Nature and Properties of Materials
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Lecture 5
Mechanical Properties of Materials 2

In this second section of the mechanical properties of materials, we will discuss a very important concept called Tensors because so far we know about this scalars and vectors. For example, when you talk about any simple numerical counting of a system you talk about below the kind of systems which are scalars. But when you talk about something in the sense of direction let us say force for example then you talk about it in terms of a vector.

But when you talk about stress okay, so we actually refer to stress not like only one direction associated with it, but if you consider for example, for a 3 dimensional space then you have you know something like 9 components of stress. So for this kind of a situation we need a kind of a more generic abstract way of describing the system and that will be in terms of tensors, okay.

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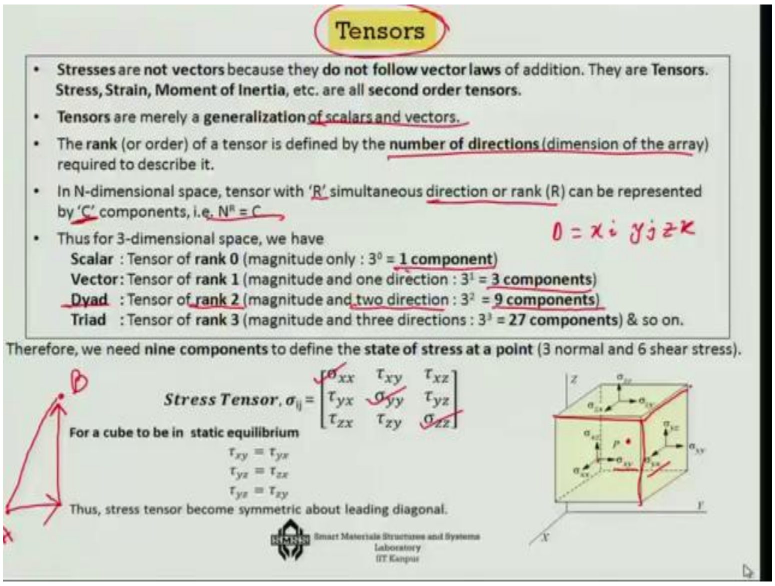

So that is what we will be doing for both stresses as well as for strain. So this is kind of a generalization 1st of all on scalars and vectors, okay. Now, how do we define the tensor for say stresses and strains before we say that, we can also show that how scalars and vectors could be special case of tensors, okay of certain ranks. The rank of a tensor is defined by the number of directions or dimensions of the array that is required to describe it, okay.

So suppose, we are talking of an N dimensional space and we are talking of a tensor with R direction or rank R. Then it can be represented by C component such that N to the power R equals to C. Now let us try to use this in terms of our day to day experience. Suppose we talk of a scalar quantity, that means say how many pens you ask me, how many pens do I have in my pocket, okay?

So suppose if I have one pen in my pocket, then this is a scalar quantity and this scalar quantity can be considered to be a tensor of rank 0 in a 3 dimensional space. In other words, N is here 3, R is here 0 and hence 3 to the power 0 that is only 1 component without any sense of direction, the number of pens that is only the number itself that is a scalar quantity.

Now suppose, we talk about that how we do move from a position 'A' and to a position 'B', so now I do not only need the distance to cover, but also I need to say that which direction I will be covering, okay. So for example you have to go this way and this way, the combination of directions in order to reach this location. So if you had this kind of a sense of direction that means you have a magnitude as well as the direction that is associated.

So it is 3 to the power 1, now in a 3 dimensional space that means you need 3 components and no wonder that any vector in a three-dimensional space you always denote it with respect to 3 unit vectors if you remember i, j and k, so that you can actually say that this distance 'D' is nothing but something like x i y j and z k, okay. So that is how you define the vector in terms of the distance, velocities, etcetera.

There are many such thing where this you know 1^{st} rank of tensor is important. When we will talk about stress, the stress is actually something which is a up rank 2, so it is a tensor of rank 2 which is also known as a dyad, okay. So that means here you need a magnitude as well as 2 directions every time. That means it is 3 square or in other words, 9 components are needed in order to define the stress at a point, okay.

So that is why I told you earlier that just a very simple generalization of a stress as a force per unit area does not really make sense because which area are we talking about. Now you have a much better definition of stress that suppose you consider a pointing and you consider a cube all around this point okay that is what we are showing here right, a cube all around this point and on that cube, so you have various surfaces on the cube, right.

And if you look at it that you need actually you know 3 normal components, okay and 6 shear components. So Sigma x x, Sigma y y, Sigma z z, 3 normal components and 6 shear components okay. Why we will need 6 here because there are some shears like Sigma x y and Sigma y x which are actually say. Shear and complementary shears are there, so you need 3 normal and the 6, total unique 9 components, okay in order to define the stress at a point.

And that is a tensor of rank 2 that is what is also known as a dyad, okay. So that just like stress is a tensor a dyad of you know that is of rank 2 tensor. Similarly, strain is also exactly the same way tensor of rank 2. So you need you know two rank 2 tensors to define the stress and the strain. Naturally, you know if you are talking about modulus of elasticity, then actually you need a rank 4 for in order to define the relationship between the 2 dyads, okay. Now there is something more, so it is about the stress the other interesting thing is the Poisson’s ratio, okay.

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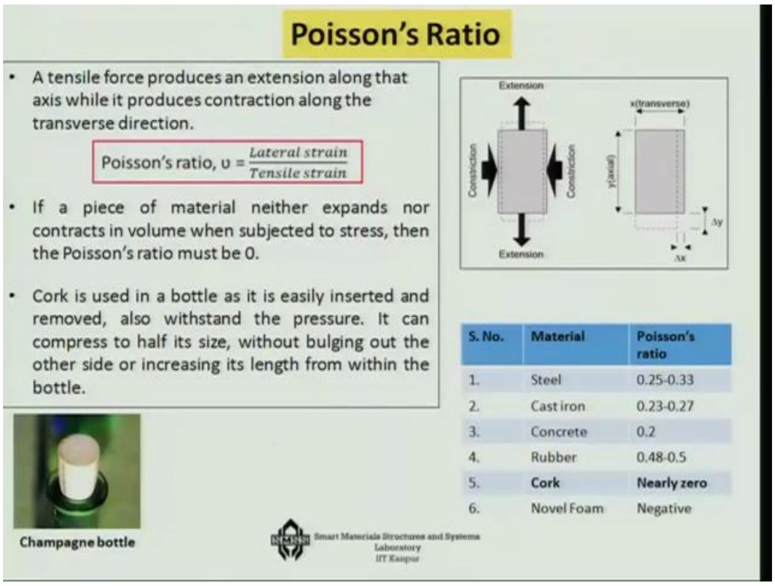

This refers to that when I am actually extending a material; let us just take this example that you are extending a material okay in one direction. Suppose we are taking the material like a dog bone shape, in a universal testing machine and I am applying tensile force, okay. This is a rubber sample and you can see how the thickness is reducing in the other direction, okay.

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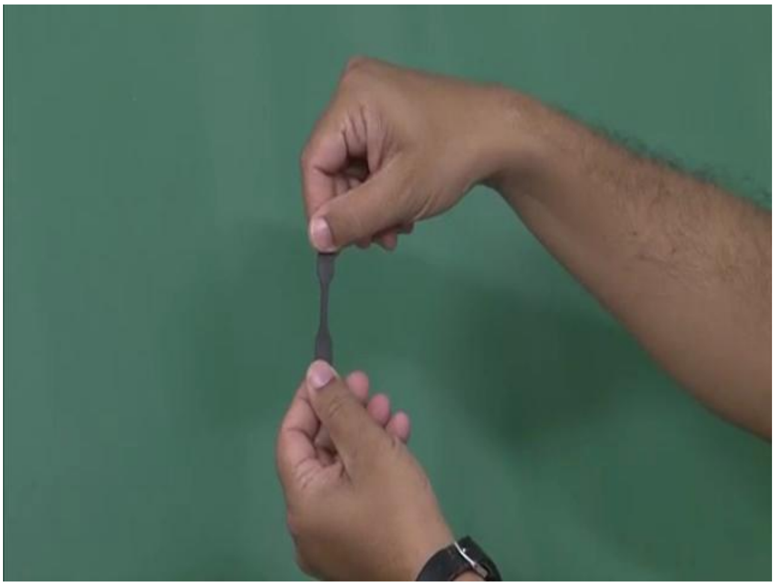

So this being a rubber imagine here, which has a Poisson's ratio close to 0.5, this shows beautifully that how it is narrowing down in the direction perpendicular to the application of the force, okay. So this nature of a material is actually depicted by what we call the Poisson's ratio of the materials. So that is the Poisson's ratio which is the ratio of the lateral strain to the tensile strain.

Now if a material does not exhibit this behavior, there are some materials which does not exhibit this behavior for example, a cork of a bottle, they generally neither expand you know in terms of when you are compressing it, nor the other way round. So they have Poisson's ratio which is close to 0. In fact, cork's Poisson's ratio is nearly close to 0.

Most of the other materials in the day to day experience that we use for example, the metals, they have Poisson's ratio between 0.25 to 0.3, etc okay, concrete has a Poisson's ratio of 0.2 and rubber has a Poisson's ratio of as I have just now shown you that has a Poisson's ratio close to 0.5. Can a material have a negative Poisson's ratio?

That means when we are actually contracting the material or expanding the materials let us say just like that rubber experience when you are expanding the material, instead of contracting can it expand on the other direction? Yes, in certain cases it can. Let me just show it to you through an example. Now, this is actually a reentrant honeycomb structure.

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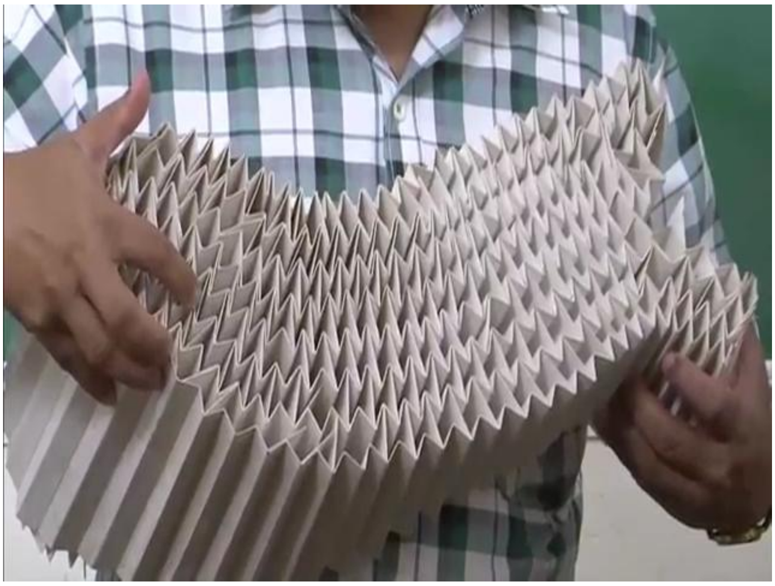

This is a This is a auxetic structure, so in this auxetic structure if you look at it that it is the each of the honeycombs instead of the honeycomb with a positive angle, they are having a negative angle, each one of them. And this is what happens in such a case? Suppose, now I am trying to expand the material, so I am expanding it in this direction. You see that the other direction, that is in the perpendicular direction instead of contracting it is actually expanding.

The more these (())(9:47) angles are actually unfolding, it is expanding more and more. And the other way when I am compressing, you can see other direction instead of expanding, it is also getting contracted in the other direction. So expansion is creating expansion, compression is creating compression that is the beauty of an auxetic structure, of a structure which is having you know a negative Poisson’s ratio.

So we have talked about you know various types of Poisson’s ratio, okay. And that definitely is a very important consideration material property consideration, when we are actually selecting a material. Now we also have talked about stress and strain, the stress and strain are related by something which is known as Hooke’s law, okay.

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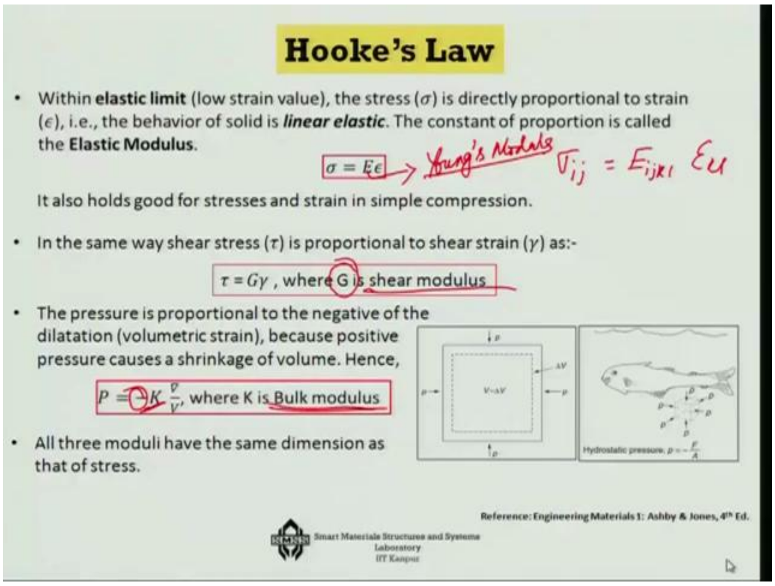

So there are of course the Hooke's laws which actually can be considered as a relationship between a single stress factor and a single strain or in a generic sense between stress tensor and a strain tensor, okay. So in that case if I consider the relationship to be a matrix relationship that means Sigma i j, equals to in that case it will be E i j k l with X i of k L, okay.

So that is a most generic Hooke's law. So basically Hooke's law is something which is relating between the stress and strain. And each stress and strain component you know within a linear elastic domain is generally found out to be a having a proportional that means you know, if you increase the stress 2 times, the strain also will increase 2 times, so there is a sense of proportionality that will that will be there between the 2, okay.

So that is you know the simplest form of the Hooke's law which we also refer that if we do it for a component, then this actually the 'E' becomes what we say as Young's modulus named after most famous physicist and physician Sir Thomas Young who had past you know made very important notes on and elastic behavior of materials.

Now, the way a normal stress component is proportional to strain component and related by Young's modulus. Similarly, if you have a shear stress component, that will be proportional to shear strain and there the modulus of elasticity is referred as shear modulus or 'G' and if you remember the case of that hydrostatic pressure case, there actually it is not happening the change in a direction, but in terms of a volume change.

So you consider it for any fluidic case, so that is why the negative sign is actually introduced here because the positive pressure causes shrinkage of volume. So you know here you get the volume change as the volumetric strain and that is related with the hydrostatic pressure and hence the modulus here is known as the Bulk modulus, so we have 3 different modulus, okay.

The elastic modulus in the 1st case it is Young's modulus, Shear modulus and Bulk modulus. All 3 modulus however have the same unit in terms of the unit of the stress because the strain is unit less, so it has the same unit as the unit of the stress, okay. So that is about the Hooke's law now and which correlates between stress and the strain and you can get actually the elastic modulus of the system.

The point is that this elastic modulus varies and this is a measure of actually the stiffness of the system okay. The way you know very easily I worked the rubber sample, with a steel sample if you give me; I may not be able to work so easily, so it has a higher stiffness that means a higher modulus of elasticity. Now this is a depicted in a beautiful chart from actually Ashby and Jones and that gives the elastic modulus for different groups of types of materials.

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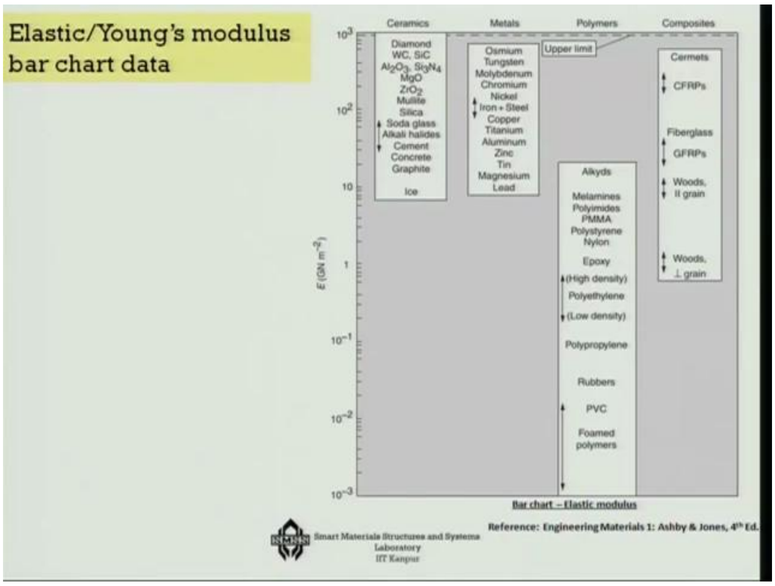

For example, if you consider ceramics, they have generally a very high elastic modulus like diamond and then they may have some you know some of the things which may have lower modulus of elasticity something like say for example the glasses or the ice or the graphite, okay the cement, they may have somewhat lower modulus of elasticity, but in general ceramics have reasonably high modulus of elasticity.

If you look at the metals, there are some metal which shows quite high modulus of elasticity for example, tungsten is one of them, chromium is another of them no matter that is why for the making the super alloy, these kind of materials are used because they deform much less with the same application of force. And then there is the Iron and steel is here for example, 200 gigapascal kind of a thing.

And then much softer ones are for example, aluminum or zinc or tin, magnesium, they are much softer in terms of the metals. Now if you look at the polymers, you can see that there is nothing here that means the polymers are a degree you know several orders of magnitude lower in terms of the stiffness. And there you know something like that is why I told you that about 2 gigapascal is something maximum that we generally observe.

And there in the higher range you get something like Poly methyl methacrylate, polystyrene, nylon, etcetera. And those which are very low are something like thing like foamed polymers, PVCs, rubbers, etc, they show a very low modulus of elasticity or in other words, the compliance is very high in such materials.

Then if you look at the composites, composites you know the natural composites are generally softer, something like woods, etc. But the man made synthetic composites actually have modulus of elasticity which is comparably the metals, CFRPs, fibreglasses, etcetera. So that is why, composites are replacing the metals you know wherever the high stiffness is required. So that is kind of a comparison chart which you can you not keep in your mind.

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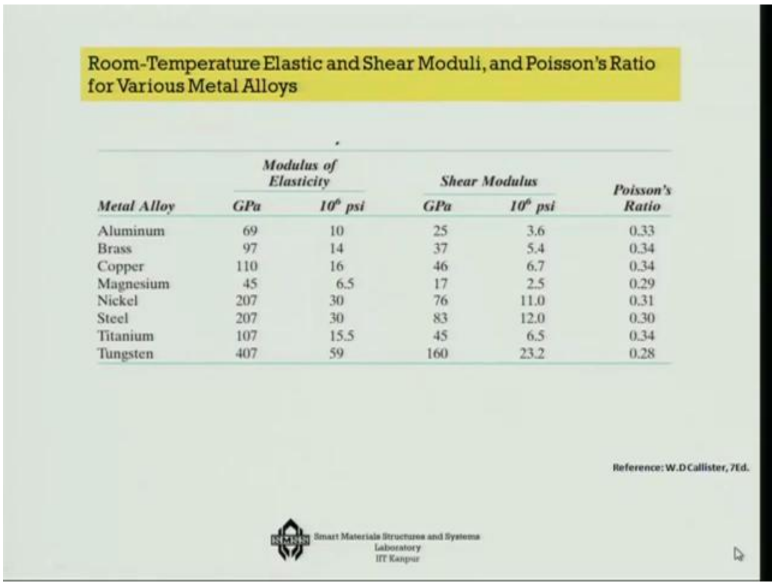
Room-Temperature Elastic and Shear Moduli, and Poisson's Ratio for Various Metal Alloys

	Modulus of Elasticity		Shear Modulus		
Metal Alloy	GPa	10^6 psi	GPa	10^6 psi	Poisson's Ratio
Aluminum	69	10	25	3.6	0.33
Brass	97	14	37	5.4	0.34
Copper	110	16	46	6.7	0.34
Magnesium	45	6.5	17	2.5	0.29
Nickel	207	30	76	11.0	0.31
Steel	207	30	83	12.0	0.30
Titanium	107	15.5	45	6.5	0.34
Tungsten	407	59	160	23.2	0.28

In terms of the absolute values here, you know ones again I have given 3 values we have given from Callister here and just for reference for example, if you consider aluminum approximately 70 gigapascal modulus of elasticity, Shear modulus is approximately 25 and Poisson's ratio is approximately 0.3, okay.

So a again in this list you would see that a high modulus of elasticity you are finding in something like tungsten, okay and the shear modulus is also high in that same range, but shear modulus is usually always lower in comparison to the you know Young's modulus of the system. What does it mean? It means that you know you can actually shear the material or twist material, deform the material in the shearing mode much easily in instead of actually elongating or compressing the material, okay.

And the Poisson's ratio you can see is not really varying is more or less in the range of something like 0.3 around that range, okay for most of the metals. So this is just for a comparison of these properties in various metals. Now, we carry out how we measure this property.

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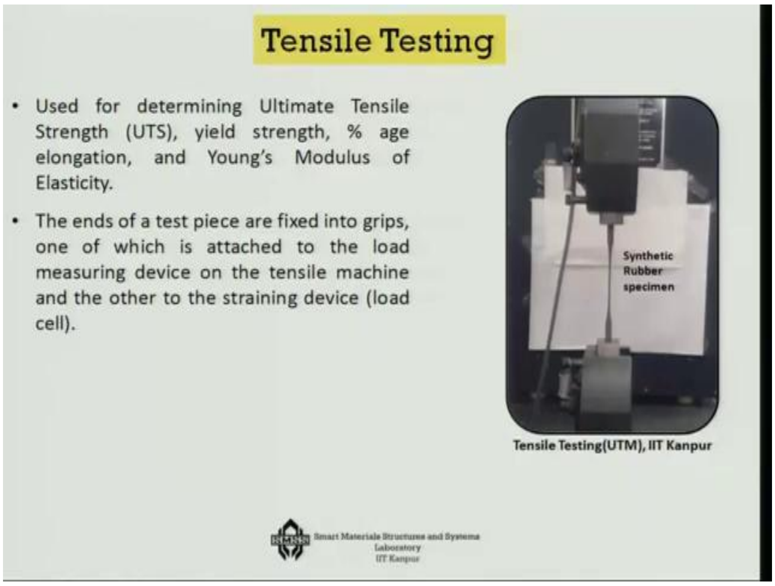

In order to measure this property, we use a tensile testing system okay in a machine a which is known as actually Universal Testing Machine in which we find our properties like for example Ultimate Tensile Strength, yield strength okay, Percentage elongation, Young's modulus of elasticity, etc okay.

So I have just not shown you that in a UTM machine how we are you know we can deform the rubber specimen, the same specimen that I have shown you okay can be used in this case.

Universal testing machine is a beautiful machine where you can actually load a sample in various ways tension, compression, Shear, not only that the loading can be made dynamic loading and which can include things like fatigue, etcetera.

So that is how we generally carry out this universal testing in a machine the tensile testing. Now if you carry out such testing of a ductile metal okay, how would you find out the stress strain diagram? And that is the engineering stress strain diagram of the material, how do you usually see with the strain diagram.

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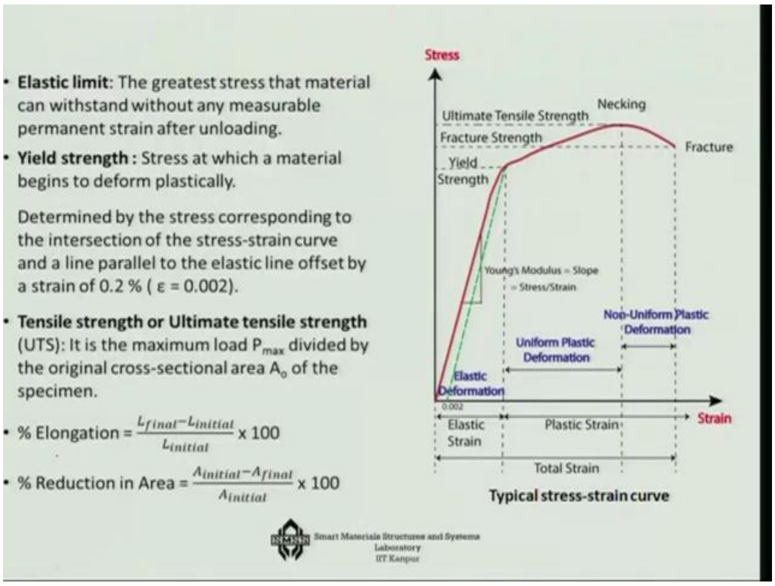

If you look at it that the stress strain diagram shows certain features. But please keep in mind that this is called a ductile metal, okay. So you will see that there is a initial region where you know the material if you actually unload the material, it will come back to the same point itself, that is known as the elastic limit okay, that is the maximum stress that withstand without any measurable permanent strain after unloading.

Now in the elastic limit of course there are 2 regions, in one region after some region it is actually perfectly linear and beyond that it is slightly non linear, but still if you release in the non linear region, it will come back to its original position that is what is the deforms the elastic limit. Now then the point comes here that is the Yield strength point. Beyond the yield strength point, if you actually deform the material, then it will start to deform plastically.

What it means is that beyond that point if you actually start to unload, there will be some amount of deformation which will always remain into the system. And that is why the yield strength position is very important because says the onset of actually plastic deformation in the system okay. So as I am increasing the load, I am going beyond the yield strength level and I am actually watching the uniform plastic deformation.

Now, if I increase the load further what is going to happen is something like a necking phenomena that would start to happen okay and then the material will go in the Fracture okay, so that is what generally you know happens for a ductile material. So in that beyond that making point what you get is non uniform plastic deformation, so the plastic in itself can be divided into 2 parts. One is the when it is uniform plastic deformation okay that is the 1st one.

And then there is non-uniform plastic deformation till failure okay. So that is what we will find into the system. And what is the point that is important for us is that up to the necking you know we get actually the Ultimate Tensile Strength, so that is the maximum load divided by original cross sectional area of the system that gives the Ultimate Tensile Strength, okay beyond that you would not aspect strength of a material because beyond that if you take the force, it is actually going to take us towards the fracture.

That is why Ultimate Tensile Strength plays a very important role in terms of the product design. And also the other 2 important factors is that what is the Percentage elongation that is the you know ratio of the difference of final to initial length change over the initial length and similarly for the area. So that actually tells us that how compliant is the material. So this is a typical stress strain curve for a ductile material.

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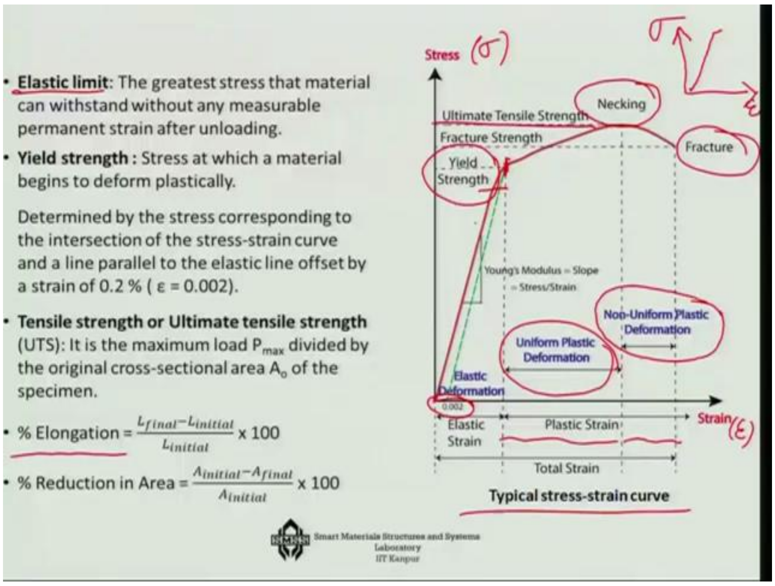

How will it look like a brittle material okay, I can just you know very in a very small qualitative manner we can draw it here that suppose this stress is denoted by Sigma and strain by Epsilon, so if I draw the Sigma Epsilon, for a brittle material is just one single line okay and maybe some point of nonlinearity, but that is it and then it stop to phase that means it will not show any plastic strain, any plastic deformation okay that is generally happens in terms of the brittle material.

The other important point here is that generally the Yield strength, if you do not find a very sharp point where this you know this is happening, we consider approximately these 0.002 level between the 2% strain level we will find out that where it is intersecting and that point is defined as the Yield Strength level because that is generally the strain level which it can take in general in metals without any permanent deformation.

But this is very specific to the metals for the things like elastomers or rubbers; this can become something like 5% to 10% okay, so depending on the material this position may change. Now then if we actually consider however the true stress strain curve that means if we consider that the area is actually changing.

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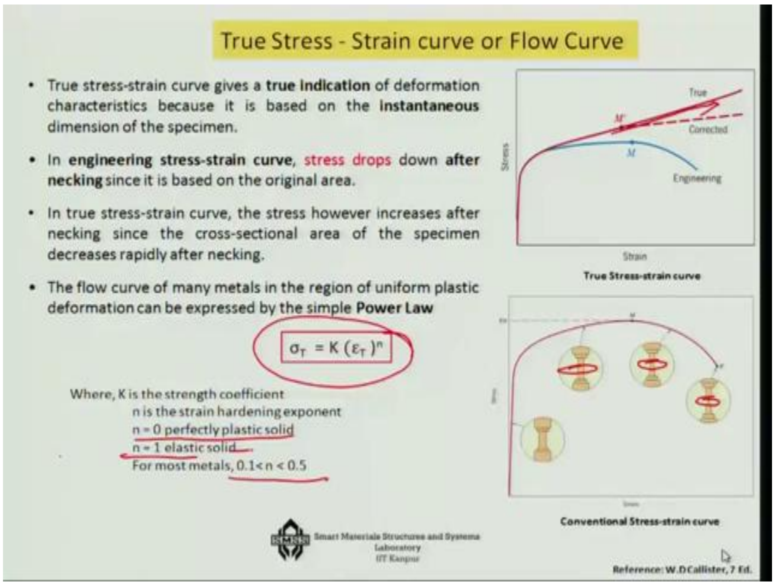

If you look at it that as you are deforming this area is actually changing, this area is becoming smaller and smaller, and the neck formation is happening, so the area is changing. And if you consider that actually area, you will see that it is not actually the stress strain curve, it is not dropping, but it is actually increasing, height is increasing because even if the force you are increasing, the area is actually decreasing at a very rapid rate so that is why the true stress actually is increasing at a very rapid rate.

Now in the uniform plastic deformation, the stress strain relationship is actually governed by a power law which is like Sigma T is K times Epsilon T to the power N, where this N is 0 for perfectly plastic solid, N is 1 for elastic solid okay. For most metals it will be in the plastic region between 0.1 to 0.5, so that is how it will behave you know in the true stress strain curve okay which will generally you know overlook actually, will only look at the stress strain curve that comes up in the UTM machine.

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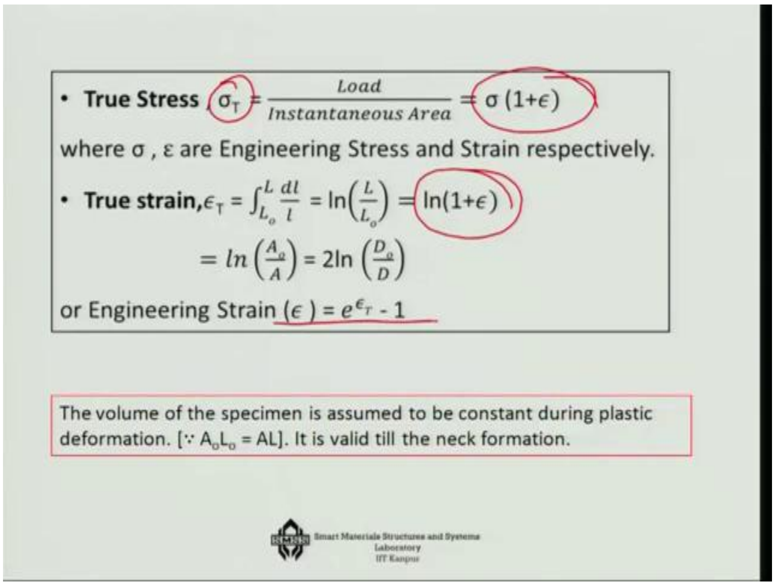

So the true stress strain curve actually true stress is denoted by this relationship which is something like the Sigma t as Sigma times 1 + Epsilon and the true strain Epsilon T is the log of 1 + Epsilon okay. And the relationship between engineering strain and true strain is governed by this relationship. So this is what you can just very simply mathematically you can actually obtain this relationship.

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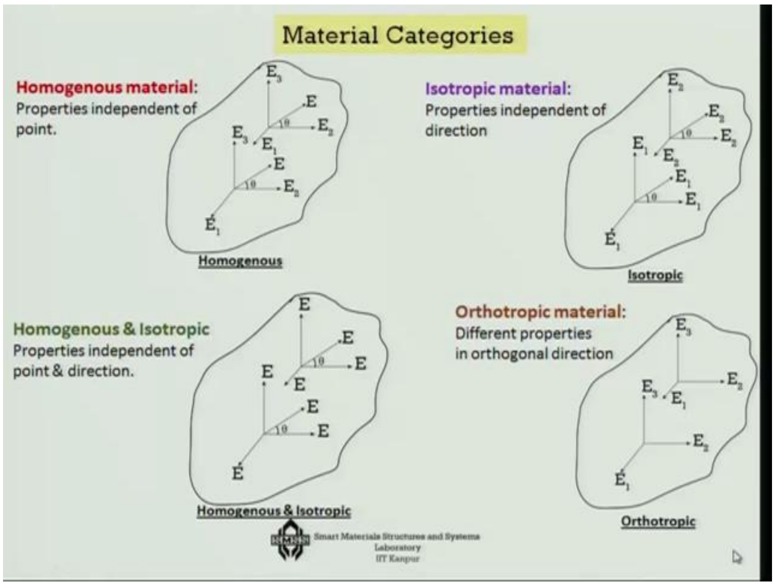

Now the other point is that the material based on the stress strain relationships are actually divided into sub categories like the homogeneous materials for which the properties are actually independent of where I am measuring the property that means, here you get a property and here you get a property, all the directions if you look at it you are going to find the same modulus of elasticity, but it varies from direction to direction.

In one direction it is E 1, in another direction it is E 2, so that is what is a homogeneous material where at different points you get the same pattern of modulus of elasticity. For isotropic, it is actually direction independent at a particular point if you consider, you get E 1 in all the directions. But if it is not homogeneous, then at a different point you get E 2 in all the directions and E 1 and E 2 are not the same, they are not the same.

If they are same, then they will become homogeneous and isotropic material. And if we consider an Orthotropic material, in that case it is slightly different in that case we will see that at different points it is homogeneous, but the modulus of elasticity varies in different directions in the sense it varies in 3 different mutually orthogonal directions that will be E 1, E 2 and E 3 and this is called General orthotropic material.

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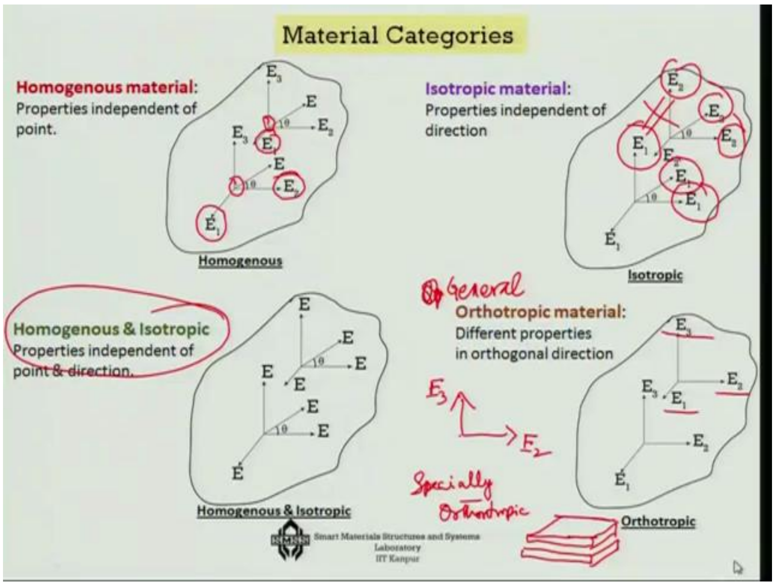

And for Specially orthotropic material you may disregard one of them and it only varies in 1 set that means something like E 2 and E 3 will vary okay, but E 1 will remain the same, so that is what happen for Specially orthotropic material. This is typically a case like you consider a laminated composite, then each lamina which is very thin in one direction are actually considered to be specially orthotropic material, so that is a very special case of a general orthotropic material.

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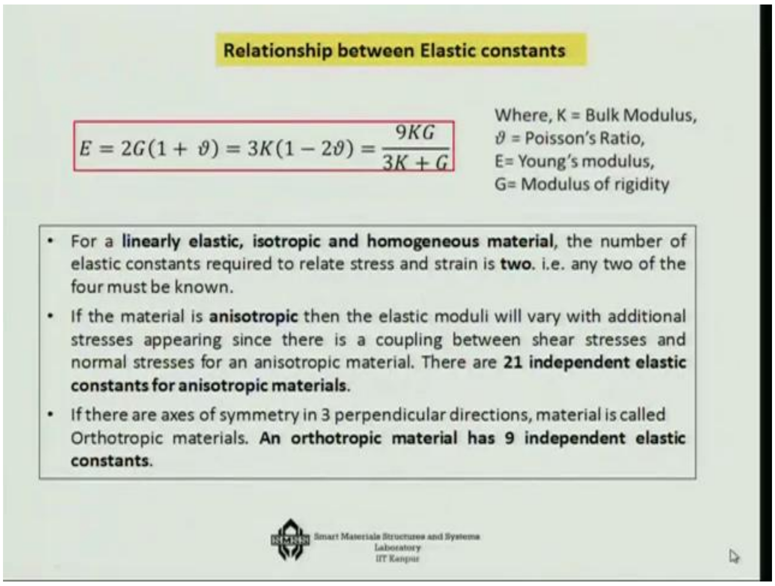

Now, the relationship, the elastic constants are actually related with each other by the relationships which is shown here that E is related to G and G is related to K okay. And hence if you know actually for a linearly elastic isotropic and homogeneous material, you need only

2 elastic constants and then the other 2 that means we have you know these 4 K, Nu, E and G. Out of them, any 2 if you give me I should be able to derive the other 2 provided that the material is linearly elastic, isotropic and homogeneous material.

And in case of anisotropic material, there are 21 independent elastic constants okay, so it is a much higher you know in terms of the variation. And if it is orthotropic material, then it has 9 independent elastic constants that will come into the picture. So naturally, the tailoring is much more for an orthotropic material. Now that is the relationship you know in terms of the actually orthotropic and anisotropic material.

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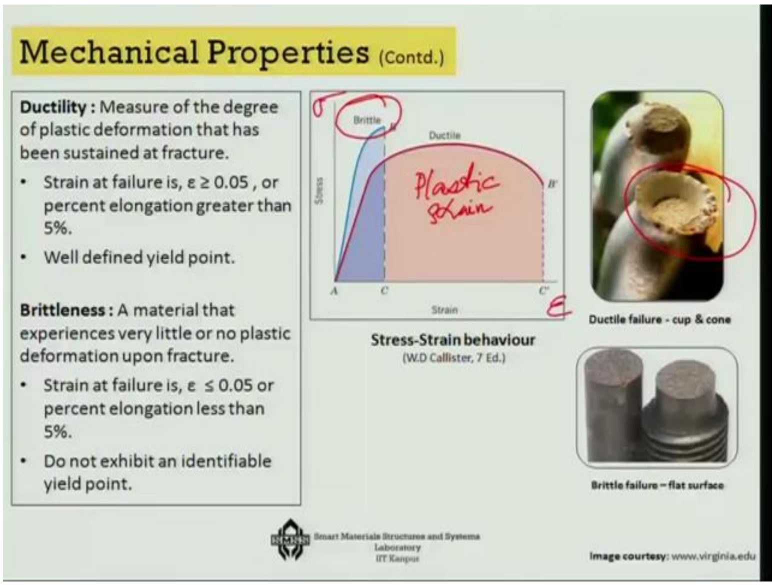

The so that is about the stress strain relationship, there are some other mechanical properties which are also important, one of them is ductility, are already talked about it that you know it is the how much you know is the capacity of how much you can draw a material under the tensile stress okay and that is very important and there is something that you will also see in terms of ductility which is known as the ductile failure, ductile sign of the neck formation of the system and the brittleness of a system.

So the difference between a brittle and a ductile system I have already drawn the stress strain curve for a brittle system and a ductile system is that precisely this ray the act you know the presence of this plastic strain that is there for a ductile material. Now, that is absent for a brittle material. So that is very important point that we have to keep in our mind while comparing between various material properties.

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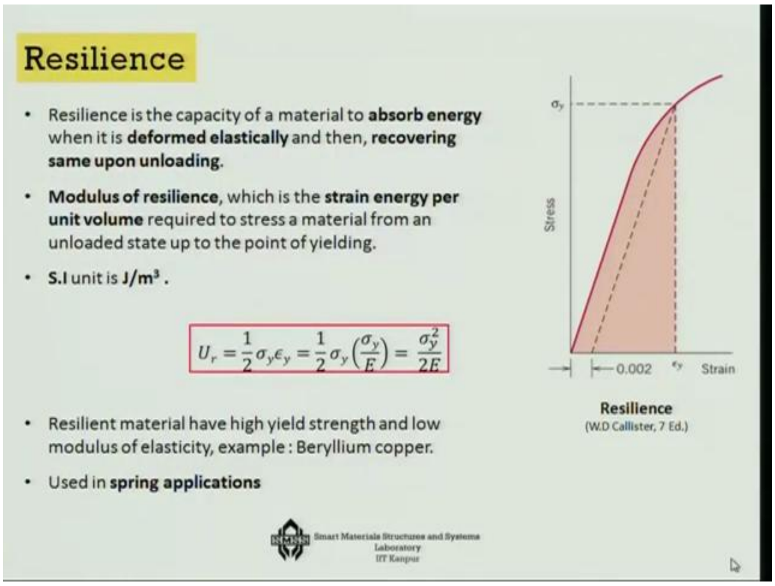

Now the other important point here is actually the resilience of a material, which is the capacity of a material to absorb the energy okay elastically, so it is in the elastic domain when it is deformed elastically you know how much of energy it can absorb. And if you consider the stress strain diagram, and if you consider the area under the stress strain diagram, you can actually find out that this is something like you know Sigma Y square over 2 E in a kind of a approximate measure.

And that is so that means if you know what is the Yield Stress and if you know what the modulus of elasticity is, you can actually compute that what is the resilience of the material in terms of modulus of resilience and how much of energy the material can absorb, which is very important in applications like spring type of applications. So the resilience is another important mechanical property. The last important property in this direction is the toughness.

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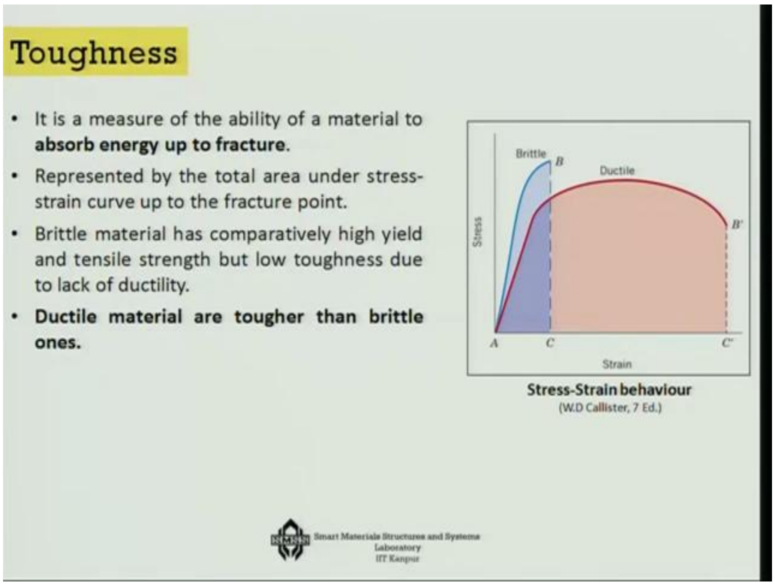

Toughness is also the area under the stress strain curve, just like the earlier case, but here you are considering the plastic region also, okay. The earlier you were confining yourself with the elastic region, now you are considering the plastic region also. In fact, the test that is very much important towards this direction are actually Izod and Charpy impact tests.

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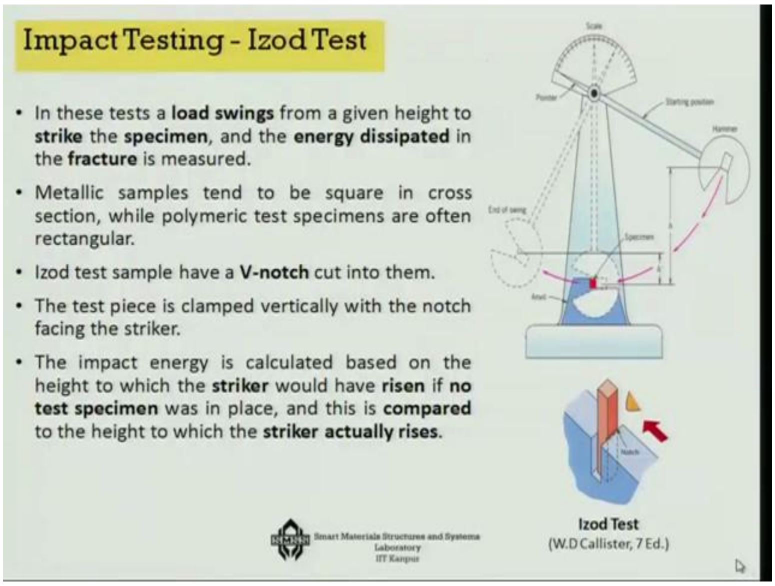

So this is an Izod impact test where actually support the sample from one position here the support is here put and you are hitting it through a pendulum and if there is no sample then it can go to a particular distance, if there is a sample then there is some energy is actually taken by it, so it cannot go you know up to that distance, so you take the ratio of the 2 return ratio we call it. And then that gives a measure of how much of energy is actually absorbed by the material and which is the measure of the toughness of the material.

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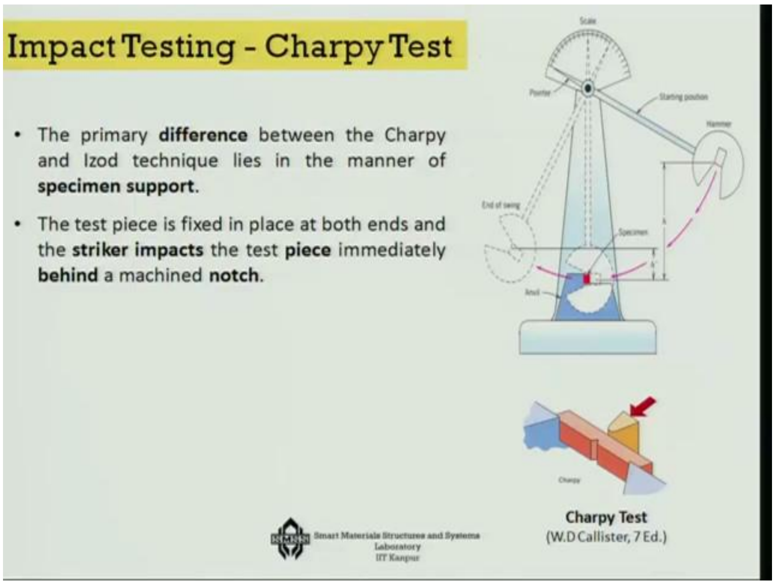

Similarly, there is another very similar test actually which is known as the Charpy test. Now the difference between the 2 tests, the Izod test and the Charpy is that in the Izod test you have you have for example if I go to the last case, you have the crack which is you make a notch actually and the notch actually faces the hammer, so the hammer faces the notch.

So you are in terms of Charpy, is you can see on the other hand the notch is on the opposite side of the hammer, that is one of the difference between the 2 and also in case of a Charpy test, you are supporting the material at 2 places instead of at one location. But both of them are very good tests which can measure that how much of energy a material can absorb and that means how tough the material is. And that actually means that you know how much the impact strength that is there in the material is.

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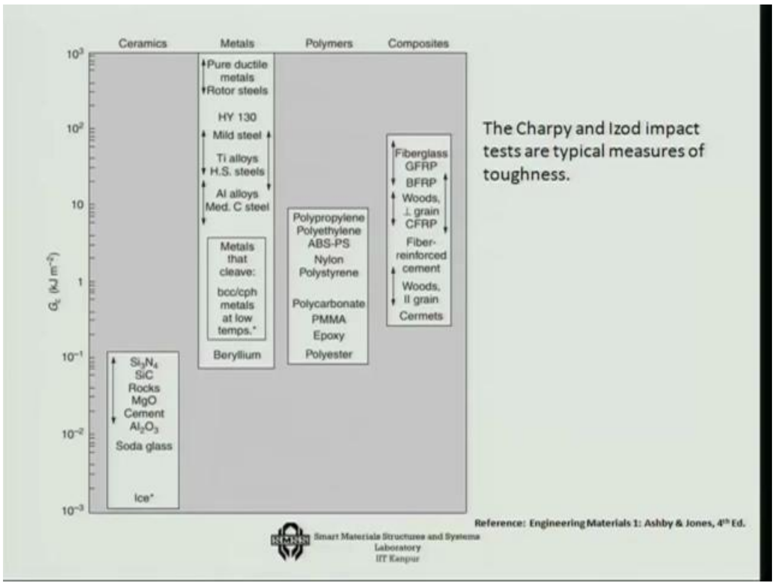

So here is some kind of a comparison from the Ashby’s table and as you can see here, if you look at it very closely that the metals are topping in the list actually because they can absorb quite a good amount of impact energy. And in fact plastics even though they deform more, they cannot absorb that much of energy, brittle materials of course are the worst here.

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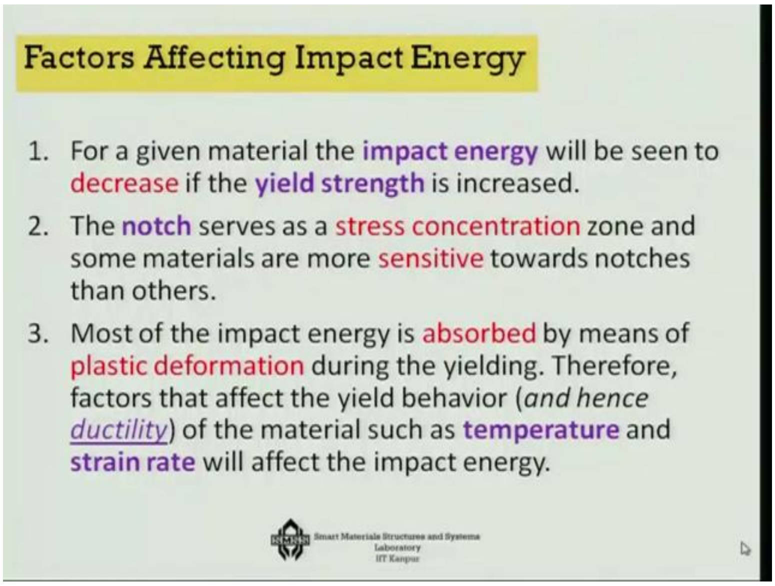

So that is what is in terms of the 3 different you know groups of materials. Then composites are somewhere in between. Now, the factors that are important in terms of the impact energy is that for a given material you know the impact energy will decrease if the Yield strength is increased, so that is generally found in the material.

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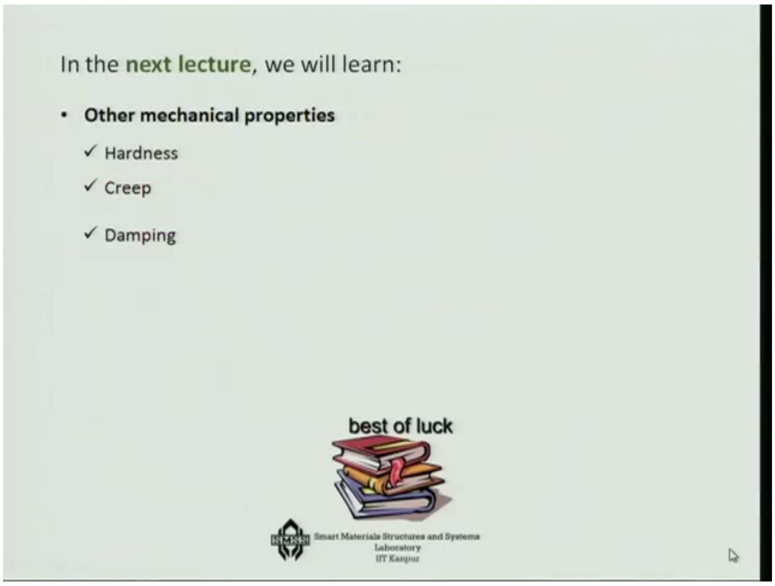

And the notch serves as a stress concentration zone, so some materials are more sensitive to notches and the most of the impact energy is actually absorbed during the plastic deformation of the material. Also, temperature and ductility plays a very important role on it. So this is where we are going to close this lecture and in the next lecture we will talk about some of few more mechanical properties that is, the hardness of the system, the creep of the system and the damping, thank you.