Nature and Properties of Materials

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Lecture 30

Cantilever Design (High Strength and Light weight)

So in the last class we have talked about the material selection for a cantilever beam and at that time our objective was to get a highly stiff but a lightweight cantilever beam. Now in this class, what we are trying to do is to see the same problem but in a different light. Now we are penny to select a material where a similar cantilever beam with circular cross section and it is loaded at its end, but now our objective is that this beam must have high strength and lightweight.

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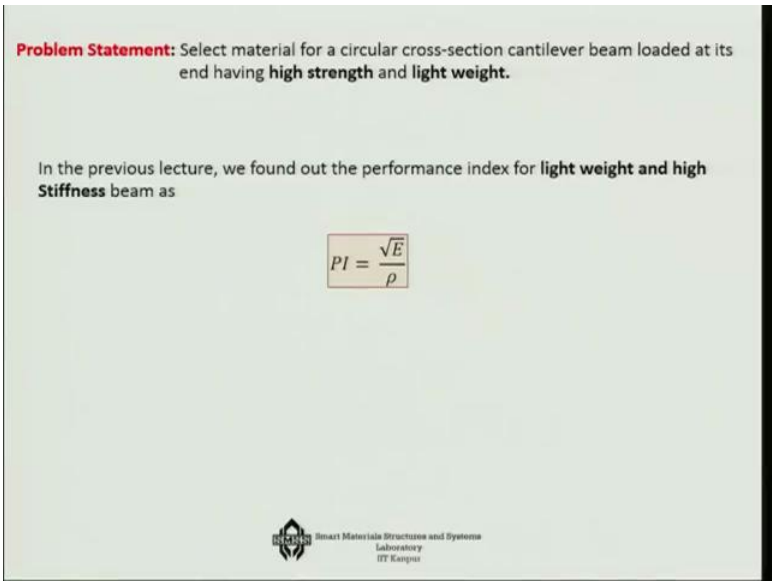

So if you remember that in the previous lecture our cantilever beam problem had the material property index which was actually Rho over square root of the in fact, performance index was square root of E over Rho. So now let us see that for this new problem in which we said that you have to have a high strength not high stiffness and lightweight, how this situation may change. So let us try to go to the board to explain this problem.

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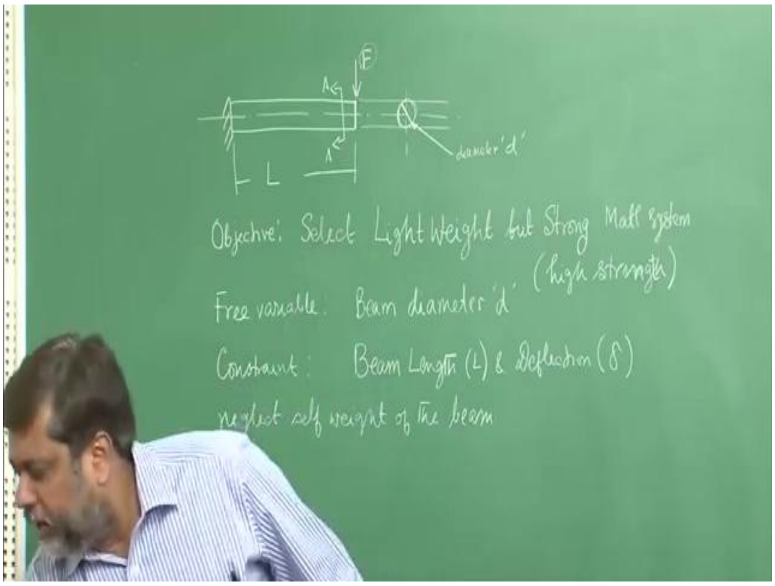

So here we are again with a cantilever beam, let us consider that the beam has uniform cross-section and if I cut any section in the beam, then I will find out that this is circular in nature okay, so the cross section is circular in nature. Let us assume that this particular the diameter let us say the diameter is d in this case okay. And this circular beam has end load F okay and our objective today is somewhat different from the last class. Now we are not bothered about the deflection, but we are bothered about the strength that the cantilever should not fail.

So our objective is that select lightweight, but strong material system that is the material system which has a high strength okay, so that is what our objective is. And keep in mind that in this case the free variable as usual is the beam diameter d, the length of the beam we cannot touch that is plane L. And what is our constraint then? Beam length L and let us say deflection of course deflection would not come into picture in this particular case, but beam length and deflection Delta.

Now, in this case the beam should not fail so that means I have to find out that what is the maximum stress that can generate in this beam due to the effect of the this force F. Let us neglect 1st of all, let us for the simple engineering analysis neglect self weight of the beam. So with this condition let us try to find out that what is the maximum stress that can come out in this particular beam. So let us try to find that out using basic basics of strength of materials.

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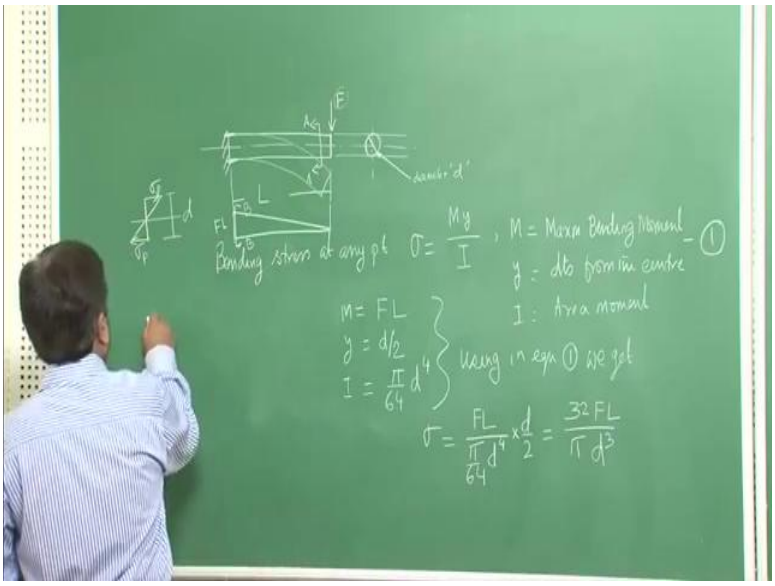

So we try to use the basics of strength of materials, we try to find out that what is the stress in this case which is critical as it is evident that the beam is undergoing bending, so it is the bending stress which is critical and that can be defined as at any point by this simple relationship that Sigma = M Y over I, where M we will consider it to be maximum bending moment and Y that is perpendicular distance from the centre and I is the area moment okay.

So with this configuration and with this situation let us call this be the 1st equation that Sigma = M Y over I. I have to find out of work that what is the maximum bending moment that is possible in this beam. So if I try to draw the bending moment diagram of the beam, now we know that maximum moment in this case will be coming at the fixed end and it will become 0 at the other end. The maximum bending moment here the magnitude is F L. Of course, the deflection of the meme is downward so which means if I consider very close to this support let us say B B.

If I look at the B B, that is the state of stress, then I am going to see that this is the bending stress in which the maximum stress is, so this is what is our the total distance that is what is the diameter d okay and the maximum stress is going to happen at this particular point okay that is the Sigma peak. So Sigma peak is going to happen maximum stress at the 2 extreme fibres at the 2 edges of the beam. Now with that understanding, we can now write down the all the numerical values that M = F L in our case okay that is the maximum bending moment, Y = d by 2 that is the farthest point and of course I = pie by 64 d to the power 4.

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So as a result, if I use all these things using in equation 1, we get the new relationship as a function of the that we get Sigma = F L over pie by 64 d to the power 4 times d by 2 that is it will come out to be 32 F L by pie d cube right. That is what the stress versus the rest of relationship, in other words I can write that d =, I can write the equation here which will be useful for us further. The d = 32 F L over pie Sigma, this is the Sigma peak, Sigma peak to the power one third that is what is my equation, right. So I get what is the maximum stress and I can express the free variable d with respect to this maximum stress that will be coming to the system.

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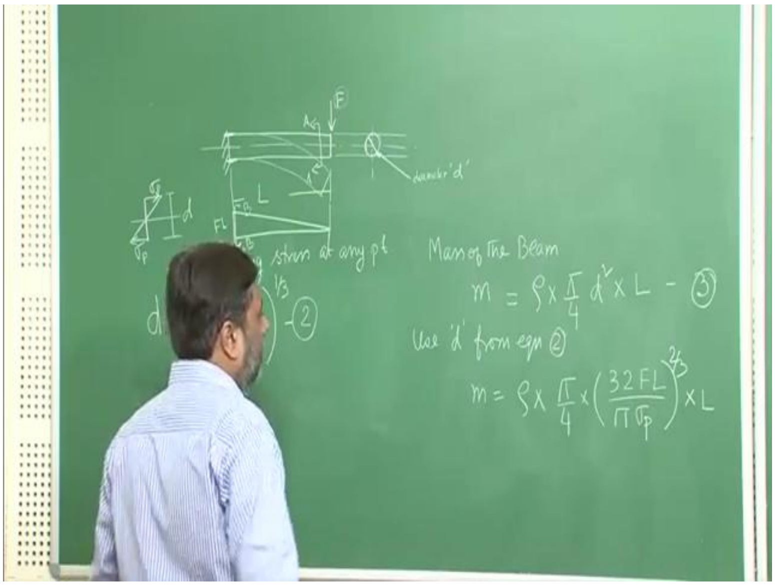

Now, our objective is to also get a lightweight beam okay, so we have to bring mass into the picture just like the last lecture, so I am erasing equation 1 now because equation 1 you have already understood and all these details related to it and I am working on the mass of the system Now, mass of the beam m = Rho into pie by 4 d square into L right, that is the mass of the beam that is what is my 3rd equation. So, I can use d from equation 2 that is the expression of d. So if I use it here then my aim will be Rho times pie by 4 times 32 FL by pie Sigma peak times we have pie by 4 d square, so d is one third, so this is two third times L, right that is what is the mass of the system.

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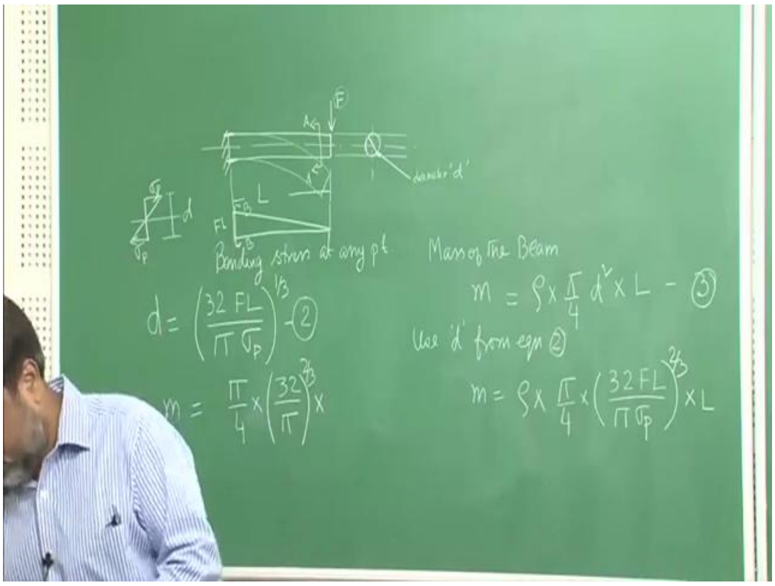

Now let us try to divide this expression as usual like the last class into several segments and an isolate that part which is related to the material property of the system. So if I try to do that, then I will get the relationship as L = pie by 4 times 32 by pie I can take it out times two third. So this is one part multiplied by let us choose another colour to write the other part that is L to the power two third and + L so 5 third.

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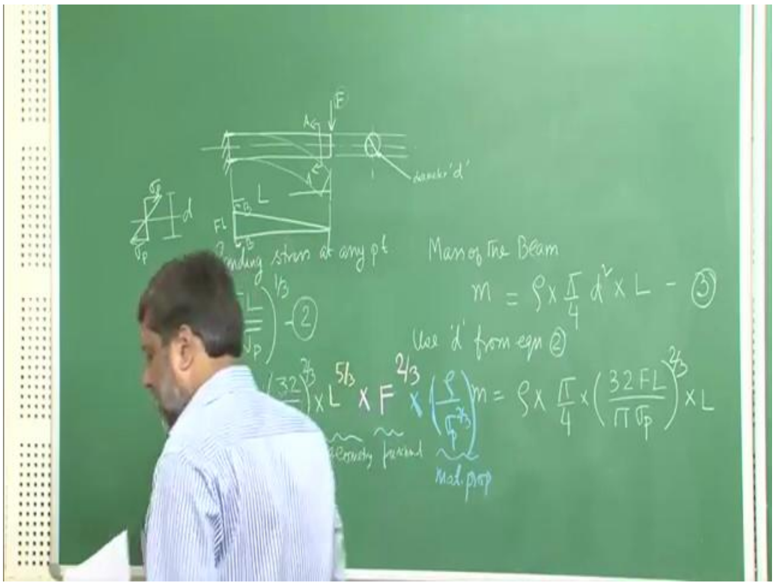

And then I put another part using another colour let us say, the functional part of the system into F to the power two third that is the other part and finally we are finally boiling down to the materials part of the system, so that we can write it as multiplied by the Rho over Sigma p to the power 2 third okay, so that is what we have that means if I try to write it down this is the constant part, this is the geometry part, this is the functional part or the loading part and the last part, this is the material property.

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Now you may tell me, what is the Sigma P how that can be the material property when Sigma p is the maximum stress that is coming to the system. And for many safe engineering designs, we can at most take the Sigma p particularly if it is linear elastic system, et cetera, Sigma p at most = Sigma y. Or in some cases if you keep a factor of safety, then you can write Sigma p = Sigma f by providing suitable factor of safety. So whatever we will say here, the performance index in this case is actually Sigma y to the power two third divided by Rho.

In other words, if you want to minimise this mass further and further, you must choose a material which has high strength and low density. Now, the relationship is not very simple that you just increase the strength 2 times and you get 2 times change in the reduction in the mass, no there is nonlinearity that exists here so it is two third of the whole thing and similarly the density. So that where we need to once again draw this kind of a logarithmic curve that suppose I say that the minimum constraint value of this is constant value is C, then I can write that log of Sigma y = 3 by 2 log of Rho + 3 by 2 log of C.

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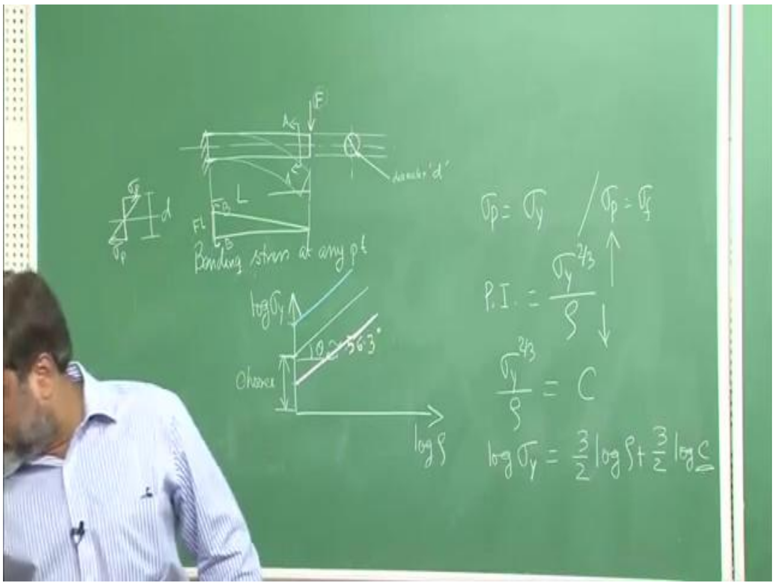

In fact, if I try to visualize this graphically, so let us erase all these parts now because we have the material property here with us, but if I try to plot this log plot of Sigma y versus log plot of Rho, then we are talking about a line here whose slope theta will be approximately 56.3 degree and of course the intercept is something that can be chosen okay, so you can choose it okay, but the slope of the line so what it means is that with certain values of C you can either start from here or you can reduce the margin or you can increase the border. No matter whatever you do that change will be here, but the slope of the line is going to be parallel.

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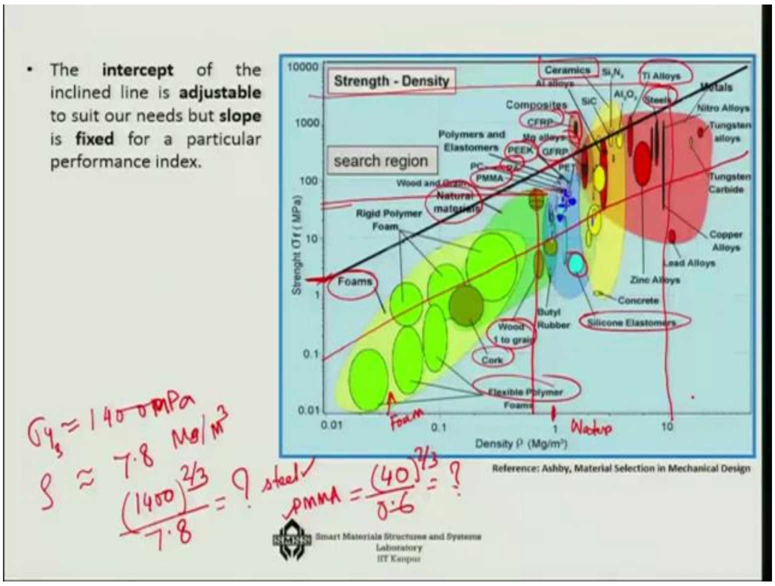

And that slope is determined by this relationship which says that that slope has to be 56.3 degree okay. So let us look into the Ashby chart that how this slope will look like. So if I now go back to the Ashby chart, now if I go back to the Ashby chart we can see that this is the strength versus the density plot and we have many possibilities here. Let us look into once again the materials, in low-density side we have materials like the foam group which has very low-density, so this is the border case 1 mega gram per meter cube that is what the water density is, so all these materials are lighter than water so these materials are possible provided my line is below.

So if I draw any line corresponding to my minimum corresponding to my minimum value of C, if my slope line goes like this then some of these corks some of the woods, flexible polymers, they come into picture otherwise they do not come into picture if I consider the C value as something like 1MPa or so, then they do not come into the picture in the current condition. And then what comes into picture gradually if you go up, we start to see at the higher density side that we have some of the high-performance polymers like PEEK polyether ether ketone, Polyanalene, PMMA these types of polymers will come.

After that we can see that the blue group, so that is the from the yellow group we go to the plastic group, so this is the this is so to say the foam group and these are mostly the natural materials as it is shown here and then we have the polymers here, some of them are high-performance polymers we get elastomers also, but elastomers also may fail below the line responding on our choice of the minimum intercept. And then above that what are we getting, we are getting CFRPs, GFRPs, etc, this is what we are getting.

Beyond that we are actually getting the ceramics and even beyond that we are getting things like titanium alloys, steel and some of the metals. But if you see I go back far away, I may get a high value of the strength, but I am also going to get a value of the density value Rho. So for example, suppose I go to steel for example, so for the steel something like you can go up to something like 1400MPa it is the Sigma y say for the steel approximately okay. So and under other hand, density is approximately is 7.8 right mega gram per meter cube.

So basically we can then use this relationship that this is 1400 to the power two third divided by 7.8. And you can see that what is the value that you are getting out of it. And similarly you can calculate this for some of the other cases like PMMAs okay, for so this is for steel, you can check it and for PMMA if you do the same thing, it will be something like let us look at PMMA, it is like the maximum value if we take for PMMA, somewhere like 10, 20, 30, 40, so 40 two third and if you look at the density, density wise it is less than water, so it is 2 3 4 5 6 about something like 0.6, what does the value come?

Just check it and see that over PMMA gives the higher performance index than the steel that is the way we choose and finally based on a particular application we consider whether we consider a very high value of Rho or not, you may actually have some kind of a cut-off value of the density and accordingly we select a particular material. This is where I will close in this particular lecture and in the next lecture we will choose another case which is not a single objective, but it has multiple constituents, thank you.