Advanced Composites
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Lecture - 17
Force and Moment Resultant (Part-II)

Hello, welcome to Advanced Composites. Today is the fifth day of the ongoing course of the, of this course which is into its third week. And what we will do is we will today have a very similar discussion on lines which we had yesterday. Yesterday we had developed an expression between force resultants, and midplane strains and curvatures. Today we will develop a similar relation between moment resultants, and midplane strains, hand midplane curvatures.

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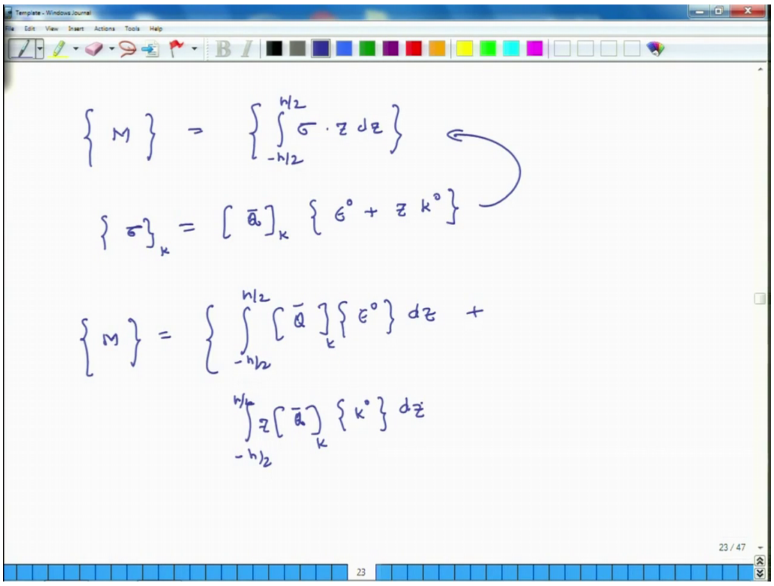

So, we will again use the similar mathematical process, but we may not necessarily go into that greater detail because the basics have already been developed earlier, but we know that moment resultant is what it is equal to integral of sigma times z times d z right, this is the basic expression.

Now, essentially these are 3 equations there is an equation for sigma x, there is an equation for sigma y and there is an equation for tau xy. And this integral happens between the limits minus h over 2 to h over 2, where h is the thickness of the plate. But we know that sigma vector is what? Is nothing but Q bar vector and it changes from one layer to a layer because the material is changing. So, we have to talk about kth layer. So, for kth layer sigma is like this, ok. So, we plug this back into this and eventually we get for moment resultant equals Q bar times epsilon naught D z plus minus h over 2, h over 2 Q bar. So, actually this k subscript should be outside and there is a z here times the curvature vector D z.

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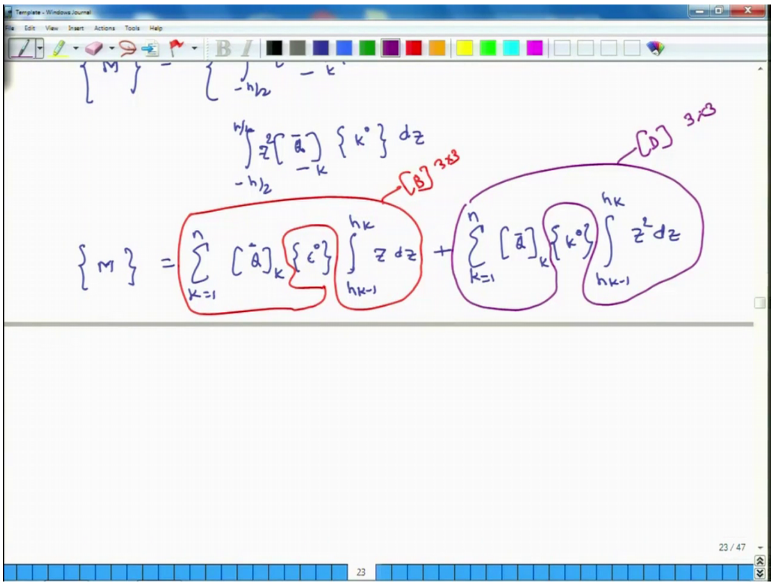

And because this Q matrix is discontinuous across different layers, so I will integrate within a layer and then as I move from one layer to other layer I will sum up things. So, I can write it as M vector is equal to, so there should be z here and there is a z square there.

So, it is a summation k is equal to 1 to n, Q bar is constant over a thickness and strain is constant over the entire thickness times integral of z d z, h k minus 1 to h k plus summation k is equal to 1 to n, Q bar for the kth layer times curvature vector times integral h k minus 1 to h k z square d z hm. So, if I do this and I add up all the terms then this entire thing we have already defined as a B matrix which is 3 by 3. But now we have a new term involving z square d z integral and which is this and this gives us a D matrix and this is also a 3 by 3 matrix, ok.

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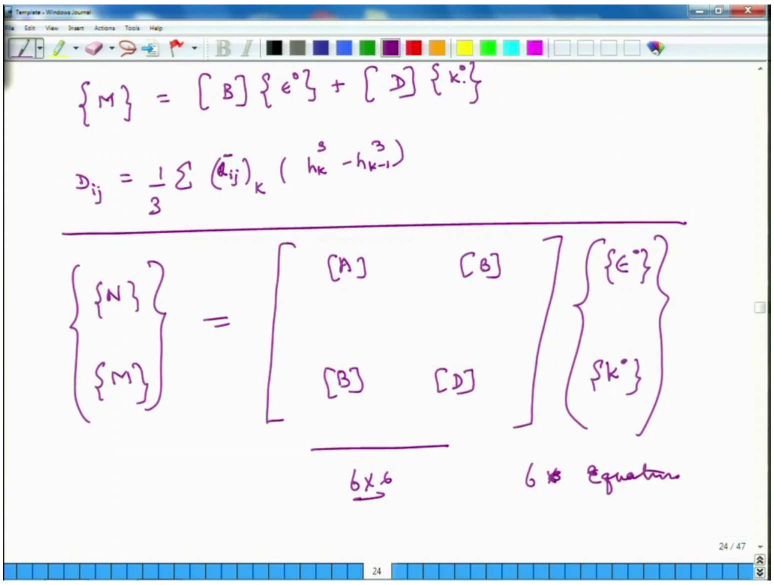

So, we say that the moment resultant vector is equal to the B matrix times the mid plane strain plus a D matrix which is again 3 by 3 times the mid plane curvature, and we have already defined the B matrix. So, we will not do that again. But we will define that D ij is equal to summation of Q ij for kth layer times h k cube minus h k minus 1 cube into 1 by 3.

Overall, we can say, so we can assemble all these relations for moment and force resultants and we can say that force resultants, and moment resultants they are what? They can be calculated from this large set of relations. So, you have mid plane strain vector and mid plane curvature vector, and this is A matrix, B matrix, B matrix and this is D matrix and all these are also vectors of size 3. So, this is a 6 by, so these are 6 equations and this is a 6 by 6 matrix, ok.

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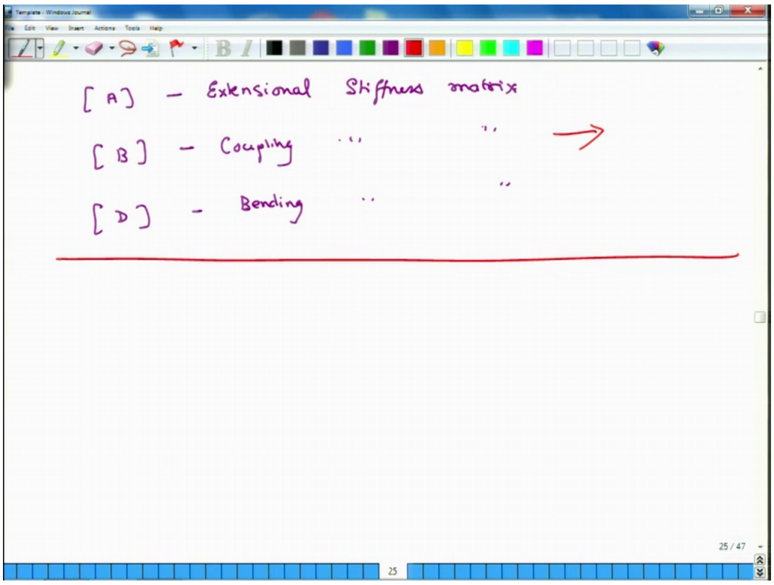

So, this is what I wanted to discuss. Now, now that we have developed relations for these we will give these matrices A, B and D some names. So, the A matrix is called extensional stiffness matrix, ok. The B matrix is called bending, no I am sorry it is called coupling stiffness matrix, and the D matrix is called bending stiffness matrix.

So, we will spend the moment trying to understand why we these things have been given these names. So, consider a case that we have a situation where B is 0, where B is 0, if B is 0 then N can be calculated if we know A and epsilon only, we do not need to know k. Essentially what it tells us is that A matrix helps us define how much the material extends by stretches or gets compressed particularly, it gets compressed and in presence of stresses extensional stresses and extensional strains. And similarly the D matrix influences how much the system bends, if the D matrix values are extremely large then the bending will be very small and vice versa. So, that is why it is bending stiffness matrix.

And the B matrix is a strange creature it is a coupling matrix and what it tells us is that if B matrix is not 0, then if we try to stretch if I apply just a plate if I have a plate and I am just trying to stretch it and if B matrix is not 0 then the material will not only get stretched, but it will also bend. And similarly if I try to bend the plate and if the B matrix is not 0 it will not only bend, but it will also get stretched. So, that is why it is called a coupling matrix. So, this is the understanding behind their nomenclature.

The next thing what we will do is we will discuss in next few minutes how do we provide names to different lamination sequences. So, nomenclature for lamination sequences.

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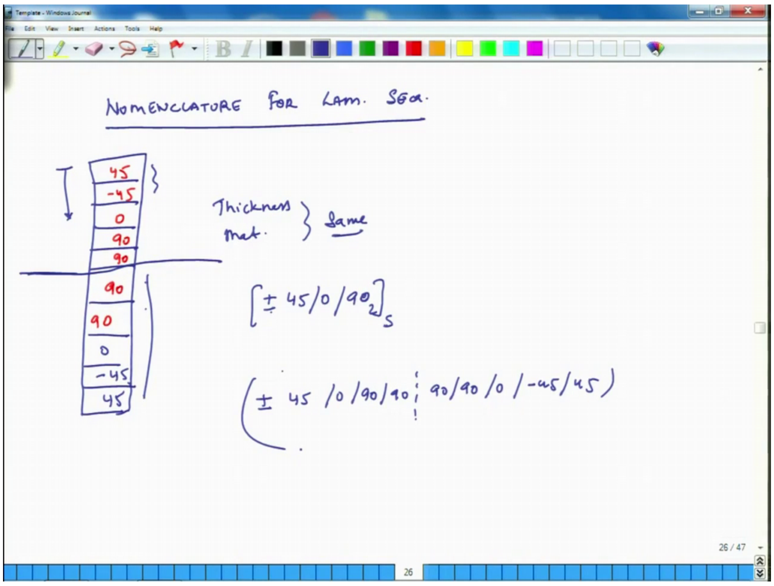

So, suppose there is this is the mid plane let us say there are several layers and let us say what I will do is I will put their angles here 45, minus 45, 0, 90, 90, 90, 0, minus 45, 45, and we will just. So, these are the orientations and we assume that thickness is, thickness and material is same for each layer. If that is the case, if that is the case then we can designate the lamination sequence of this. So, we always start from top and we go down. So, we say 45 and then there is a minus 45 next to it, so we say plus and minus 45 we will just rectify this. I think I had drawn it or correct, but then suppose this is the lamination sequence then these two are plus 45 and minus 45, so I write plus 45 and minus 45 then there is a 0 degree layer. So, I separated by a slash.

And then there are four 90 degree layers but this is the mid plane, this is the mid plane and mid plane is passing through this thing. So, I write 90 and then all other layers are just mirrored because this is the plane of symmetry, so I just say symmetric. In a long way this, if the moment I write s essentially what it means is that the lamination sequence is plus 45, minus 45, 0, 90, and there are two layers above the mid plane, so this is 92. So, 90, 90, this is the mid plane 90, 90, 0, minus 45, and 45 this is what it means, ok.

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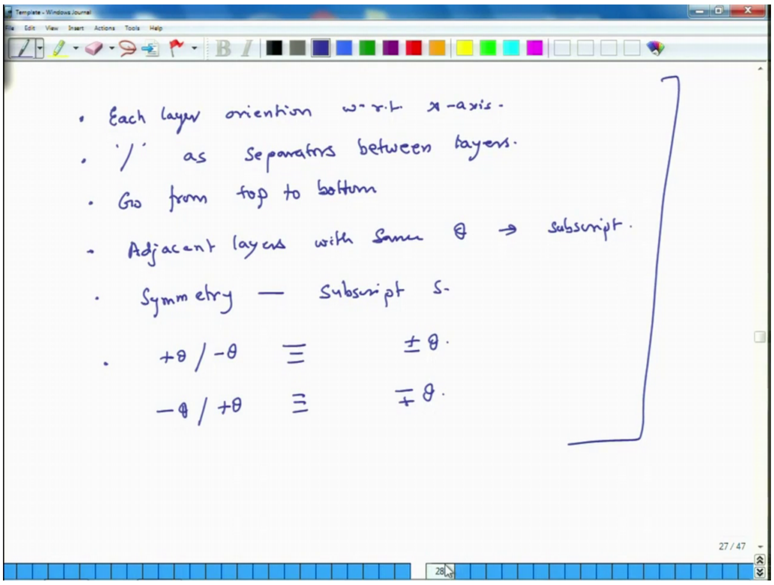

So, we will very quickly write down some rules. So, whenever we express it such a format we assume that we say that each layer orientation is assigned with respect to x-axis. So, this x-axis does not change it is a universal axis. Then we use these as separators between layers. We always go from top to the bottom, what else? If there are two layers next to each other and they have the same orientation, so adjacent layers with same theta then we give a subscript. If the laminate is symmetric about mid plane then we use a subscript S and we can also use plus theta, minus theta this can also be written as plus minus theta. Similarly if the lamination sequence has a negative theta and plus theta, then this can also be written as minus plus theta.

So, these are some broad principles through which we determine lamination, we prescribe lamination sequence.

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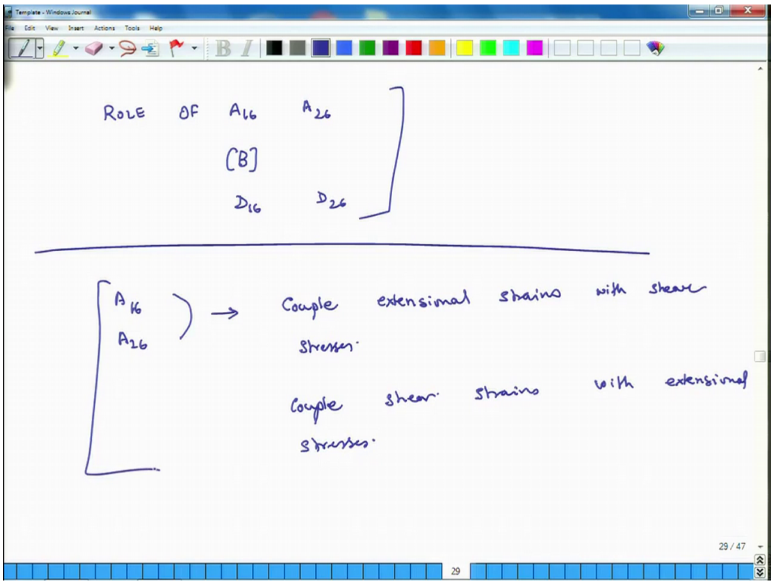

The next thing we will start discussing is role of A 16, A 26, B matrix entire and D 16 and D 26 these elements, ok. Now, detailed discussion on these elements we have already had in our earlier lectures in the introductory course, but for purposes of completeness and just a quick review we will discuss these things and explain their influence.

So, what does A 16 and A 26 do? A 16 and A 26, these two elements couple, extensional strains with shear stresses and they also couple shear strains with extensional stresses. So, what does this mean? It means that if B matrix is 0, in such a situation suppose you have a system when A 16 and A 26 are not 0 and then in that case if you subject the plate to a pure tension stress it will exhibit not only extensional stresses, but it will also shear. And if you expose it to pure shear stress it will not only get a shear strain, but it will also experience or exhibit extensional strains. So, this is about A 16 and A 26, ok.

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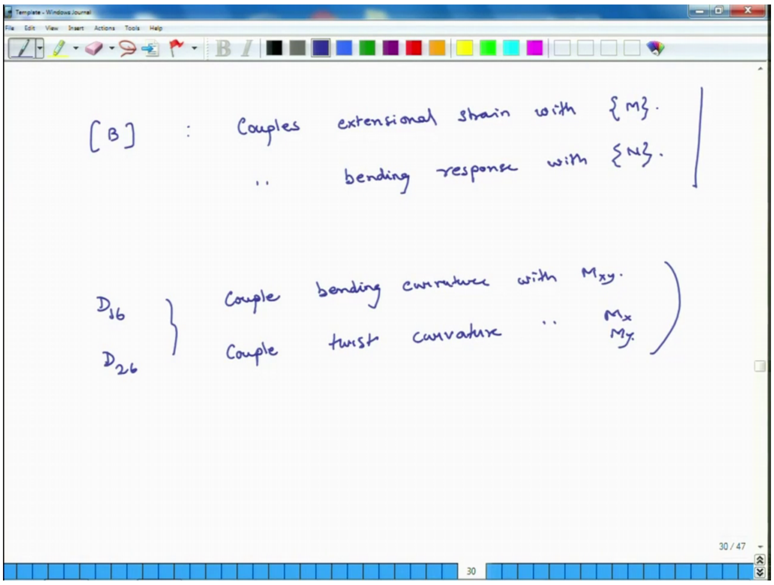

Then the B matrix, B matrix it couples extensional strain with moment resultants, and it also couples bending response with force resultants, ok. So, what does this mean in plain words? That if the B matrix is not 0 and I suppose pull the plate then the plate will not only get stretched, but it will also bend and if the B matrix is not 0. And if I try to bend the plate and if B matrix is not 0 then it will not only bend, but it will also get a stretched or compressed. So, that is why it is a coupling matrix.

And then when finally, we look at D 16 and D 26. So, both these terms the couple bending curvature with M xy, s M xy is a twisting thing, ok. So, it bending curvature and they also couple twist curvature with M x and M y. So, what does this mean? So, just to show this suppose I have a plate and I try to bend it, yes I try to bend it and if D 16 and D 26 is not 0 it will not only bend, but it will also do this type of thing twisting. And if I try to twist it, it will not only get twisted, but it will also do this thing bending. So, it couples bending and twisting responses of the system. So, D 16 and D 26 are of this nature.

So, this is the overview of this all A, B, D matrices and something special about some of these particular members A 16, A 26, the B matrix and D 16 and D 26. And tomorrow we will have some further discussion on these terms and we will learn how to make these elements 0 or minimize these values. So, that concludes our discussion for today. And I look forward to seeing you tomorrow.

Thank you.