Advanced Composites
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Lecture - 10
Specially Orthotropic Material Under Plane Stress

Hello, welcome to Advanced Composites today is the fourth day of the ongoing which is second week of this course. Yesterday, we just finished our discussion on the number of elastic constants required for specially orthotropic material and we found that the number of elastic constants which are required to characterize such materials is 9. And then, we developed relationships between stresses and tensor strains for such materials and we also developed relationships between stresses and engineering strains for such materials.

What we will do today is, we will discuss similar stress, strain relationships for several other types of materials and what we will start with is a material known as transversely isotropic.

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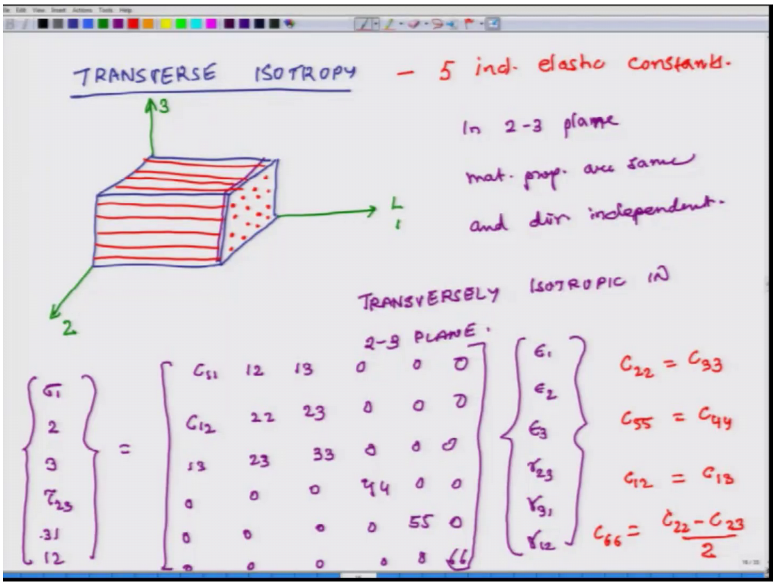

So, we are going to discuss Transverse isotropy, transverse isotropy. So, I will make a picture first and this is the block of material, fibers are running in the direction of all these red lines. And if I look at it from one particular end, I see the ends of fibers and that is pretty much it and here, this is my L axes or you can also call it axes number 1, this is my second material axes and this is my third material axes.

So, now this material if I cut a plane, if I cut a slab of this material like this and then if I try to find it is modulus in the 2 direction and 3 direction, it will be same right. I mean that is how because the material is oriented like this and because of that in the 2, 3 plane, in 2, 3 plane the material properties are same and direction independent, right. So, they are direction independent. So that means, this material is transversely isotropic in 2, 3 plane, in 2, 3 plane.

Now, the stress strain relationships for an orthotropic material were this 1, 2, 3, tow 2, 3, 3, 1, 1, 2 and then we had these elastic constants in terms of c's and we had strain epsilon 1, epsilon 2, epsilon 3, gamma 2 3, gamma 3 1 and gamma 1 2. So, what are the elastic constants? It is c 11, 12, 13, 0, 0, 0; c 12, 22, 23, 0, 0, 0, 1 3, 2 3, 3 3, zeros and then I have three zeros here in all the remaining rows and then I have 4 4, 0, 0, 0, 5 5, 0, 0, 0, 6 6.

If this material is transversely isotropic in the 2, 3 plane, then what are the relationships between different material constants. It means, that if I pull this material in two direction or if I pull this material in three direction, it is Young's modulus or not Young's modulus, elastic modulus should be same which means c 22 should be same as c 33. This is one; similarly, if I exert a shear stress on this in the 1 3 plane, 1 3 plane, so this is 1 and 3, 1, 3 plane or in 2, 3 plane, then again the shear moduli will be same and what that means is that c 55 should be same as c 44. This is based on the type of material we are talking about here and similarly the c 12 and c 13 reflect the Poisson effect. So, c 12 is equal to c 13. And finally, and we will not discuss the proof of this, this last component c 66, this can be mathematically expressed as c 22 minus c 23 divided by 2.

So, what does that mean, that instead of nine independent elastic constants we now have only five independent elastic constants and the remaining four can be found from these four equations. So, transversely isotropic material which is 3 dimensional, not very thin which is 3 dimensional in nature has 5 independent elastic constants, five independent elastic constants this is important to understand we will consider another class of materials isotropic.

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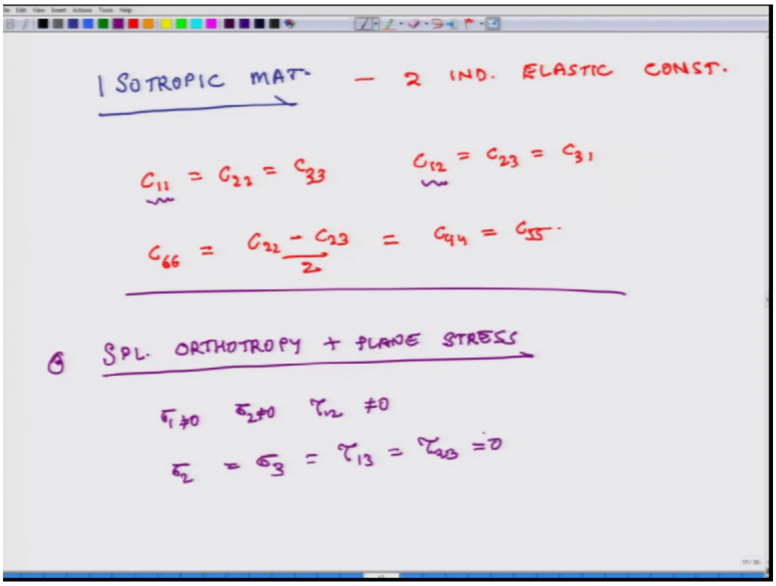

And without going into detail proof, isotropic materials have only two independent elastic constants. They have only two independent elastic constants. So, what are the further simplifications c 11 is equal to c 22 is equal to c 33, c 12 is equal to c 23 is equal to c 3 1 and then c 66 is equal to c 22 minus c 23 divided by 2 and this is equal to c 44 is equal to c 55. So, these are the mathematical relationships. So, you have only two independent elastic constants ok; only two independent. So, suppose you know only c 11 and c 12, then you can compute everything ok, you can compute all the other remaining seven constants.

So, isotropic materials have only two elastic constants. The last case, we will do is a special Orthotropy in plane stress state. So, special Orthotropy plus plane stress, plane stress condition. So, we have already talked about what is a special Orthotropy if it is materialize, specially orthotropic it has nine constants, but then on top of that we are saying that it also has plane stress condition. So, when there is plane stress condition, what does that mean, that stresses are acting only in one plane?

So, what does that mean? So, you will have only suppose sigma 1, sigma 2 and tow 1 2; all other stresses sigma 1 3 is out of the 1, 2 plane; sigma 2 3 is out of 1 2 plane; sigma 3 3 is again out of 1, 2 plane. So, we have only stresses in a particular plane. Now, it could be 1, 2 plane or 2, 3 plane or 3, 1 plane. So, in this case we are just considering 1, 2 plane. So, if that is the case then, we say all other stresses are non existence. So, these are non zeros and other stresses sigma 2 equals sigma 3 equals tow 1, 3 equals tow 2, 3, these are zeros.

Now, if that is the case if tow 1, 3 is 0.

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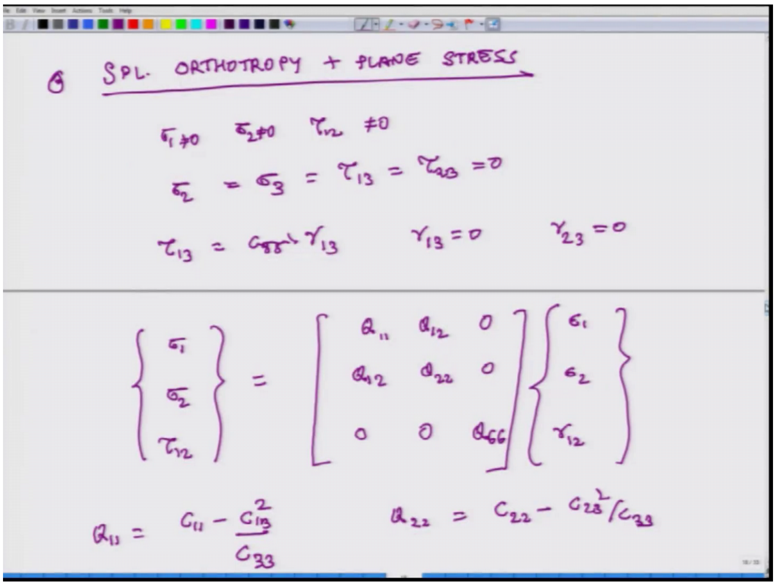

Then tow 1 3 is equal to what? It is equal to c 55 times gamma 1 3 right and if tow 1, 3 is 0 then, whatever the value of c 55 d gamma 1 3 is going to be 0. So, it tells us that gamma 1 3 is equal to 0. So, c 55 we do not care whether we can find it out or not ok; does not matter we do not need it because gamma one 3 is 0 and tow 1 3, 0.

Similarly, gamma 2 3 is also 0 because tow 2 3 is 0; gamma 2 3 is also 0 ok. So, we have nine constants and that number of constants because of that, it just comes down to seven because we do not need c for 5 and c 44. But this number of constants, it comes down further. So, again I mean if you do all the math in a organized way, essentially what you end up with is sigma 1, sigma 2 and tow 1 2 and that equals epsilon 1 epsilon 2 gamma 1 2 times. So, this is equal to this strain vector times stiffness matrix and this stiffness matrix, we will call Q matrix and I will define what are these values, ok.

So, this is the Q matrix and what are these values. So, Q 11 is equal to c 11 minus c 1 3 is square by c 33. So, if you do the mathematics by this you will get these relations Q 22 is equal to c 22 minus c 23 square by c 33.

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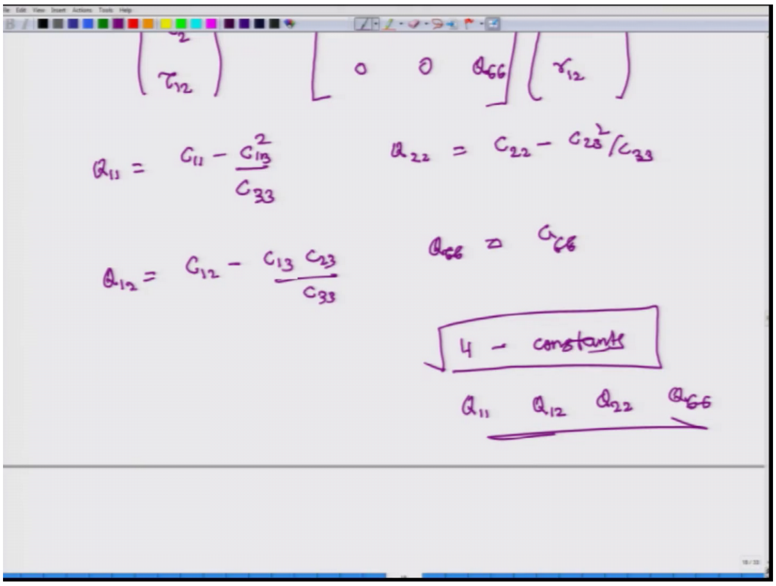

And then, Q 12.is equal to c 12 minus c 1 3, c 23 by c 33 and Q 66 is equal to c 66. So, if there is a material which is specially orthotropic and in a plane stress situation, then we need only we need only four constants. It may have more independent elastic constants, but we do not need other constants. We only need four elastic constants and these are Q 11, Q 12, Q 22 and Q 66. These are the four elastic constants we need and we can do the whole mathematics and mechanics of these materials correctly. If that is the case, it may have other materials like I discussed earlier, it may have c 55 and c 66. But because it is a plane stress situation, we do not need these. So, they are irrelevant to us now instead of 9, we need only 4 constants and these are these four Q's,

So, this is the conclusion of our discussion for today. What we have discussed today are several types of materials. We showed that starting from especially orthotropic material which has 9 constants. We showed that material which is transversely isotropic that is number of independent elastic constant it comes down to 5. If the number of if the material is isotropic in nature, then we know it only two elastic constants. So, they have only two independent elastic constants. And if the material is thin and it is having a plane stress condition, plane stress condition, if it is thin and it has a plane stress condition then, because the strains in shear strains in 3 and 1 directions are 0 and also because sigma or stress in the third direction as 0. Because of this, we need only four elastic constants independent, elastic constants and we have defined those elastic constants in terms of the Q matrix.

So, what we will do tomorrow is, we will learn how to transform these stresses from the principle material access system to any arbitrary access system. And we will learn how to transform this. So, that we can if we have a specially orthotropic material in the status of plane stress, then we should be able to predict the stresses and strains in the system as long as we know the Q matrix.

So, that is what we will do tomorrow. And then till then, have a great day.

Thank you.