Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology Kanpur

Lecture - 11
Stress and Strain Transformation

Hello, welcome to Advanced Composites, today is the 5-th day of this course and we will continue our discussion on the number of elastic constants required for especially orthotropic materials when they are subjected to a plane stress condition. So, the other day we had shown that we need total of 4 elastic constants in such situations and we had expressed the relationship between the stress vectors.

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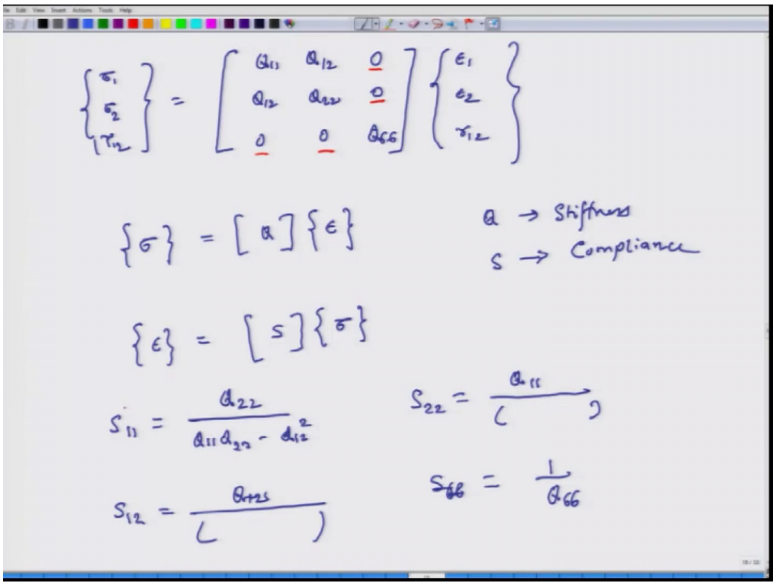

So, sigma 1, sigma 2 and tow 1 2 and the strain vector through the stiffness matrix Q; so this would be epsilon 1, epsilon 2, gamma 1 2 and we have Q 11, Q 12, 0 Q 12, Q 22 0, 0, 0, Q 66.

If you remember this stress strain relation is for especially orthotropic material and the relations actually indeed show that because they show that the terms in the third column and in the third row the first 2 terms in the third column and the first 2 terms in the third row they are indeed 0. Had they not been 0, then extensionally stresses would have also generated shear strains and vice versa. But, because they are 0 it is ensured mathematically that extensional stresses will only generate shear extensional strains and shear stresses will only generate shear strains. I can abbreviate this entire equation in the following form.

So, I just write sigma so, which denotes this vector. So, this equals Q matrix which is the stiffness matrix times the strain vector. I can also multiply both the sides of this equation by inverse of Q matrix and what that gives me is that if I want to compute a strain in terms of a stresses, then the inverse of q matrix will help me compute that. So, inverse of Q matrix is called S matrix and that represents the compliance matrix. So, Q represents stiffness and S represents compliance.

So, the strain vector equals the compliance matrix times the stress vector and what are the values of different element of this matrix? So, S 11 equals Q 22 by Q 11, Q 22, minus Q 12 whole square. S 22 equals Q 11 divided by the same denominator and S 1 2 equals Q 12 divided by the same denominator. And then finally, S 6 6 is equal to 1 over Q 66 and then once again Q 1 6, S 1 6, S 2 6 are 0 in the compliance matrix as well.

The last sets of equations which I will discuss in context of these materials are relationships between engineering constants and the Q or S elements. So, what are engineering constants?

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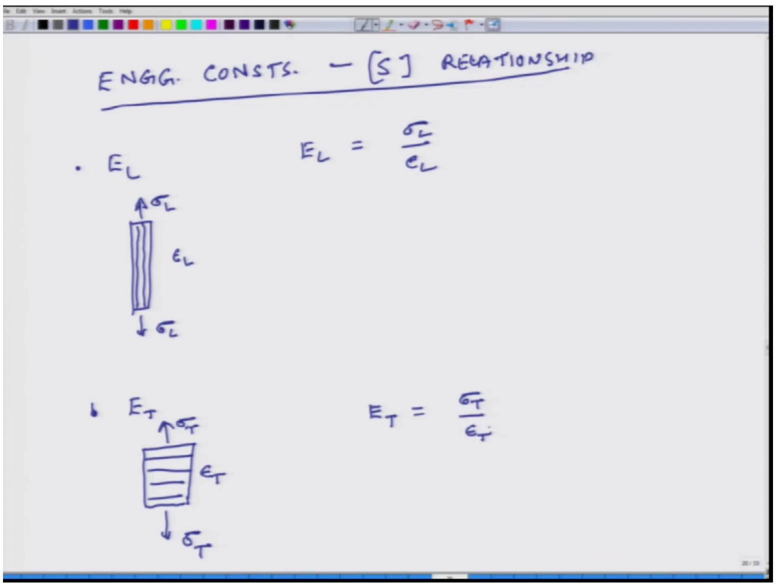

So, we are going to discuss engineering constants and how they are related to S matrix. So, this relationship we will discuss. So, first we will list what these engineering constants are. So, the first one is E L and these are engineering constants because by doing simple engineering experiments we can measure these.

So, what is E L? E L is if you have a sample and the fibers are in this direction and you do a tensile test on it. So, you apply sigma L and as you apply sigma L, it generates epsilon L. So, then E L is equal to sigma L over epsilon L. So, that is how you calculate or determine experimentally E L that is an engineering constant.

Other engineering constant is E T. So, what do you do here? You have again a similar sample, but fibers are aligned in the transverse direction and again you subjected to a tensile test. So, you apply this tensile stress. And in this case it is sigma T consequentially the material exhibits a strain in the transverse direction as epsilon T and E T is equal to sigma T by epsilon T the third engineering constant is.

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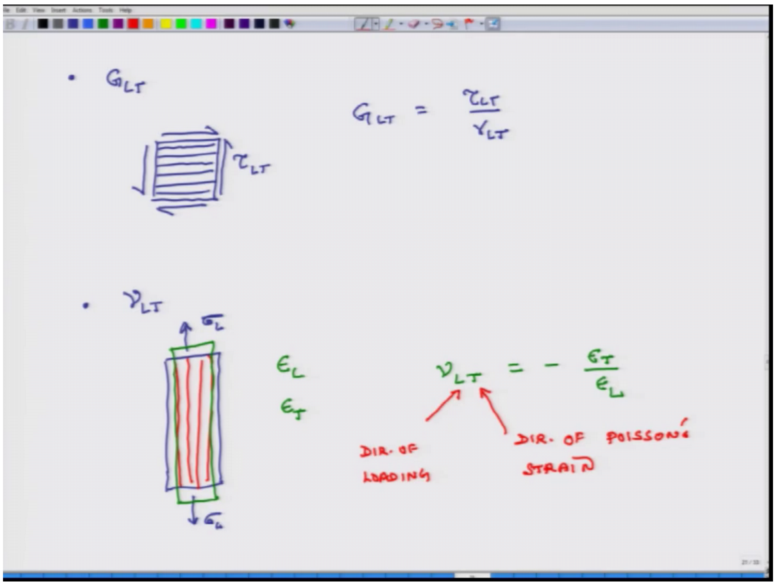

G L T and how do you measure G L T? You take a sample and you subjected to some shear test shear stresses. So, this you can do either on a rectangular slab, but it can be more easily done on pipes and you can subject them to torsion, so that is much more easy to.

So, you subjected to shear stress tow lt. So, the fibers in this case are again aligned to the L direction and because of this sample undergo some shear strain. So, G L T is equal to tow L T divided by the engineering shear strain which is G L T. And the last engineering constant is Poisson's ratio new L T Poisson's ratio new L T. So, what you do? To measure it you take a tensile test specimen, here the fibers are running in the loading direction. So, you subject this material to sigma L and as this material gets tested it becomes longer, but it also becomes thinner.

So, using may strain gauges you measure the longitudinal strain and you measure the transverse strain. And then your Poisson's ratio L T is equal to negative of transverse strain by longitudinal strain. And this Poisson's ratio L T new L T, it has 2 subscripts. The first subscript L is the direction of loading and the second subscript T is the direction of Poisson's strain.

So, first subscript says that you pull it in the L direction and the second subscript says that when you are pulling it in the L direction measure the Poisson strain which is in the transverse direction. And then take the ratio of E T over E L and you will get new L T ok. So, these are 4 engineering constants for any specially orthotropic material which is subjected to plane stress state now if we have to compute the S matrix or Q matrix what do we do? So, again there are mathematical relationships which we had discussed in our earlier class and I will just simply write these expressions, so S 11.

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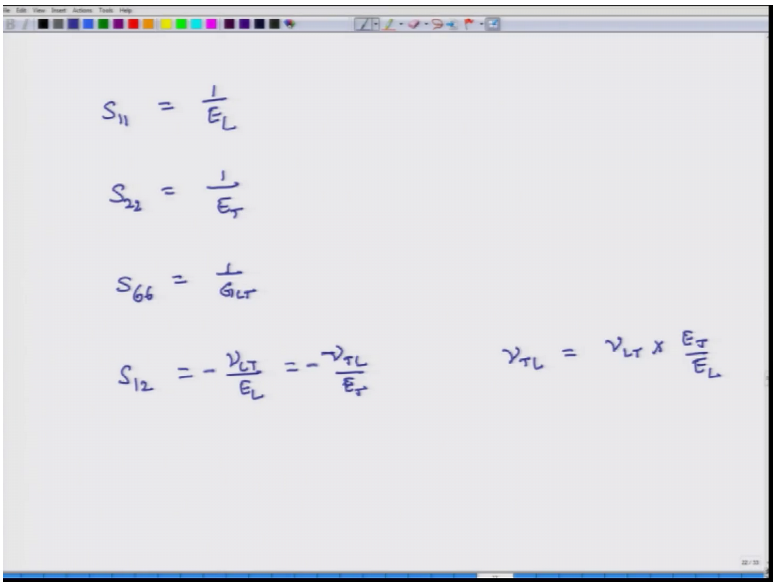

Equals 1 over E L, S 22 equals 1 over E T, S 6 6 equals 1 over G L T and the last one is S 1 2 equals new L T divided by E L and this is same as new T L divided by E T and new T L equals new L T into E T over E L. So, these are the stress the relationships between the

S matrix and engineering constants and we have already explained the mathematical relationships between S S and Q S. So, if I have to compute Q S from engineering constants I can now use these relations to compute Q S as well.

So, now we will move to the next topic.

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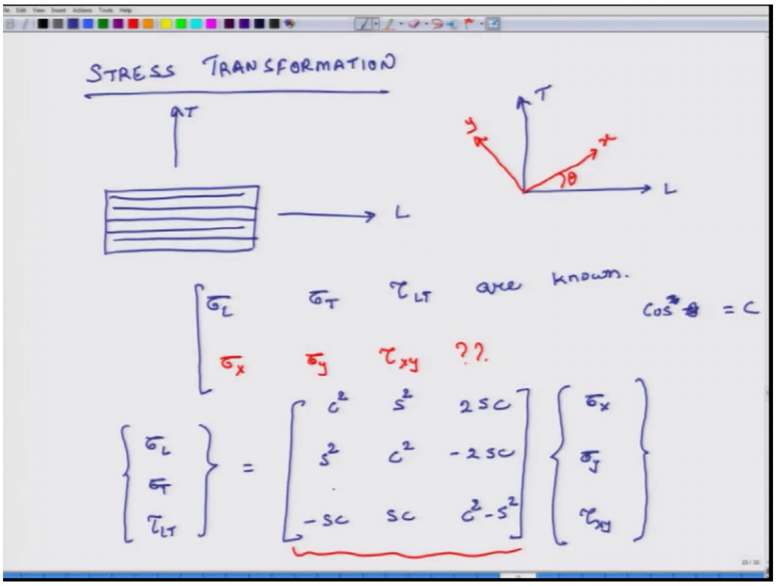

And that is about a stress transformation. So, we will talk about a stress transformation and also we will talk about a strain transformation. So, what is the question? The question is that suppose I have a material and let us say this is my L direction and that is the T direction. So, if it is the L direction then fibers are like this, then the question is. So, let us draw this separately this is L, this is T.

So, the question so, the question is that suppose sigma L, sigma T and tow L T are known. Suppose I know these values for whatever reason, then if I rotate my coordinate system if I rotate my coordinate system to an x y coordinate system like this. So, this is my x and this is my y and I have rotated basically by an angle theta in the counter clockwise direction, then can I calculate sigma x sigma y and tow x y what are the values of sigma x, sigma y and tow x y?

Now, this is not new engineering all of you have done this in your engineering courses and also we covered this transformation in the last class. So, I am going to directly write down these the transformation relations. So, the transformation relation is.

That sigma L, sigma T, tow L T equals there is a transformation matrix and we will write down the elements of that transformation matrix and that times sigma x, sigma y, tow x y. So, what are the elements on this transformation matrix? It is a 3 by 3 square matrix and the elements are cosine square theta. So, cosine theta I am just writing it as C ok. The second element is sin square theta, the third element is twice of sin theta n cosine theta.

Then second row sin square theta, cosine square theta and minus 2 sin theta cosine theta and then the last row is minus sin theta cosine theta, sin theta cosine theta and cos square theta minus sin square theta. So, if we know sigma L, sigma T and tow L T all we have to take do is take the inverse of this matrix basically I have to move this matrix to the another side.

So, I multiplied both sides by inverse of this matrix and from that I can calculate sigma x sigma y and tow x y. Likewise the relationship for strain now remember this is the stress, this is the relationship for a stress. If we can use a exactly the same equations for transforming strains, if we used tensor strains, but if we have to use engineering strains then the relationship transformation equations become somewhat different.

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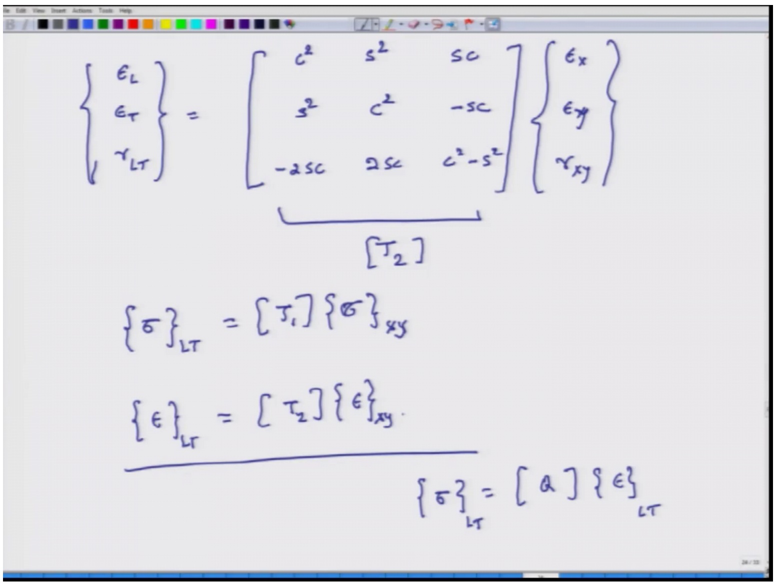

So, for engineering strains this is the relation epsilon 1, epsilon 2, gamma 1 2 is equal to. So, actually I will not write epsilon 1, epsilon 2, gamma 1 2, it will be epsilon l. So, this is equal to epsilon x, epsilon y and gamma x y. And the transformation matrix is C square, S square, S C, minus S C, C square, S square, minus 2 S C, 2 S C and cosine square minus sin square theta.

So, I call this matrix as transformation matrix number 1, T 1 and I call this matrix as transformation matrix the second matrix transformation matrix. So, in short I can say that sigma in L T coordinate system is equal to T 1 times sigma n x y coordinates system. And for transforming strains, L T coordinate system equals T 2 times strain vector measured in x y coordinates system. So, these are the 2 equations.

So, now we move to another thing and here the question is. So, our original relations between a stresses and strains where using L T coordinate system. So, our original relation was that between stresses was this a stress and strain was this, but in both these cases stress was in L T coordinate system and strain was also in L T coordinate system. Because, we had always said that this relation is for especially orthotropic materials which means the material axes are aligned with the loading axes.

But, the question is can we have a similar relation for generally orthotropic material. So, that is something we will discuss now, so for a generally orthotropic material so that is what we are interested in finding out so stress.

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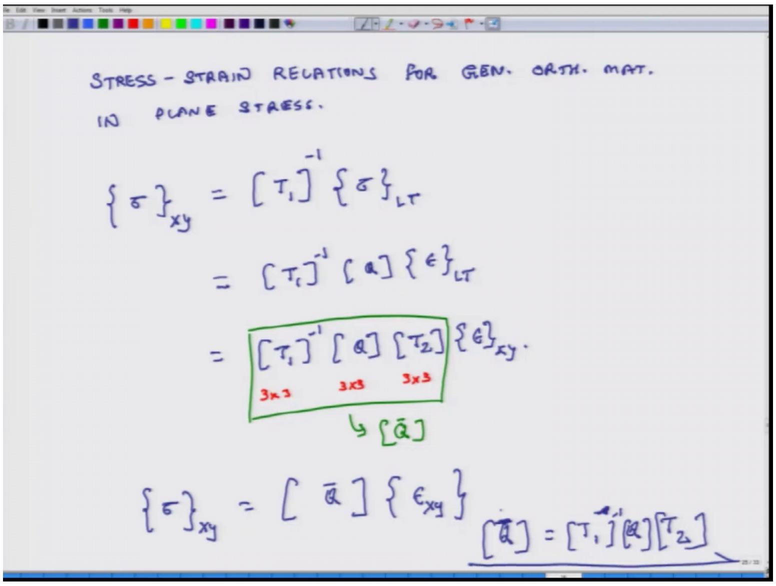

A strain relations for generally ortho material in plane stress plane stress is a still there. So, that is what we want to develop so, first we write an expression for sigma x y. So, what is sigma x y? This is equal to inverse of T 1 times sigma measured in L T coordinate frame and now I am going to express sigma L T in terms of strains. So, this is equal to inverse of T 1 and sigma L T is what? Q matrix times a strain vector in L T coordinate frame and now I am going to transform the strain vector in L T coordinate frame to strain in x y coordinate frame.

So, this is equal to T 1 inverse Q T 2 epsilon in xy coordinate frame. So, this is the 3 by 3 matrix, this is a 3 by 3 matrix, this is a 3 by 3 matrix. So, all these 3 matrices can be multiplied together and I call that Q by Q bar matrix and that will also be a 3 by 3 matrix. So, my relation becomes sigma in xy coordinate frame is equal to a Q bar matrix, it is not Q matrix, it is a Q bar matrix. So, it has a bar on top of it, times epsilon x y. Where Q bar equals T 1 inverse Q T 2. So, this should be inverse T 2.

So, this concludes our discussion for today and what we had shown is that using this transformation I can compute a Q bar matrix which connects a stresses in especially orthotropic plate, which is loaded in plane stress condition with in a stresses and strains in x y frames. So, that is where I would like to conclude our discussion for today and tomorrow we will continue this discussion further and we will develop this concept even further.

Thank you.