Advanced Composites
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Lecture – 12
Transformation of Stiffness and Compliance Matrices

Hello, welcome to Advanced Composites, today is the last day of the second week of this particular course. Yesterday, we just developed relation between stresses and strains I and specially orthotropic material loaded in a plane stress condition, in a situation, in a general reference frame. So, there we were not specifically aligned to a particular material access system, we were just making sure that the material is orthotropic. And it is thin and because it is thin, it will have a plane stress condition and for that condition the relationship between a stresses and strains in a general x y coordinate system.

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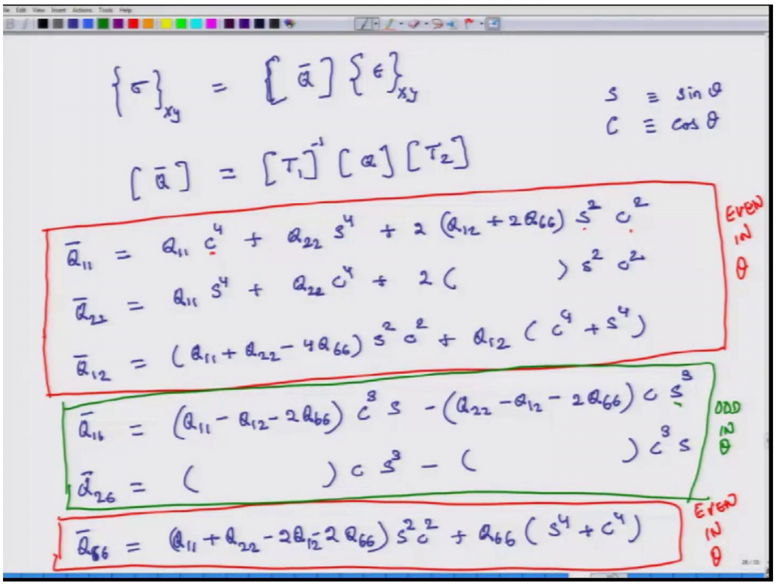

We had expressed was this sigma with respect to x y coordinate system equals Q bar matrix times the strain vector with respect to x y coordinate system. And to recap Q bar matrix is nothing but inverse of transformation matrix T 1 times Q matrix times transformation matrix T 2.

So, we know what exactly T 1 matrix is. So, we can take it's inverse, we know what is T 2 matrix and if we do all the computation the relationship between Q bars and Q S come out like this. So, Q 11 bar equals Q V 1 1 times cosine 4 theta, C means cosine and S means sin theta. So, Q 11 is equal to Q 11 bar times cosine 4 theta plus Q 22 sin 4 theta plus 2 Q 12 plus 2 Q 66 times sin square theta cosine square theta.

Similarly, Q 22 bar is equal to Q 11 sin 4 theta plus Q 22 cosine 4 theta plus 2 times this entire expression times sin square theta cosine square theta and Q 12 bar equals sin square theta. So, Q 12 bar I S equal to Q 12, Q 11 plus Q 22 minus 4 Q 66 times sin square theta cosine square theta plus Q 12 cosine 4 theta plus sin 4 theta.

Now, the Q matrix was having Q 16 and Q 26 has 0, but in the Q bar matrix Q 16 bar is not 0. So, I will write down the expression for that. So, that equals Q 11 minus Q 12 minus 2 Q 66 cosine cube theta sin minus Q 22 minus Q 12 minus 2 Q 66 cosine theta sin cube theta and Q 26 bar equals same thing.

But, it is cosine theta sin cube theta minus same terms in this bracket and this is cosine cube theta sin theta and finally, the expression for Q 66 bar, so that equals Q 11 plus Q 22 minus twice Q 12 minus twice Q 66 sin square cosine square plus Q 66 times sin 4 theta plus cosine 4 theta ok. And we will spend a couple of minutes looking at these expressions and I will make some observations Q 11 and Q 22 and also Q 12 their values do not change if I change the value of theta from plus theta to minus theta.

How are they dependent on theta? They are dependent on cosine 4 theta. So, if change from cosine 4 theta to cosine 4 of minus theta it is the same value sin 4 theta sin square theta cosine square theta. The same is also true for Q 66, it is also true for Q 66, it is value also does not change if I switch my theta to minus theta; so if theta is 30 degrees and in the other case if it is minus 30 degrees. So, values of Q 11 bar Q 22 bar Q 12 bar and Q 66 bar they do not change they remain same.

But so, these are even functions of theta. So, these are even in theta, this is also even in theta, but Q 16 bar and Q 26 bar let us look at these. So, when I change from theta degrees to minus theta degrees cosine cube does not change because cosine of positive theta and negative theta is the same. But, sin theta changes the sin theta changes to negative sin theta similarly the sin cube theta also changes. So, for this reason Q 16 bar.

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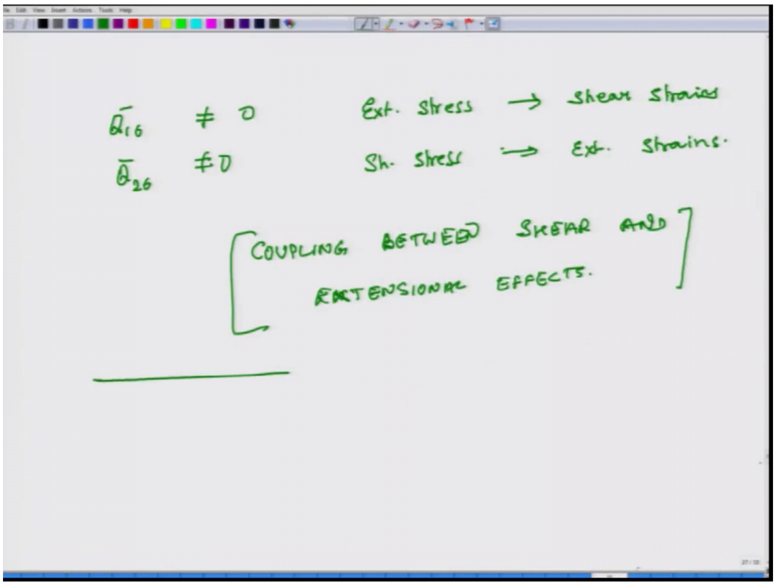

And Q 26 bar are odd functions in theta the other thing I would like to say is that because Q 16 bar and Q 26 bar they are not necessarily 0. This mathematically implies that extensional strains or extensional stresses they can generate shear strains. And shear stresses they can generate extensional strains and vice versa.

This was not the case for specially thermo orthotropic material because, Q 16 and Q 26 bar 0, but when is we move to the land of general orthotropy, then there is coupling between shear and extensional effects ok.

So, if you have a rectangular sample and you pull it, it will not only become longer, but it will also deform and it is angles at the corners will change from 90 degrees to something different. And it is important to note that the direction of this coupling is sensitive to theta. So, if you change from positive theta to negative theta, it is direction reverses, it is direction reverses because Q 16 and Q 26 bar are odd functions of theta.

So, it is important to get our conventions right. So, whenever we measure theta this is the convention we follow otherwise, we will not get our shear strains and things like or we will not get our solutions correct because of the sensitivity of Q 16 bar and Q 26 bar to the direction of theta. So, we should be clear on our conventions which are consistent with the conventions we have used in this formulation. So, I can once again write.

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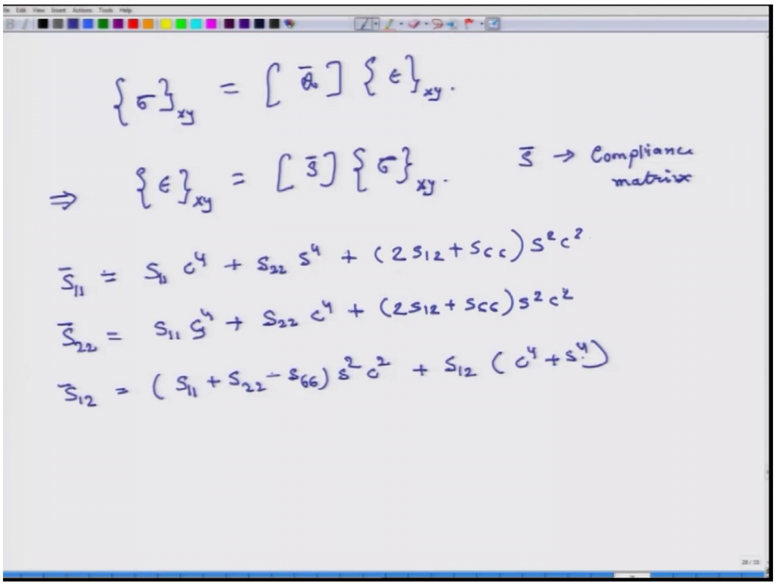

My stress vector in x y coordinate system as Q bar matrix times a strain vector if I multiply both sides of this equation by inverse of Q bar matrix then I can express strain in terms of a stiff in terms of a compliance matrix and stress. So, this gives me strain vector is equal to S bar matrix times a stress vector ok. So, stress bar is compliance matrix and it is nothing but inverse of Q bar.

So, the mathematical relation between different Q bars and S bars is this. So, this is equal to S 11 C 4 plus S 22 S 4 plus 2 S 12 plus S 66 S square C square. S 22 bar equals S 11 S 4 plus S 22 C 4 plus 2 this entire thing S square C square. And S 12 bar equals S 11 plus S 22 S 66 S square C square plus S 12 times C 4 plus S 4.

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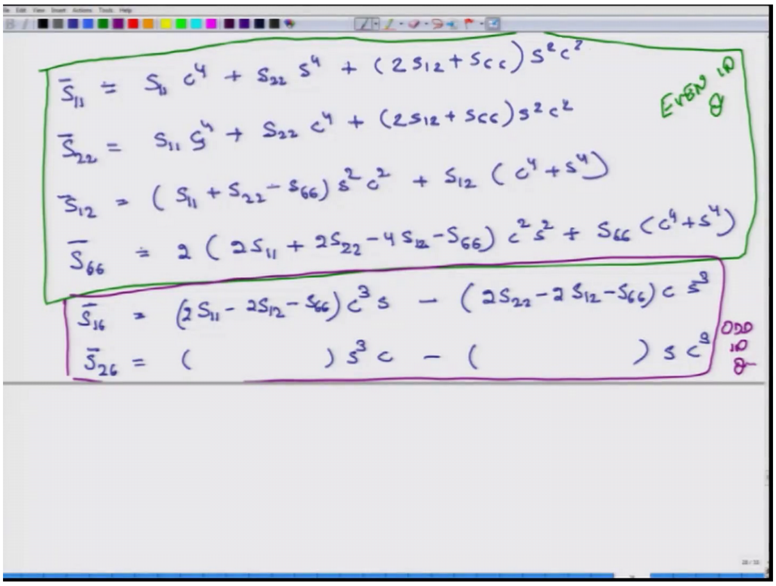

And then S 66 is equal to twice of twice of S 11 plus 2 of S 22 minus 4 of S 12 minus S 66. Cosine square sin square plus S 66 cosine 4 plus sin 4 S 1 6 bar equals 2 S 11 minus 2 S 12 minus S 66 cosine cube theta sin minus 2 S 22 minus 2 S 12. So, these are small error here this is 2 in bracket ok. So, it is 2 minus S 6 cosine sin cube and S 2 6 bar equals the same thing times sin cube cosine minus this thing sin cosine cube.

So, once again we see that S 11 bar S 22 bar S 12 bar and S 66 bar they are all even in theta. While these 2 guys S 1 6 bar and S 2 6 bar they are all odd in theta. So, it is important to understand it, lastly I will write the stress strain relationships between stress and a strain relationships in terms of engineering constants. So, those relationships are.

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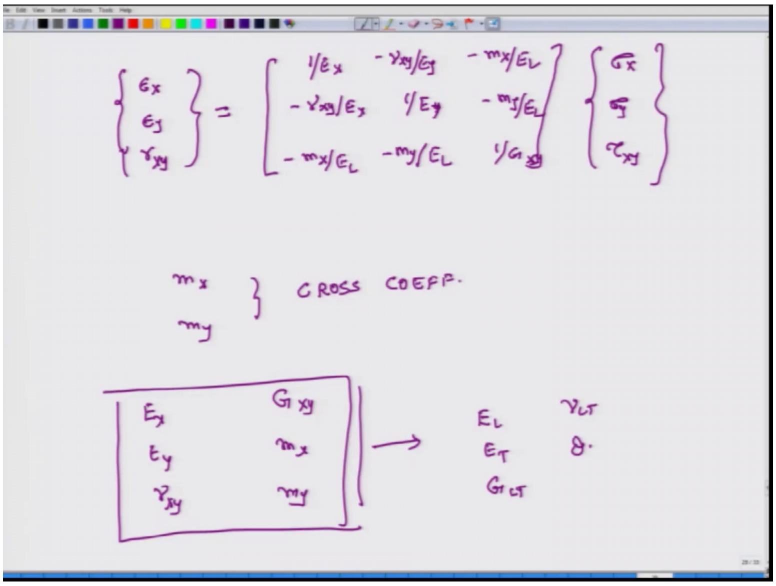

Given here epsilon x epsilon y gamma x y equals 1 over E x minus nu x y over E y minus m x over E L minus nu x y over E x 1 over E y and minus m by over E l, and this is minus m x over E L minus m y over E L and 1 over G x y times sigma x sigma y tow x y.

So, these are the stress strain relationship for a generally orthotropic material which is in a state of plane stress in terms of engineering constants you know engineering stiffness constants of the material which are E x E y E L m x m y and so, on and so, forth. So, here m x and m y, if these terms were 0 then they will not be any shear extensional coupling. So, these are the cross coefficients these are called cross coefficients and what do they do? They once again they couple the extensional response to the shear response and vice versa.

The relationships for E x E y nu x y G x y m x m y; so these 6, we have already explained these relationships in our previous course. So, you please go and refer to that and you will find the relation so, that E x E y nu x y g x y m x and m y all of these things can be calculated in terms of E L E T G L T nu L T and the transformation angle theta.

So, even though these are 6 constants, but they can be expressed in terms of is 4 fundamentally Independent constants for this orthotropic material under plane stress state.

So, that concludes our discussion for today and also the content which I wanted to cover in this week. I hope you have a wonderful weekend if you have any questions please do send us emails and next week we will move on to another topic where we will start discussing the behavior of laminates in terms of these constants and how we can relate stresses and strains in case of laminates.

So, with that I conclude our discussion. Have a great day and a wonderful weekend.

Thank you.