Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 15
Stress - Strain Variation along Laminate Thickness

Hello, welcome to Advanced Composites. Today is the third day of the ongoing week and what we plan to do today is extend our discussion on stress strain relationships and we will explore how stresses and strains vary and change as we move along the direction of the thickness of a laminate. This is the first thing we will go and discuss and then later we will start developing relations between forces moments and mid plane curvatures and mid plane strains.

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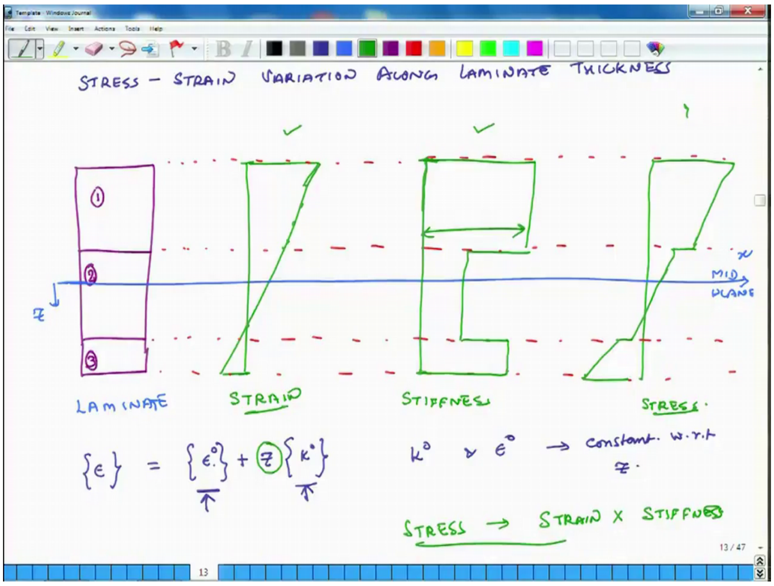

So, the first point of today's agenda is the stress strain variation along laminate thickness, ok. So, let us consider a laminate and for purposes of simplicity we will just make a laminate which has 3 layers. So, this is layer number 1, layer number 2 and layer number 3, so the top layer second layer and the bottom layer.

And then we have this laminate and this is the mid plane. So, this is my mid plane and this is the z direction, and if I go in this direction this is the x direction. Now, so this is my laminate. Now, the first thing we will explore is how does strain change along the thickness of the laminate.

So, we will just make some dotted lines for purposes of reference, these are reference lines. Now, let us consider the fact that epsilon is equal to the epsilon vector which is the strain vector is equal to the mid plane strain vector plus z times the mid plane curvature vector, ok. What it means is, so this portion and this curvature portion they do not change. So, k naught and epsilon naught, they are constant with respect to z. If x and y is the same then is z is changing they do not change.

So, if x and y is same and all what I am changing is I am moving up and down the thickness of the laminate. What that tells me is that this mid plane strain is not going to change across the thickness and mid plane curvature is not going to across the change across the thickness, but because there is a term z involving curvature what will happen is that the strain will change linearly across the thickness. So, if this is the thickness it may be that strain is very large number here and it may be doing this, ok, it varies linearly across the thickness of the plate. This 0 point may or may not be at the mid plane, it all depends; it depends on the actual values, but the variation of strain will be linear across the thickness, ok. So, that is there. So, this is how a strain changes across the thickness.

The next thing is we will let us see how is a stresses change across the thickness. But before we do that we have to see how stiffness changes across the thickness. So, again let us say this vertical line represents thickness. Now, consider the fact that the top layer, the first layer has one stiffness and this value it does not change along its entire thickness. It is the same material, so its stiffness is not changing, but the moment I move from layer number 1 to layer number 2 there is a different material. So, the value will change, and again it will remain constant across the second layer.

And then again as I move from layer number 2 to layer number 3 it may again change. So, stiffness behaves in this way. It is constant within a layer and as I move from one layer to other layer it jumps, and it can assume some different value. And the value can be calculated by calculating the q bar matrix, and the relations for that q bar matrix we have already discussed in previous lectures. And finally, we will look at how stresses change.

Now, stress is what? A stress is in general I mean broadly speaking you can say is a multiple of strain times stiffness right strain times. So, basically you multiply this and this plot and what you will get is the plot for stress, ok. So, what happens? The first for the first layer I am multiplying all these values of strains with some constant number which is this, and as I am multiplying this with this constant number the value will decrease because here it is a decreasing trend so value will decrease. And then when the next layer comes I have to multiply it by a lesser number and value decreases further. And then I again multiply it by another constant and maybe it may become like this.

Student: Sir, you do not (Refer Time: 07:42) stress either strain stress.

I am sorry. So, this is stress. So, overall observation is strain varies linearly across the entire thickness of the plate the stiffness is constant in each layer, but it jumps from one value to other as I move from one layer to other and stresses are linearly changing within a layer, but as I move from one layer to other layer they can jump, they can jump. So, actually this last segment is not accurate. So, it will again jump like this, ok.

So, stresses are linearly varying within a particular layer, and when I move from one layer to other they jump to some other value. So, stresses are continuous across the thickness, stiffness is discontinuous across layers but constant within a layer and stresses are discontinuous across layers, but continuously varying and linearly varying within a particular layer. And this understanding is very important because when we do advanced calculations on composites, we will use this understanding to understand the physics of several important problems. So, this is the first part of what we wanted to do today. Now, we will go to the next topic and which is relations for entire laminate, relations for entire laminate.

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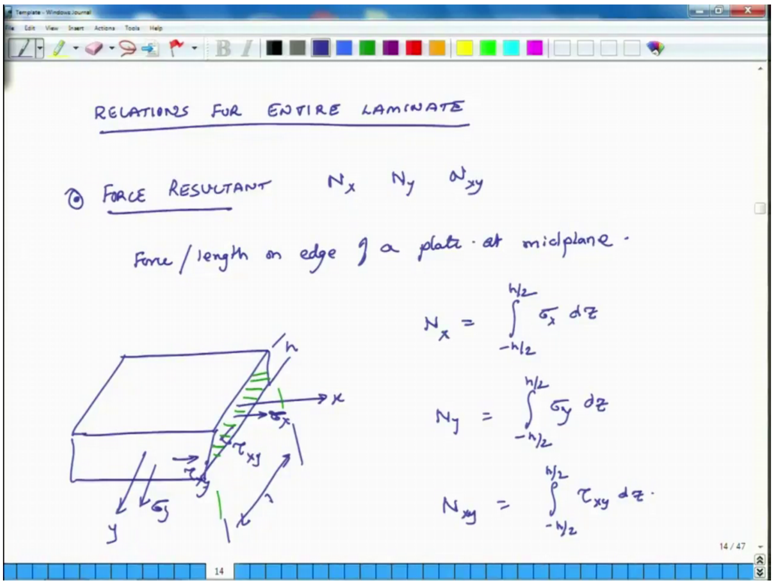

So, what we will develop now is we will start developing relations at the plate level. We, till so far we have been just been talking about the stress strain relations at the level of a particular layer. Now, before we start developing this we will introduce two terminologies one is called force resultant, and there are 3 types of force resultants N x, N y and N xy. So, what is a force resultant? It is force per unit length on edge of a plate. So, add the mid plane, this is important.

So, whenever we depict the force resultant we only draw the mid plane and we depict on on this. So, physically what does it mean? Suppose I have a plate, now if this is a plate and that is my x axis, this is the y axis and the vertical one is z axis and now we will we will be very conscious that whatever our axis system is it will go through the mid plane of the plate, ok. So, in this direction I have sigma x, in this direction I have sigma y and I have shear stress in these directions.

Now, what is N x. It is the force per unit length on the edge of the plate at mid plate. What that means, is that if I integrate this sigma x on this entire surface, ok. If I integrate this sigma x on this entire surface what will I get? I will get one the total thing which I will get is force, right. If I integrate stress on a surface, surface has area so I will get force, but what I am interested in is finding is how much force exists per unit length of the plate. So, suppose this is the length of the plate then that roughly you know in an approximate sense divided by the length of the plate is N x, but more precise definition of N x will be mathematically precise definition N x is equal to sigma x d z minus h by 2 to h by 2, ok; this is.

So, what does it effectively means is that if the plate has a length this dimension is unity then N x is the force per unit length of the plate. And h is the thickness of the plate, h is the thickness of the plate, this is h. Similarly N y is equal to sigma y d z and N xy is equal to integral from minus h by 2 to h by 2 tau xy d z. So, let us look at these.

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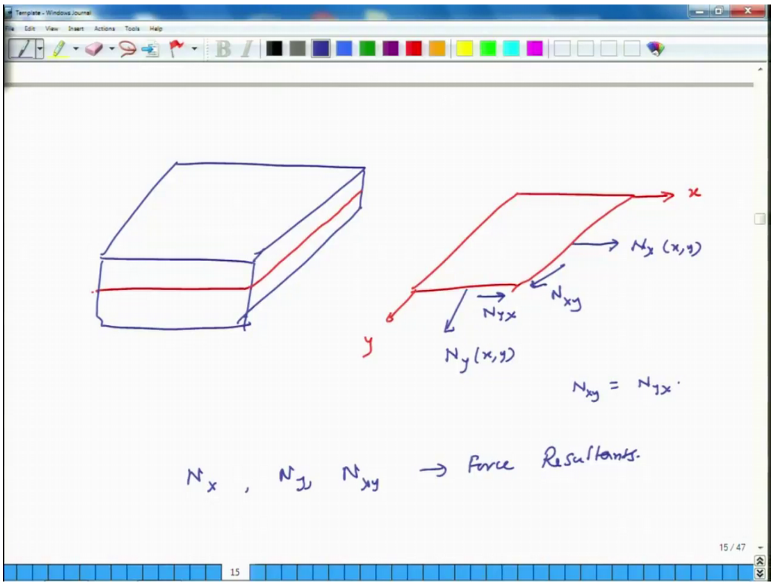

So, if I have to depict these I will depict them at the mid plane. So, again suppose this is the plate and let us say this is the mid plane of the plate, then if I have to depict N x, N y, N xy I will only depict them at the mid plane I do not depict them on the entire surface because I have already integrated them in the z direction. So, the surface is gone, ok.

So, I will just draw the mid plane of the plate. So, this depicts the mid plane and if this is my x axis, this is my y axis, and the downward axis is z, then N x is acting in this direction, N y is acting in this direction and this N x and N y it can change its a function of x and y, this is a function of x and y, ok. Because here the integration is has happened only in the z direction, ok. And, what is N xy? N xy is this thing. So, this is N xy and this is N y x and because tau xy is equal to tau yx. So, N xy equals N y x. So, this is these are force resultants. So, N x, N y, N xy, are force resultants. Next we will look at Moment Resultants.

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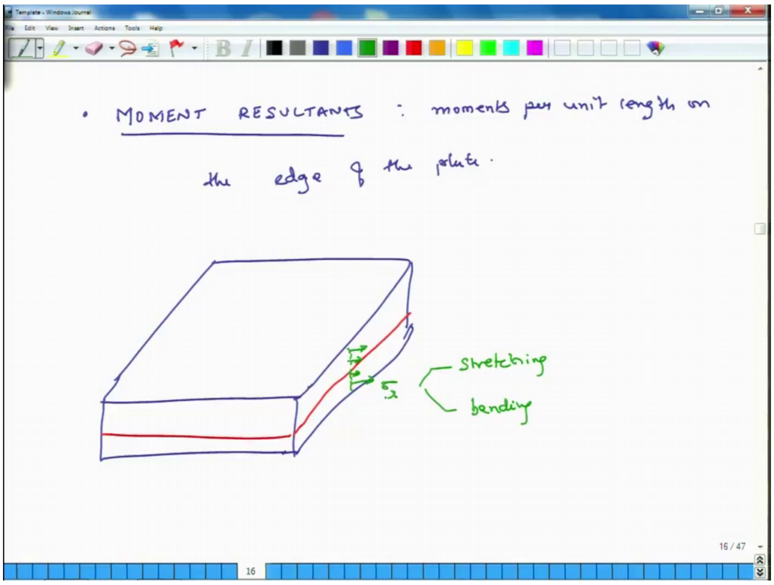

So, this is the other term we should understand before we start developing relations between at the plate level, so moment resultants, ok. So, these are moments per unit length on the edge of the plate, ok. So, let us understand what these mean.

So, suppose I have a plate and of course, this is my mid plane, this is the mid plane then if I have let us say if I draw vertical line on this, now it may have some sigma x and this sigma x may be varying on this. And as sigma x varies what will sigma x do? It will cause the plate to stretch and because it is not constant across the whole thickness it will also cause the plate to bend. So, this sigma x which is varying in the z direction which is varying in the z direction along this vertical line it causes two things, it causes stretching and it causes bending. Now, the stretching part we have already talked about by talking about N x, but we have not talked about bending, ok.

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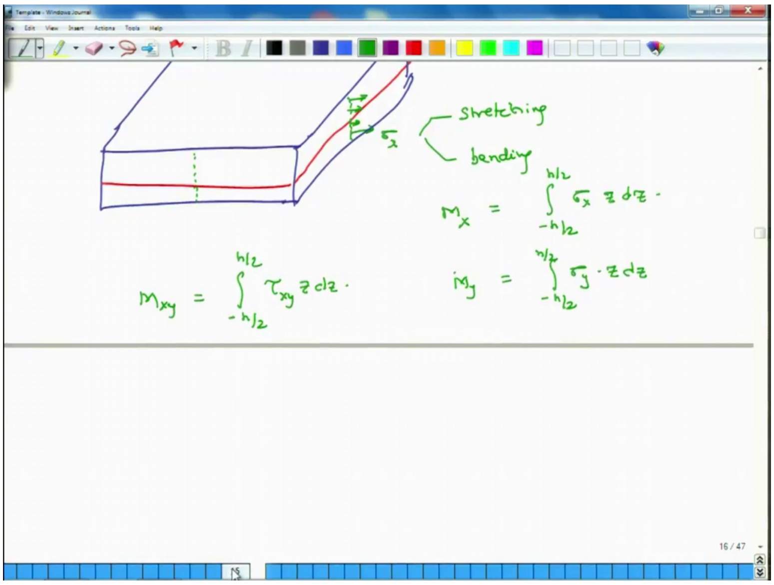

So, to understand how much the plate is going to experience the bending moment we will define a term moment resultant M x. And what is M x? M x is integral of minus h by 2 to h by 2 sigma x, z d z. This is the moment experienced by the plate in the x direction per unit length per unit length, and.

Student: (Refer Time: 18:49).

Yeah, similarly we have on the on this plane M y M y. So, this is moment resultant in the y direction and that is defined as minus h by 2 to h by 2, sigma y times z d z and there is also a moment because of the shear stress and that causes the plate to twist and that we can write it as M xy is equal to integral of minus h by 2 to h by 2 tau xy z d z. So, these are the 3 moment resultants.

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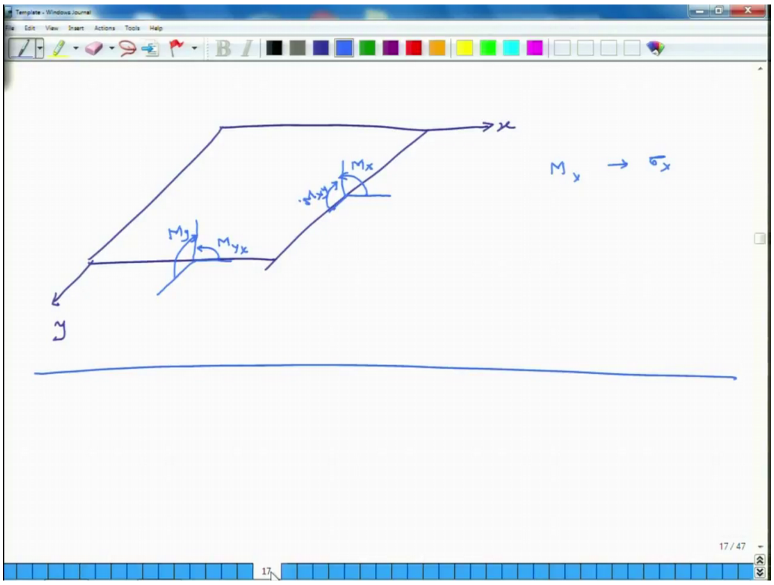

Now, let us depict them on the mid plane because we always depict these resultants on the mid plane. So, suppose this is the mid plane of the plate this is my x axis, this is the y axis, and the x is going downwards into the screen is the z axis, then my M x, ok. So, what is M x? M x is acting like this. So, this is positive M x, ok, this is positive M x. So, why do I call it M x? Because, so M x, we call it M x because it is related to sigma x and it is exiting on the x plane, it is acting on the x plane. This is, this is the, it is acting on the M x plane. Similarly let us see how M y x. So, this is the way this is positive M y; this is the convention for positive M y and finally, we will look at M x y, ok.

So, if the there is shear then this is M xy, this is positive M xy and this is positive direction. So, this is positive M xy actually strictly speaking this should be M yx, but because tau xy is same as tau y x then M xy is equal to M yx. So, these are the conventions M x acts on the x plane, M y acts on the M y plane, the direction of M x is counter clockwise rotating around y axis, and we call it M x because it is born of sigma x. We call M y as M y, M y as M y because it is born of sigma y and M xy is born of sigma xy or tau xy. So, that is why we name it like this.

So, this is the conclusion for today's discussion. With this background now that we have understood about force resultants and moment resultants. What we will do tomorrow is we will move one step forward and we will develop mathematical relations between strains, curvatures, and these resultant entities which are force and moment resultants.

So, that is what we will start developing starting tomorrow, and for that I look forward to seeing you tomorrow.

Thank you. Bye.