let say this is my x axis, and this is position x this is x plus delta x, and ux plus delta x t times area let say the cross section of this element is a. So, ux plus delta x comma t times area. So, that is the flux which is going out. If I am integrating it over time t then the total flux which goes out, is u times a times t and then whatever is coming in I have the similar relationship. So, please remember that these are lower case use, not capitals times a times delta t. So, this is the basic form of continuity equation.

Now, this is nothing, but the volume of element v and I can take it as VT divided by delta x, which is the length of element. So, this is my length then the total volume divided by delta x is cross sectional area. So, I can re express this relation as delta t equals u x plus delta x t minus u x t VT over delta x times delta t or I can divide this whole thing by delta time by time delta t.

What I get is I get in the denominator t and this I get again delta time. So, this thing goes away. So, I can erase it, and another transformation I can do is I can erase that delta x and bring it inside. So, what we are doing is, that we are taking this delta x inside the parentheses and as a consequence this is my modified expression for continuity. Now remember that delta x is a finite change or finite length of the material volume, and delta t is again finite time interval.

Now, what I can do is that there is nothing which stops me from shrinking the size of this material volume to as small as possible. So, I can make this delta x as small as possible, in the limit that the length is going down to 0. And is the same thing for delta t also that because, as this x becomes smaller and smaller the change of volume will also happen over an extremely small period of time. So, both these things become extremely small, and as a consequence essentially what this does is that I get d tau over d t equals del u which is a position and time dependent entity which is u over del x times VT.

So, that is my continuity equation. What it change says is that the velocity gradient in the x direction is, directly influences in a positive way the change rate of change of volume of the material element which we are observing. So, if the velocity gradient positive volume rate of change of volume is also going up and so on and so forth. So, that is the continuity equation. The next equation we are going to discuss is the gas law actually it is differential form.

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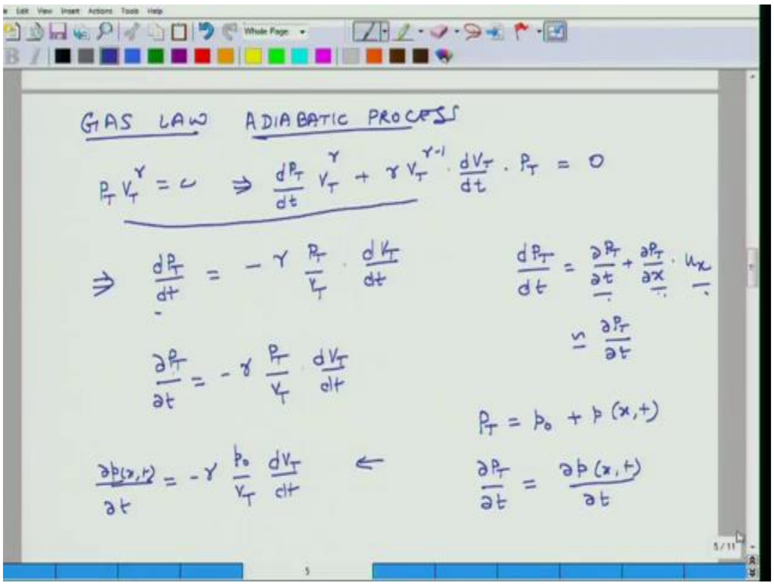

So, because this material volume is expanding in contracting because it as different pressures at both ends remember, I mean there is pressure p as a function of x and t on the left hand and p as a function of x plus delta x and t on the right hand. So, it will have different pressures on different faces, and because the pressures are different that piece of gas will either expand or contract and because these pressures are changing very rapidly, the expansion and contraction is also happening of the gas volume also happens extremely rapidly and as consequence.

What happens is that there is very little time for the heat which is generated or absorbed because of this consequence, because of this expansion and contraction this heat which is generated or absorbed it is very a difficult for heat to either come out from the system to the outside or it is very difficult for heat from outside to come in to the system. Because the time scales involved with fluctuations of these pressures and volumes are extremely small. So, in a sense what is happening is that this is the process very similar to adiabatic process.

So, for an adiabatic process we know that total change total pressure in the gas and it is volume to the power of gamma is constant and this I can express it in differential form like this or I can further reform modify this as derivative of pressure with time equals minus gamma PT over VT times total derivative of volume with respect to time. Now, this total derivative of pressure with respect to time, I can break it up into it is partials.

This is partial with respect to t plus partial with respect to x times dx over dt and once again because dx over dt as discussed earlier is nothing but velocity. So, I can erase it and I can call it u, and for the same reason that this is a small entity this is small entity this is small entity and then products of these two small entities are much smaller than this one. This can be approximated as partial of PT over time. So, the right left hand side of this equation becomes partial of PT with respect to time equals minus negative of gamma PT over VT times dvT over dt the next thing we do is we realise that pt is equal to p naught plus p of x and t. So, partial of PT over time is nothing, but partial of pressure fluctuation over time.

So, my gas law again gets modified as it is left hand side gets modified, as partial of p with respect to x with respect to tau, with respect to x and t is equal to minus gamma PT over VT dvt over dt. What else I also know that PT equals p naught plus p of x and t and this lower case p is extremely small compared to p naught

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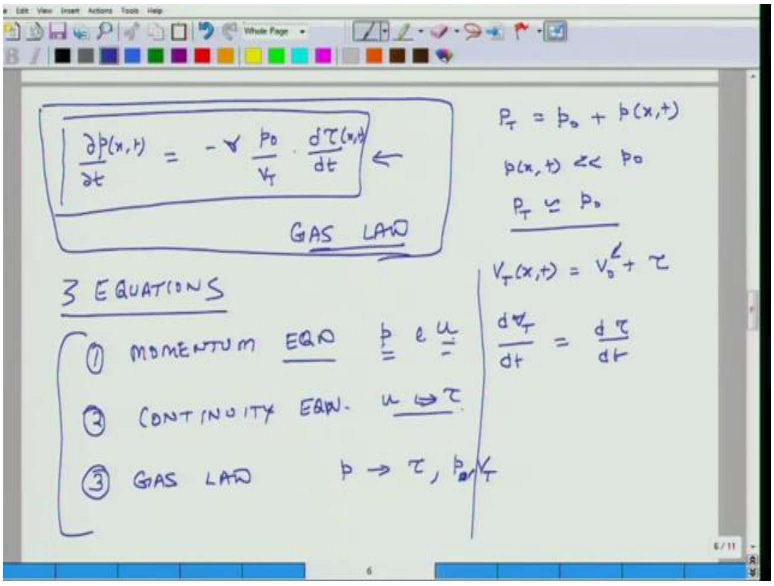

So, PT is approximately equal to p naught. So, I take this relation and put it back into this one. So, what I do is I erase this and I replace it by p naught. The other thing is that, VT which is the final, volume of the gas element which is a function of position and time is nothing, but original volume plus change in volume because this is constant. So, what it means is derivative of volume with respect to time is nothing, but derivative of change in volume with respect to time.

So, my gas law becomes partial of p with respect to time, and pressure is the function of x and t is nothing, but minus gamma p naught over VT times rate of change of volume with respect to time. So, this is my gas law, and again here tau is again a function of position and time and this gas law is actually a differential form of the adiabatic gas equation. The integral form of adiabatic gas equation was this and what we have done is we have differentiated it and also embedded some of the assumptions which we started with and this is what we have come to.

So, here we have assumed that the process the process of compression and expansion of gas is adiabatic. Had this process being a different one then we would have a started with a different initial equation. So, here we started with PT, VT to the power of gamma equal c, but if in case we the reality said that it was an isothermal system then we would have started with the different equation and we would have got in the same thing, but this assumption that the process is adiabatic, it actually is very consistent with our actual measurements of velocity of sound and because of this our assumption that this process, which the gas follows as it propagates in form of sound as sound propagates through the medium the gas obeys the adiabatic process. This particular assumption is borne out by significant amount of experimental data. So, this is the gas law, we are going to use in developing the wave equation.

So, what we have done till so far is we have developed three equations the first one is the momentum equation and that connects pressure and velocity and also density, but changes in pressure to changes in velocity. The second one is the continuity equation, and that connects velocity gradients with time variation of volume. Actually in this one density or a third variable does not come into picture. And the third one is the gas law, it is differential form and this connects time rate of change of pressure partial derivative pressure with respect to time, and it connects it with rate of change of volume and of course, the other two parameters involved are ambient pressure, and overall volume.

So, these are the three equations, we have three variables lower case p, lower case u and lower case tau. So, using these three equations, we solve them we can get all the three variables using this three equations. What we will do in the next lecture or module is we are actually going to eliminate two of these variables and develop an equation for pressure single, one variable equation pressure and also one variable equation for velocity. That is what we are going to term as the wave equation. So, thank you very much and look forward to seeing you tomorrow.

Thank you.