Advanced Composites
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Lecture - 13
Strain - Displacement Relations

Hello, welcome to Advanced Composites. Today is the beginning of the 3rd week of this course. And what we will cover over the period of this week is that we will learn how to develop stress-strain relations not at necessarily at the ply level, which we had discussed in the last week, but at the laminate level.

Now, you know that a laminate is basically a collection of several layers of composite materials. So, you can have a laminate, which has 10 layers, 20 layers or whatever. And what we are interested in is that if I apply an external stress on this laminate, what kind of strains and curvatures are produced in the laminate. And then also what kind of strains and stresses are experienced in such a situation at each layer of the laminate. So that is what we are interested in finding and that is what we will develop over the course of this week.

We had done this exercise in a great detail in the introductory course, but for purposes of completeness, we will again revisit this particular part of the course. And then from next week onwards, we will move into advanced areas of composites. So, the process for developing relations between stresses and strains at laminate level is a three step process.

The first step will constitute developing strain displacement relations, when the plate experiences some bending. So, suppose we have a plate, and we are trying to bend it. So, when I bend it, I apply some bending force or bending moment. And when it happens, then the plate develops a curvature. And as it develops a curvature, it also experiences some strains and curvatures. So, we will try to develop relations between these strains and curvatures and the displacement at each point in the plate. So, we will develop strain displacement relations. [vocalised-noise].

The second step will be that we will link these strains with stresses at the level of a ply, so that is the second step that we will develop stress-strain relations at a ply level or at the level of a lamina. And finally, we will develop relations between stresses and strains for the entire laminate. So, this is a three step process.

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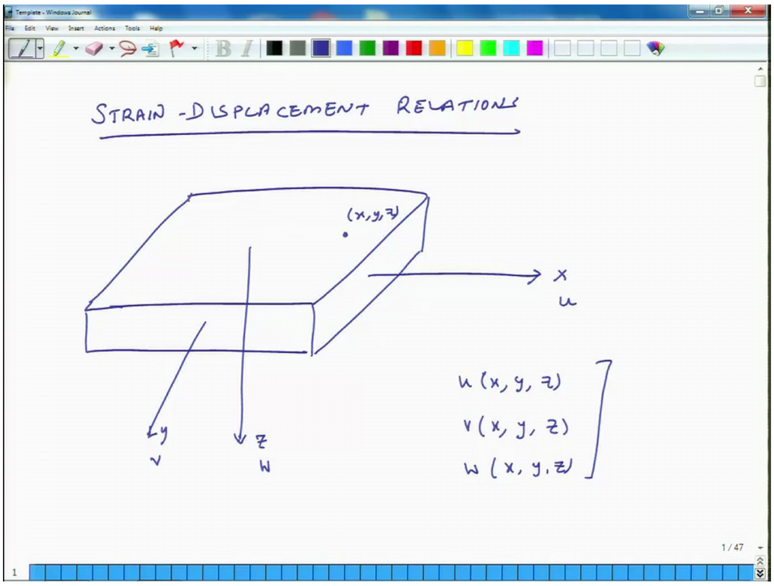

So, the first step is as I suggested that we will develop the strain displacement relations. So, as so this consider this plate. So, this is a laminate, it has several layers. And let us say that this is my axis system x, this is the y-axis, and this is the z-axis. So, the z-axis is going downwards. And the displacement in x is u, displacement in y direction is v and the displacement in z direction is w. So, if I consider any point and its coordinates are x, y, and z.

Then once the plate bends, then it experiences a displacement, which is u is a function of x, y, and z; v is also a function of x, y, and z; and w, which is the displacement in the z direction is x, y, and z. So, what we intend to do is that we want to develop relations between u, v and w. We want to relate these two strains in the plate strains in the plate ok. So, this is the problem at hand.

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Now, before we start doing that, I wanted to make two things clear. Basically, these are related to two assumptions, which we make. Typically, if there is a plate, consider a plate, and it has several layers of composites. So, let us say this is a book. And we can consider this book as a plate. And each of these pages, we could consider these as a lamina or a layer of composites.

Now, if I do not glue or bond these individual layers together and if I bend a plate. So, initially when I have the plate, let us say this is my x-axis, this is my y-axis, and this is the z-axis. So, this edge of the plate is 90 degrees. This edge of the at the corner of the plate is at 90 degrees to the xy, x and y-axis right. And it is parallel to the z-axis. And if these layers are not attached to each other rigidly, then when I bend the plate, when I bend the plate, this axis also bends and this not this axis, this edge of the plate is no longer at 90 degrees to is no longer I am sorry, it is no longer, parallel to the z-axis. There is an angle, which gets developed. And this happens, when there is no bonding between the layers, when there is no bonding between the layers. So, and that helps, so that is there. But, if there is a perfectly rigid bond between all the individual layers, and then when I bend it, then this edge of the plate will remain parallel to the you know, it will remain parallel.

And it will remain the angle between this edge of the plate, this edge of the plate, and this edge of the plate it will no longer change, if it is rigid. So, and if there is not a perfect bond, if there is no bond or if the per bond is not rigid enough, then when I bend the plate this angle between this edge of the plate, and these other two edges of the plate, this will change. And as a consequence if the bond is not perfect, so we will just note it down.

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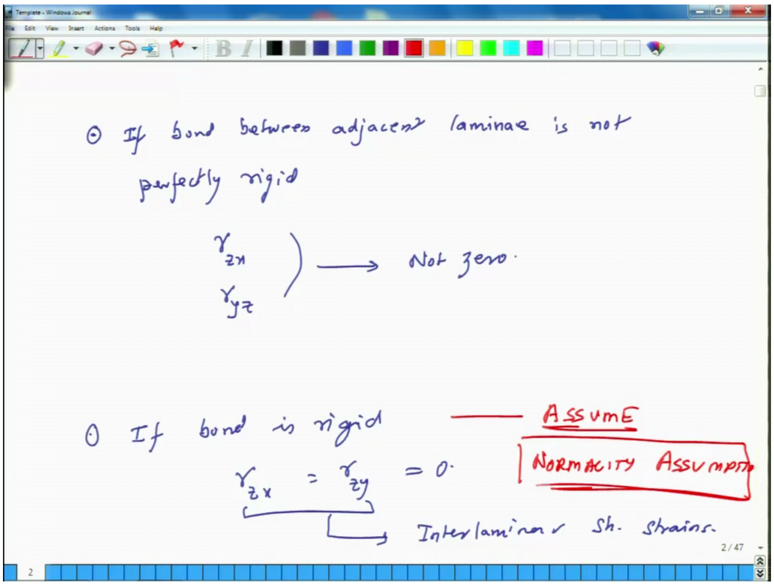

If bond between adjacent laminae or layers is not perfectly rigid, then what will we have? We will have a shear strain, because things each layer will slide over another slide other layer. And we will have shear strains gamma z x and gamma y z, they will not be equal to, so they will not be 0. So, they will be not 0.

Now, the theory which we are going to use is that, we will assume that the bond between individual layers is perfectly rigid. And when that happens, so if the bond is rigid, then gamma z x equals gamma z y is equal to 0. These shear strains are called inter laminar shear strains. So, we assume that the bond between adjacent layers is perfectly rigid and as a consequence of that the inter laminar shear strain that is the shear strain between two adjacent layer is identically 0. This is one thing we will assume. So, this is something we are going to assume.

And when that happens, when we assume that, then when the plate bends, so initially the angle between this edge, this edge, and this vertical edge is 90 degrees. And when the plate bends, even then the angle between this edge, this edge, and this edge still remains 90 degrees. So, this is an assumption. And this is we call it the normality assumption. So, we this is one assumption about plates.

So, what we are developing is a classical lamination theory, you know theory for classical known as classical lamination theory, classical lamination theory. So, this is the normality assumption. So, we make the normality assumption. The other assumption we make is that, suppose this is a plate. And when I put some stuff on it, when I put some weight on it, and I also apply some bending moment, then this thickness, the thickness of the plate, even after bending, it does not change ok.

And if this thickness does not change, if this thickness does not change, and that is a realistic assumption, because when the plate bends, the thickness will not change. There will be curvatures and things like that, but overall this thickness is not going to change. However, if I load the plate on this side, and also press it from this side, then the plate may get compressed or if I pull it, then it may extend, but that is not happening. So, we have just putting some normal forces, and that does not lead to a lot of contraction or expansion of the thickness of the plate. And if the thickness of the plate is not changing, then remember our axis system is that in the z direction we call it w.

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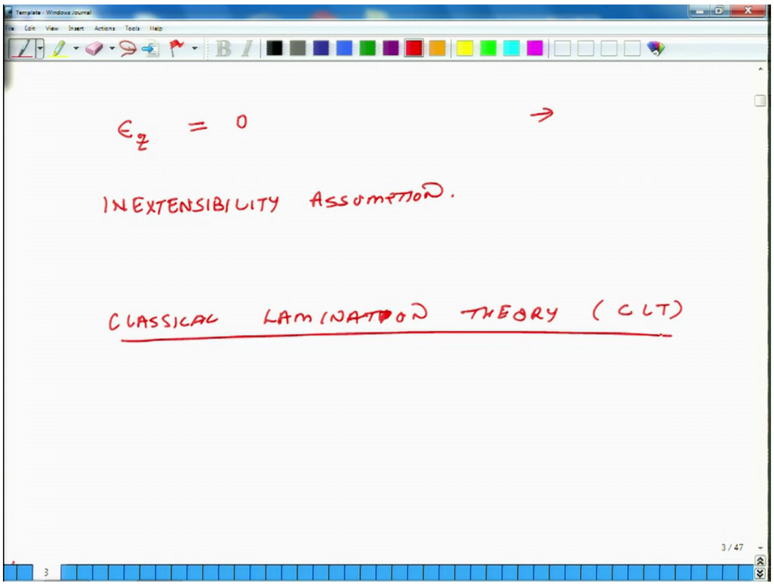

So, if the thickness is not changing, then epsilon z that is the strain extensional strain in the z direction is 0 ok. So, this is so this is another assumption. And this is called the inextensibility assumption. So, we use we are making two assumptions. One is the assumption of inextensibility that epsilon z is 0. And the other one is the assumption of normality.

So, from inextensibility assumption, we assume that epsilon z is 0. And from the assumption of normality, we assume, we come up with the relation that gamma x z and gamma y z, they are 0. So, these are the two assumptions. And using these two assumptions, we will develop a theory for bending of plates, which is known as classical lamination theory. C, it is sometimes abbreviated as CLT. So, this is the theory we are going to develop.

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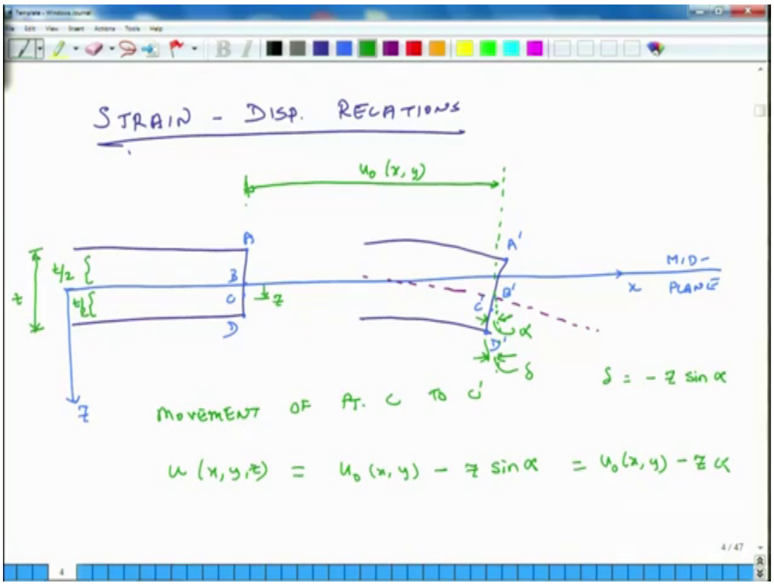

So, now we start developing this theory. And the first step in developing this theory is that, we will develop strain displacement relations ok. So, we consider a plate. And when we look at it from the side, suppose there is a plate, and I am looking it at from the end. So, I only see this surface ok. And let us say this is the mid plane of the plate. So, this is the mid plane.

So, remember it is not the neutral axis. It is the mid plane of the plate. So, if the plate is of thickness t, I have t plus t over to material up, and t over to material down. So, this is there. And let us say this axis is x-axis, and my z-axis is in this direction ok. And I have, I put some points on this. So, I put point A, B, C, and D. And the distance between B and C is z. And the overall thickness of the plate is t. And this is t by 2, and this is t by 2.

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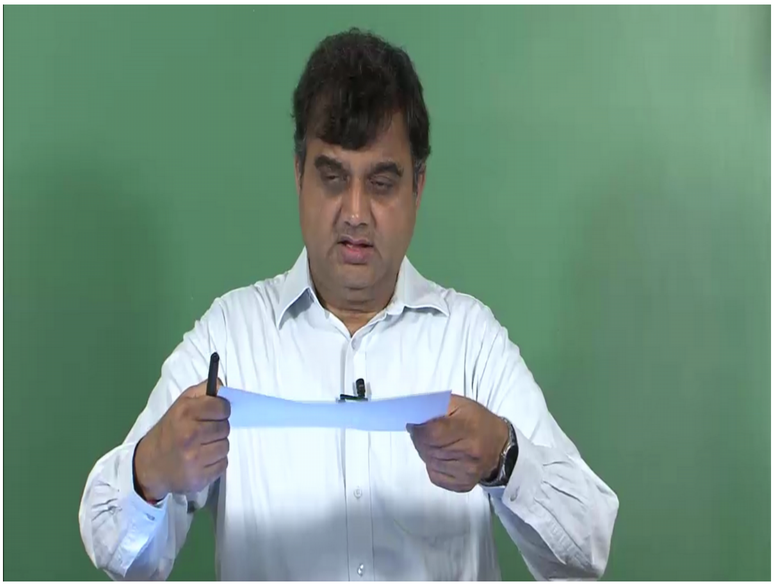

Now, what I do to this plate is suppose this is the plate, what I do to this plate is I pull it I pull it, stretch it. And as I stretch it, the plate will experience a displacement in the x direction, so that displacement is u and I also bend it so it will experience a displacement in x direction, and also a displacement in z direction, which is w.

So, when I pull it, and also bend it, let us say it this edge A D, this edge A D it moves out by some distance, and it also rotates. So, these edge A D, so that when I bend it, the plate becomes something like this ok. So, the displaced positions have A prime; this is the mid plane B prime; this is the point C prime; and this is point D prime ok. And again in the deformed configuration, this is the mid plane of the plate.

And what happens is that I have pulled it outwards. So, the midpoint of the plate, which is B, it moves to B prime and that displacement is called let us say we call it u 0, u 0 ok. And u 0 is a function of x and y, because it will vary from one point to other point in the plate. So, it depends on x and y. It does not depend on z, because u 0 is the displacement of the point on the plate, which is located the mid plane. And mid plane is always located at z equals 0. So, it does not depend on z, but it can vary from x and with respect to x and y ok.

So, the plate gets stressed by a distance u 0. And also this thing plate is rotating, and as a consequence, when it rotates let us say this angle of rotation this angle is alpha; so it rotates by some angle ok. So, what happens to point C, the movement of the points the what is the how much has point moved by so movement of point C to C prime, how much is had moved by, it has moved by a total distance u. And u is a function of x, y, and z hm.

And that how much has so again let us look at this point C. C has moved to C prime, what has happened to it, it has moved out by u 0, but it had because of rotation, it has also come in by this distance, let us call this distance delta hm. So, it has come inwards, it has moved out by u 0. And it has come inwards, because of rotation by this distance called delta.

And what is we can calculate delta. Delta is equal to how much it is minus z times sine of alpha ok. So, we can write it as z times sine of alpha. And if these rotations are not significant, they are not very large numbers. If alpha is a small number, then I can also write it as u 0, x, y minus z times alpha, because sine alpha is same as this thing.

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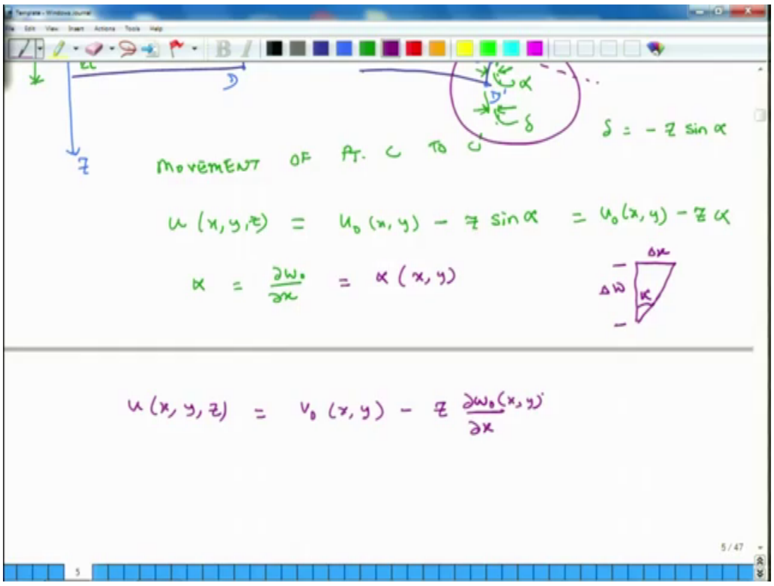

And we can also say that alpha. So, what is alpha, alpha is nothing but del w over del x ok. If you how do I get this, see if I look at this thing in detail, then what do I get, if I make a triangle in this area, then this triangle is something like this. So, this is alpha; this is delta x; and this is w or change in w. So, delta w over delta x is alpha ok, if other things are constant. And again we are computing it, this alpha in terms of mid plane deflection w naught, so alpha is also a function of x and y only alpha, because alpha depends on w naught. And w naught is the deflection of the mid plane of the plate. So, alpha depends only on the mid plane deflection of the plate. So, with that we can say that u, x, y, z equals u naught x, y minus, z times del w naught over del x. And w naught is a function of only x and y.

So, this concludes our discussion for today. Tomorrow, we will extend this discussion. And we will actually develop strain displacement relations. And please review this. And if you get confused, please do shoot us emails. And once again, we will meet tomorrow, and we will continue this discussion. And we will finally develop strain displacement relations for the classical lamination theory.

Thank you very much, bye.