Advanced Composites
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Lecture - 16
Force and Moment Resultant (Part-I)

Hello, welcome to Advanced Composites. Today is the 4th day of the ongoing week which is the third week of the course, and what we plan to do today is develop relations at the plate level using classical lamination theory between force resultants, moment resultants, and mid plane strains, and curvatures. So, that is the purpose or objective of our today's class. And for that first we will develop or I will lay down the convention for numbering the layers and also identifying the z coordinates of laminated composite.

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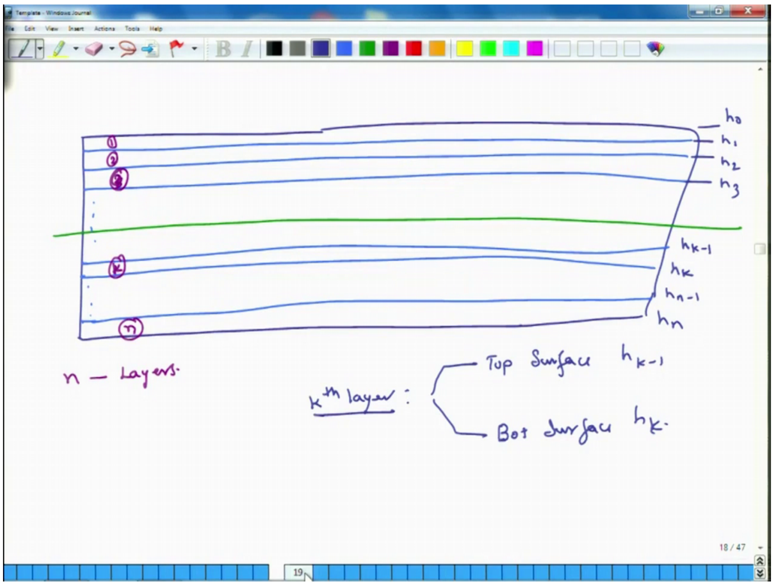

So, consider a laminate and let us say it has, this is the mid plane of the composite, and let us say it has several layers. So, this is the first layer, second layer, third layer and so on and so forth. And let us say this is the kth layer and then and then we could go and this is my nth layer.

So, first let us number these layer number 1, 2, 3, kth layer. So, this is nth layer. So, total number of layers is n. Then let us look at the quadrate system. So, if the overall thickness is t, then the top layers coordinate system is h 1, then this is h 2, then this is h 3, then this is h 4, and I keep on going down. So, for the kth layer the top surface has a z coordinate of h k minus 1, and the bottom layer bottom surface has a coordinate of h k, and similarly the top surface of nth layer has a coordinate h n minus 1 and the bottom surface has a coordinate h n.

Student: (Refer Time: 02:54).

I am sorry yes you are right. So, there is a error. So, the top surface is h 0, h 1, h 2, h 3 and so on and so forth, so this is there. So, for kth layer its top surface has a coordinate h k minus 1 and its bottom surface has a coordinate h k. So, this is the convention we are going to follow. With this convention in mind now we start developing the relation between N, M, mid plane strain, and mid plane curvatures.

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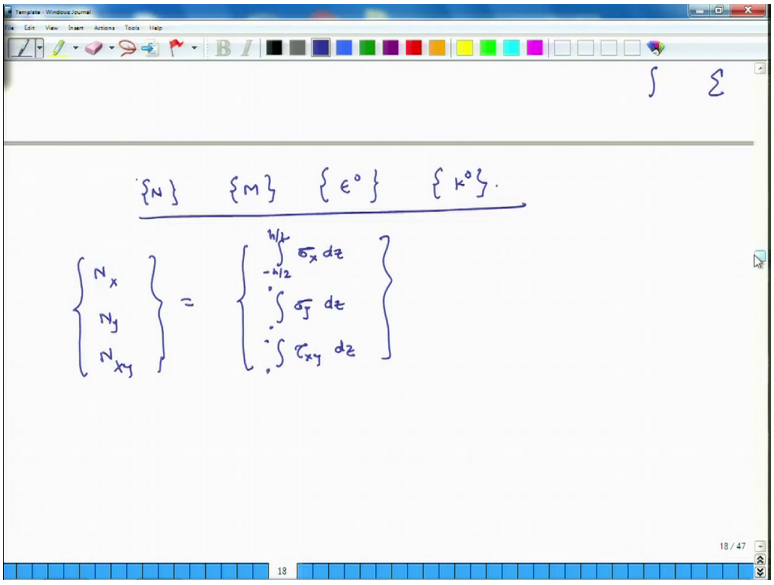

So, this, these so we will link these entities we will first develop consider the force resultants. So, the force resultant N x, N y, N z, excuse me, N xy. We have defined these, these 3 things as integral of sigma x d z, integral of sigma y d z, integral of tau xy d z and in all these things it is from minus h over 2 to plus h over 2 and same things for other two integrals, ok.

Now, we know and we have explained this also earlier that as I move from one layer to other layer within a layer the stress is continuous, but as I move from one layer to other layer the stress jumps. And whenever there is a jump happening I cannot use this kind of an integral right rather. So, within a layer I can integrate a function, but as it moves from one layer to other layer I have to use instead of integral I have to use a summation sign. And if there are 10 layers then I have to sum this thing 10 times, right.

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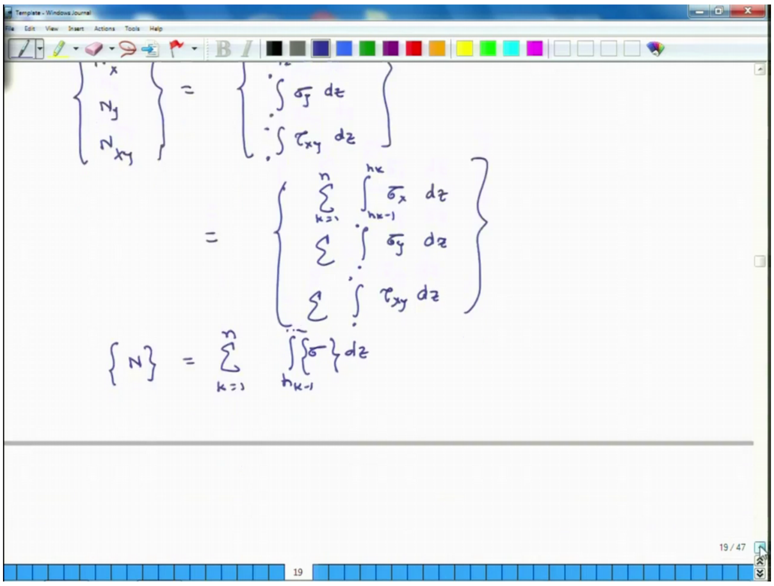

So, for the entire laminate strictly speaking this relation has to be modified and it has to be modified like this, sigma y d y, tau xy, d z. So, these should be d z, and if there are n layers then this integral has to be from this summation has to be from k is equal to 1 to n and the summation and the integral for kth layer will be from h k minus 1 to h k, right. Similarly, for the next equation also the same things will happen.

But we also know that, ok. So, this is there and the next thing I can do is I can abbreviate this I can abbreviate. So, left side I can just write it as N vector rather than writing N x, N y and N xy, and the right side I can write it as. So, I am bringing this summation sign outside and inside I have minus no, h k minus 1 to h k and sigma vector excuse me d z. This is how I can express this mathematically. And the summation is happening for k is equal to 1 to n layers. So, I have written the same equation, but written it in a more compact form that is all I have done.

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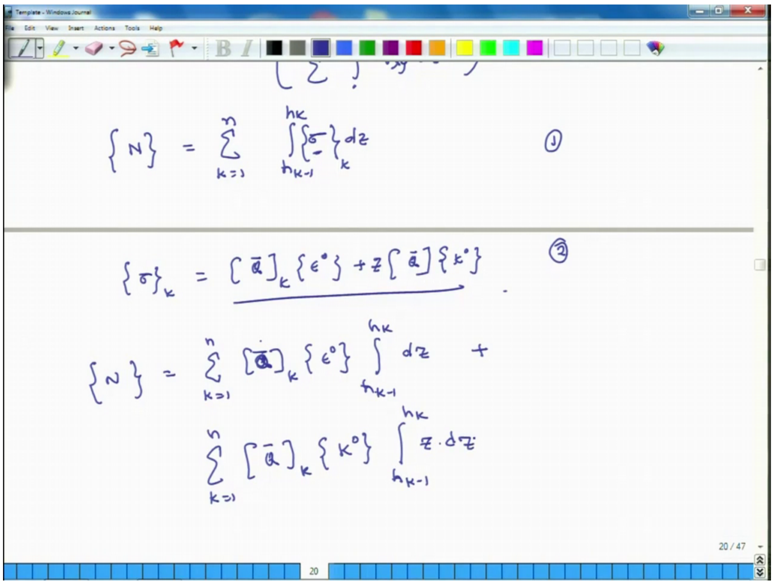

We also know that is stress. Now, this is stress evaluated for the kth layer evaluated and k is changing from 1 to n. So, we know that stress for the kth layer is what, is nothing but Q bar matrix for the kth layer times mid plane strain plus Q bar matrix for the kth layer times mid plane curvature and this entire thing is to be multiplied by z, ok. So, let us call this equation 1 and let us call this equation 2. So, combining 1 and 2 what we get is N, vector N, which is N x, N y, N xy; this is equal to summation of. So, what I am doing is instead of sigma I am putting this entire thing here, ok.

So, what I get is summation k is equal to 1 to n and Q bar matrix is common. So, I take it out, epsilon naught is also constant along the thickness of the plate. So, that also I can pull it out. So, there has to be an integral here h k minus 1 to h k, d z plus the second component k is equal to 1 to n Q bar matrix for kth layer times curvature at the mid plane times integral h k minus 1 to h k z times d z.

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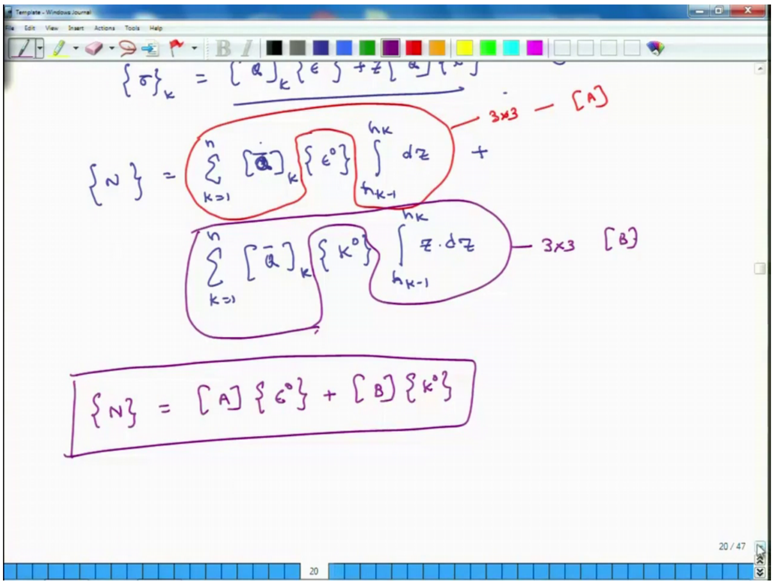

So, this entire thing is a 3 by 3 matrix, because Q bar is a 3 by 3 matrix, and this I call a A matrix. And this entire thing is another 3 by 3 matrix because again Q bar is a 3 by 3 matrix and I call it a B matrix.

So, A matrix is multiplied by a vector which is having 3 elements strain vector and B matrix is being multiplied by the curvature vector. So, I can write this entire relation as that N is equal to A matrix times a strain vector plus B matrix times curvature vector. So, this is an important relation and it links mid plane strains and mid plane curvatures to force resultants.

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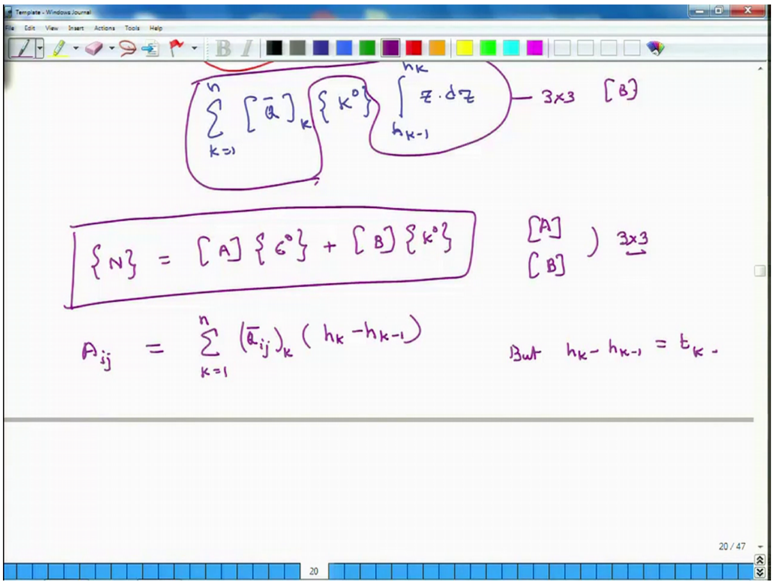

And what we will do is we will define, so A and B these are 3 by 3 matrices. So, we will define what are there elements and based on these definitions we can say that A ij is equal to summation of Q ij bar for the kth matrix times h k minus h k minus 1, and I am integrating it or summing it from k is equal to 1 to n, ok. But we know that h k minus h k minus 1 is the thickness of the kth layer, h k minus h k 1 is the thickness of the kth layer.

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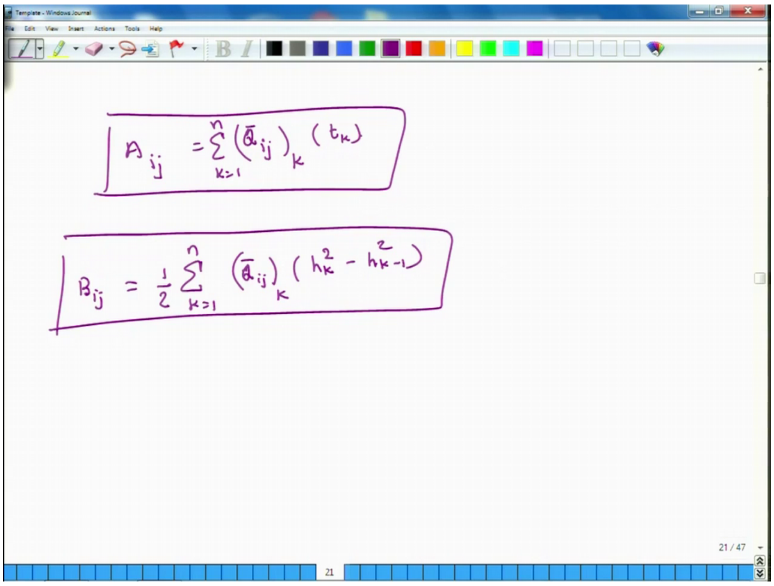

So, I can say that A ij is equal to Q ij bar and I am summing it up for k is equal to 1 to n. So, this is for kth layer times t k, again t is thickness of the kth layer. So, this is the definition of A matrix.

Likewise B ij is equal to summation of Q ij bar times h k square minus h k minus 1 square and this, and there is a factor of half k is equal to 1 to n and this Q ij is also for the kth layer. And this square divided by 2 comes because I am here integrating z times d z, z times d z. So, I get z square by 2 and when I take its limit I get h k square minus h k minus 1 square, ok. So, that is what I get. So, this is my definition for the B matrix.

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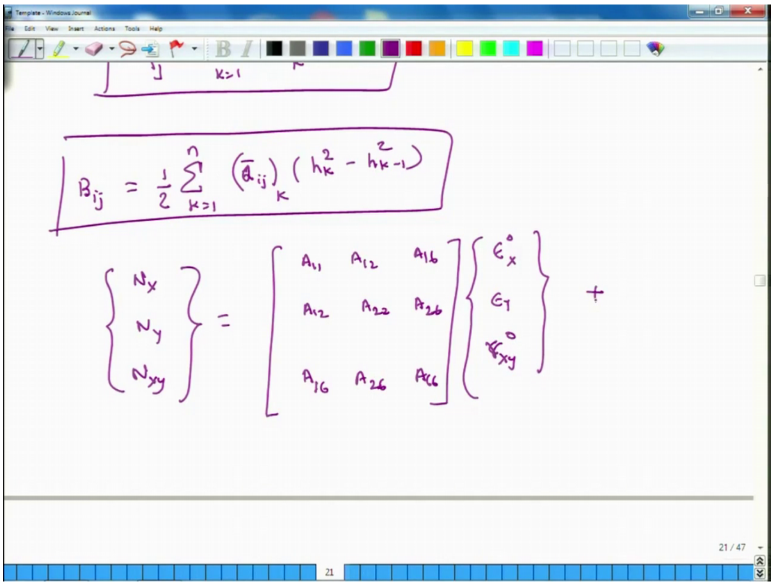

So, overall I can write it as N x, N y, and N xy, equals A 11, A 12, A 16; A 12, A 22, A 26; A 16, A 26, A 66 times mid plane strains.

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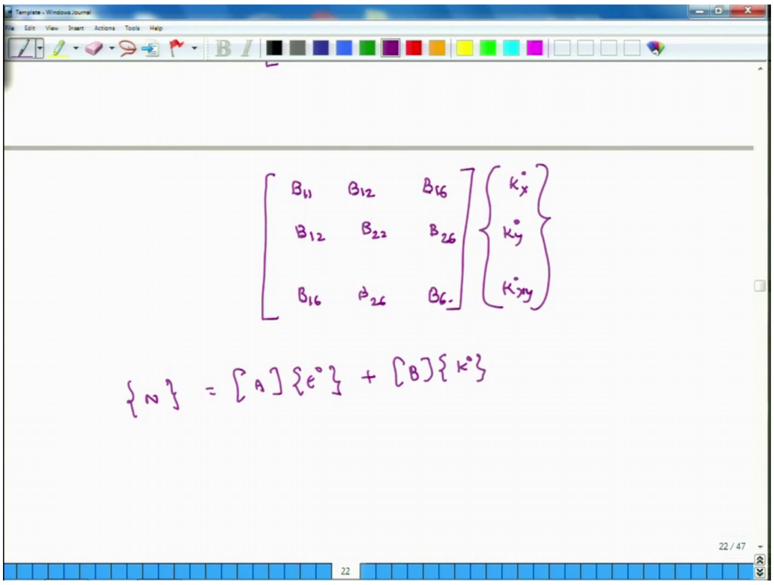

Plus, the B matrix, B 12, B 22, B 26; B 16, B 26, and B 66, ok. So, these are the relations or in brief I can write the same relation as N equals A times epsilon naught plus B times mid plane curvature strains, ok. So, this completes our discussion for developing a relation between N and epsilon naught and k naught. Very similarly we will develop a similar relation with for moment resultants, but that we will do tomorrow because the time for today's class is over. So, I look forward to seeing you tomorrow.

Thank you.