Advanced Composites
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Lecture - 14
Relations for Stress and Strain along Thickness of Laminate

Hello welcome to Advanced Composites. Today is the second day of the ongoing week which is the third week of this course. Yesterday, we just started our journey in terms of developing strain displacement relations in context of classical lamination theory and the first relation we had developed was that for u which is the displacement of a point on the plate in the x direction and we had expressed this u displacement in terms of u naught. Which is the displacement of point corresponding point with same x and y coordinates on the mid plane of the plate and also the slope of the plate that is partial derivative of w with respect to x.

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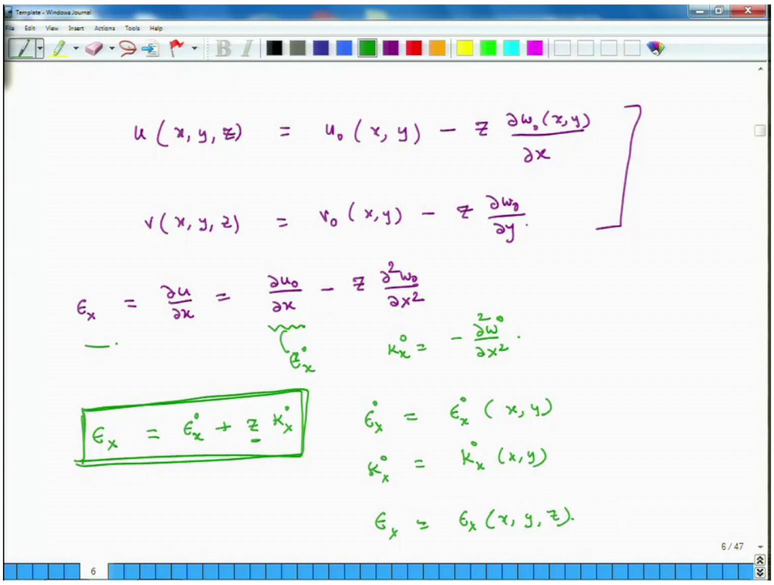

So, the relation which we had developed was u which can vary. So, it is u which is a function of x and y and z is nothing but u naught which is the mid plane displacement minus z times delta partial derivative of w and w is a function of x and y. So, it is because it is being calculated or measured at mid plane. So, partial derivative of w naught with respect to x. So, this is the relation for u.

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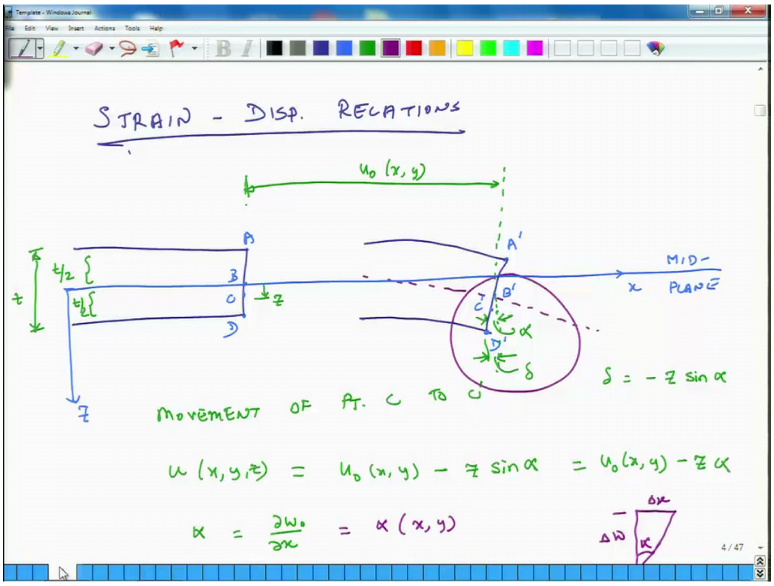

Likewise, this plate right now I bent it in the, I applied forces so, that it develops a curvature which is delta w over delta x. I can also bend the plate in the other direction and using similar third process, I can develop another relation for v; which is the displacement in the y direction. So, that is x v which is a function of x y z is equal to v naught which is the mid plane displacement located at coordinates x and y minus z del partial derivative of w with respect to y. So, these are the relations between u and v in terms of mid plane displacements u naught and w naught ok.

Now, we know that strain in x direction, epsilon x is equal to del u over del x. So, when I do this and I differentiate the entire thing I get del u naught over del x minus z del 2 w naught over del x square. Now, I call this thing as epsilon naught x which is the mid plane strain in the x direction and I call k x which is the curvature of the mid plane of the plate and I define it in such a way in such a way, so, I can write the relation that epsilon x is equal to mid plane strain in the x direction plus z times curvature of the plate at the mid plane.

Now, for this remember here epsilon x is mid plane strain it depends only on x y. So, if I have to expand it, it is a function of only x and y. Similarly, k x which is the curvature of the plate it only depends on x and y. So, it is a function of x and y only, but because I have this z in this relation, the overall strain for any point which is away from the mid plane is actually a function of x, y and z ok. So, this is the relation I developed from the first from this relation.

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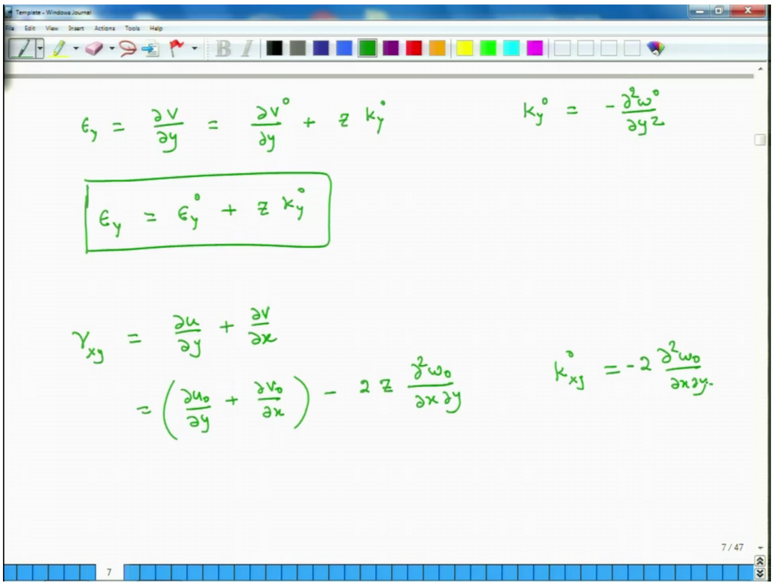

Likewise, I can also develop a relation that epsilon y that is equal to partial of v with respect to y and if I do similar mathematics what I get is del v at the mid plane with respect to y plus z times k y which is the curvature in the y direction. Where, k y in the y direction is defined as minus second derivative of mid plane deflection in the z direction with respect to y. So, I can also express this relation as epsilon y is equal to epsilon y at the mid plane plus the z coordinate of the point times k y which is the curvature in the mid plane direction and finally, we will also develop relations for the shear strain. So, we know that gamma x y equals gamma x y equals what? It is del u over del y plus del v over del x and if I do the entire mathematics again what I get is del u naught over del y plus del v naught over del x minus 2 z del 2 of the second derivative of w naught with respect to x and y.

And then I define a shear curvature. So, k x y naught is equal to negative of twice second derivative of w naught with respect to x and y.

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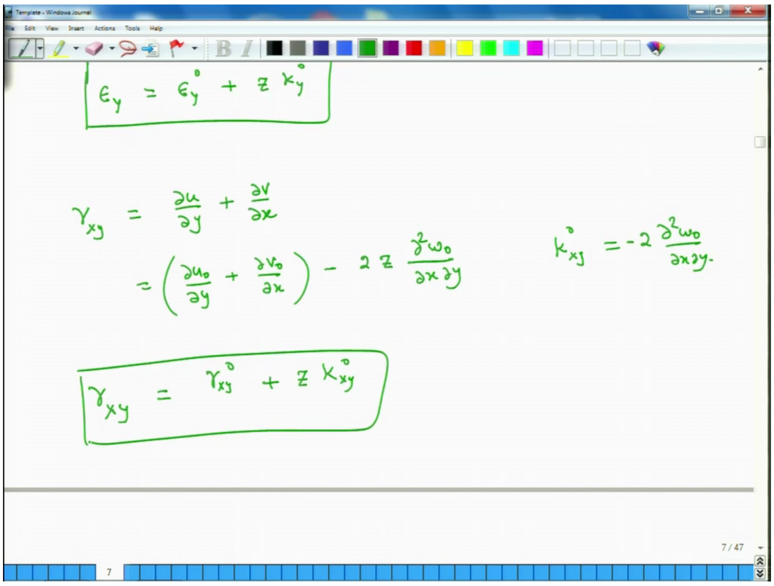

So, I get my final relation for shear strain and that is equal to mid plane strain component plus z times k x y. So, what we will do is we will summarize these strain displacement relations.

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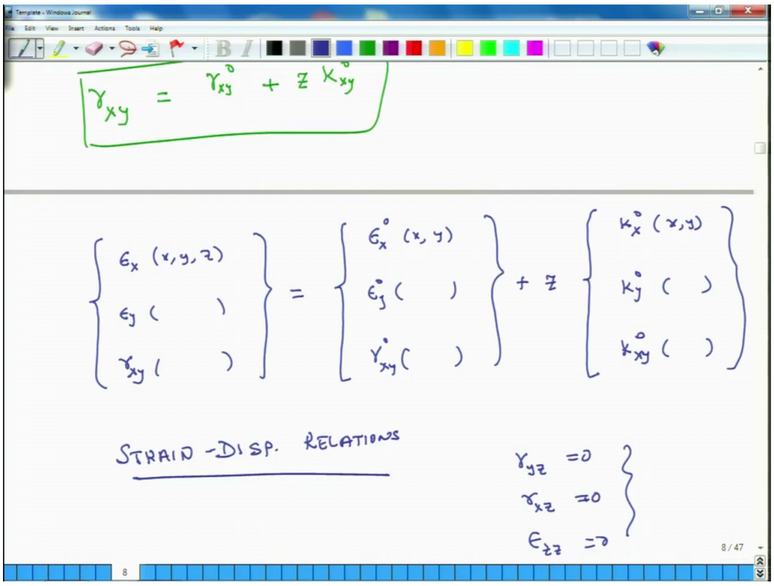

So, I can say that epsilon x epsilon y gamma x y and all these are functions of x y and z. These are equal to their mid plane components; epsilon x evaluated at mid plane, epsilon y evaluated at mid plane and gamma x y evaluated at mid plane plus the value of z coordinate times mid plane curvatures k x naught again evaluated mid plane k y naught evaluated mid plane and k x y naught evaluated at mid plane. So, these are the strain displacement relations.

Why are they strain displacement relations? Because on the left side, I have a strain and on the right side I have mid plane strains and mid plane curvatures and I know that mid plane strain epsilon x is del u over del x epsilon y naught is del v over del del y and gamma x y naught is del u naught over del v plus del v naught over del x and likewise we have definitions of curvatures also in terms of mid plane deflections which is w naught.

And then there so, these are three strains and then the what are the other three strains? The other three strains are gamma y z equals 0, gamma x z equals 0 and epsilon z z equals 0 and these are based because of our assumptions. The first two system strains are 0 because of the normality assumption and the last strain epsilon z z is 0 because of the assumption of in extensibility. So, this concludes our discussion on strain displacement relations and what we will do next is we will see that how these strains can be related to now stresses.

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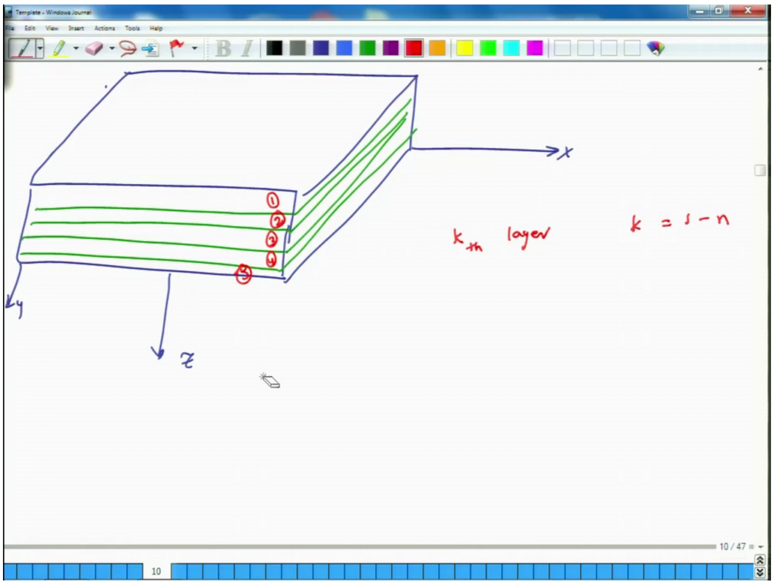

So, if I have a laminate and let us say this is x axis, this is my y axis and downwards is z axis and let us say there are several layers and likewise I have similar lines visible from the other side and let us say I number these layers. Let us say I call this layer number 1 2

3 4 and 5. So, this is a 5 layer laminate and what I am interested in fact, is in finding the stress let us say in some layer.

So, I am interested in finding stresses in kth layer, where k could be an index anywhere between 1 to n. If there are n layers in the laminate then k will be anywhere between 1 to n. So, suppose I am interested in finding out then what do I do? What I do is we know that what is the strain and stress relationship for any layer. If I know epsilon x, epsilon y and gamma x y for any layer then for that particular layer, I can write sigma x, sigma y and tau x y and this is we are discussing for kth layer.

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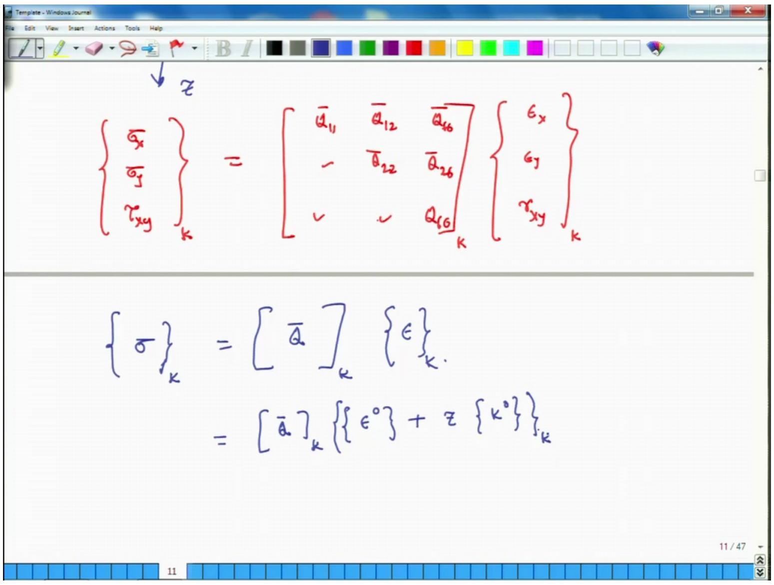

So, we will put a subscript k; this is equal to; so, for each layer there will be a Q bar matrix there will be a Q bar matrix. So, it will have Q 1 1 bar Q 1 2 bar Q 1 6 bar Q 2 2 bar Q 2 6 bar and Q 66 bar and then these are all this is a symmetric matrix.

So, I do not need to write other elements. So, this stress is equal to Q bar matrix times the strain. So, if I am interested in finding stress at any point then what do I do? I have calculated strains at that point and I multiply the strains with the proper Q bar matrix. So, what is this? So, this is epsilon x, epsilon y and gamma x y and these strains are varying from layer to layer right. So, this is for kth layer and this is also for kth layer.

So in short, I can write that my stress vector for the kth layer is equal to the product of the stiffness matrix for kth layer times the strain vector for kth layer. But we know that Q bar matrix, I mean this changes but this strain vector is nothing but different components, it has a mid plane component and a bending component. So, this I can express it as the mid plane strain and mid plane strain does not change from one layer to other. It is at the mid plane plus z times k mid plane curvature and the value of z is for the case layer.

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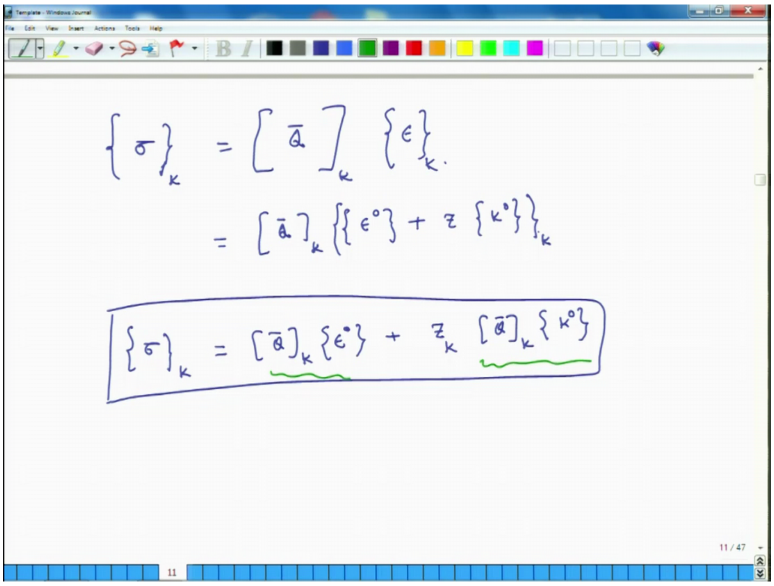

So, I can write sigma for kth layer is Q bar matrix for kth layer times the mid plane strain vector plus z for kth layer because curvature is again independent of layers times Q bar matrix for kth layer times curvature at the mid plane. So, this relation will help us calculate strain stresses at any point in a given layer. So, what do I do? Suppose, there is a particular point in a given layer, first step is that I find out its value of z based on our coordinate system.

So, once I calculate find out its z value then I if I know the mid plane strain and mid plane curvatures then all I have to do is multiply that mid plane strain by Q bar for that particular layer. So, I will get the first component and then I multiply Q bar with the mid plane curvature and I also multiply it with the z coordinate and I get and I add these up and I will get the stresses at that point in the laminate ok. So, this is one thing. The other thing I wanted to discuss is that how do stresses and strains variation of stresses and strains along the thickness.

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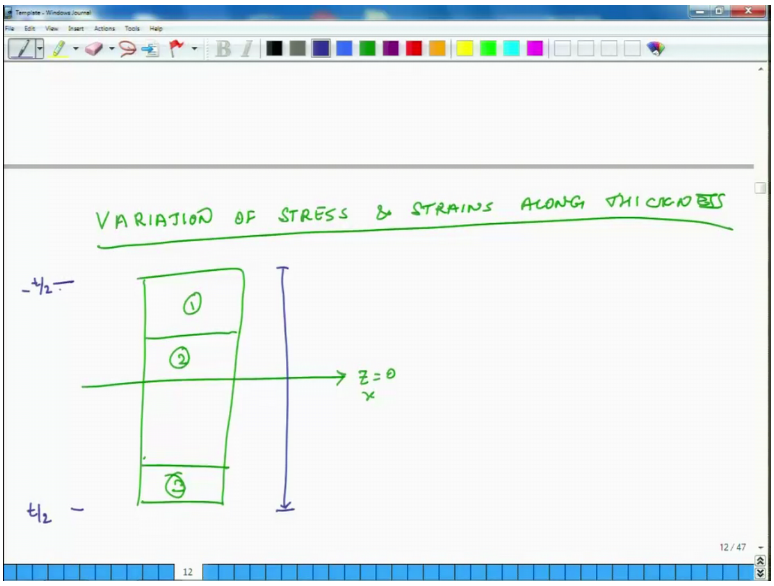

How do they change? So, what essentially what we will discuss is that suppose there is a laminate and let us say it has 3 layers and this is the mid plane. So, this is z is equal to 0 and this is the x direction and at the mid plane z is equal to 0 and this has let us say it has 3 layers layer number 1, 2 and 3, then what happens as I travel from here to here? How does a stress change? How does a strain change and how does the stiffness of the layer change as I move from z is equal to so, suppose this is t by 2 and this is another t by 2 ok.

So, the overall thickness of the laminate is t, then as I move from top surface of the composite to the bottom surface of the composite, how is strain changing across the thickness and how is the stress changing across the thickness?

So, this is what we will discuss in our next class and also we will then further advance this theory and we will go to the third stage of this overall process which will involve developing relations at the laminate level between forces, moments, strains and curvatures. So, that is the conclusion for today's lecture and I look forward to seeing you tomorrow.

Thank you.