Advanced Composites
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Lecture - 18
Important observation Related to [A], [B] and [D] Matrices

Hello, welcome to Advanced Composites. Today is the last day of the ongoing week which is third week of this course and what we will conclude a our discussion over for this entire week is by making some important observations related to A B and D matrices, specifically which are named as extensional mat extensional stiffness matrix coupling stiffness matrix and bending stiffness matrix.

Now, whenever we start designing these composites in a lot of cases, we would like our response of the plate to be less complicated and in of such a way that there is minimum shear extension coupling a minimum amount of coupling between bending and extension and an minimum amount of coupling between bending and twisting.

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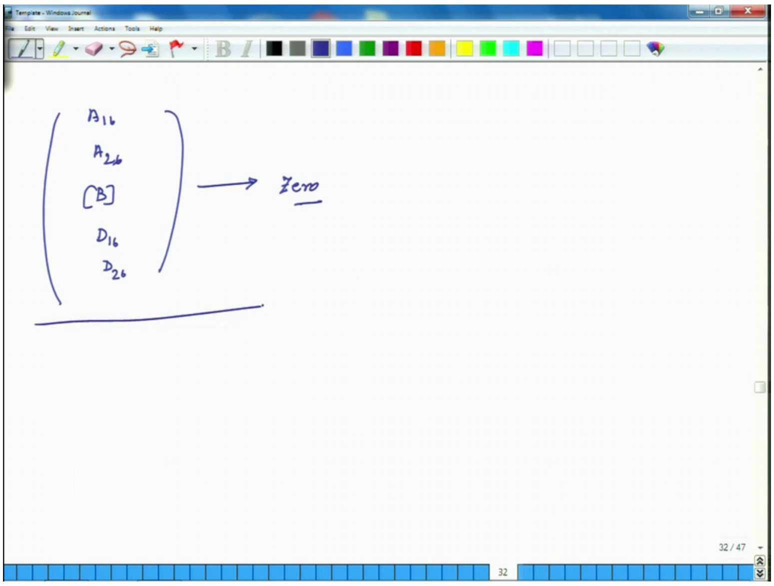

And if that has to happen then we have to ensure that A 1 6 A 2 6 the B matrix and D 1 6 and D 2 6 they should be zero.

If A 1 6 and A 2 6 is zero then they will be no coupling between shear and extension if the B matrix is zero then there will be no may coupling between bending response and the in plane response of the system and if D 1 6 and D 2 6 are zero. Then there will be no coupling between the bending and twisting response of the system. And for purposes of reliable strong composites we want these responses to be as little as possible. So, ideally a laminate in a lot of cases we would like to design such that A 1 6 A 2 6 the B matrix D1 6 and D 2 6 all of them are zero.

So, now we will learn some tricks how to ensure this thing is accomplished.

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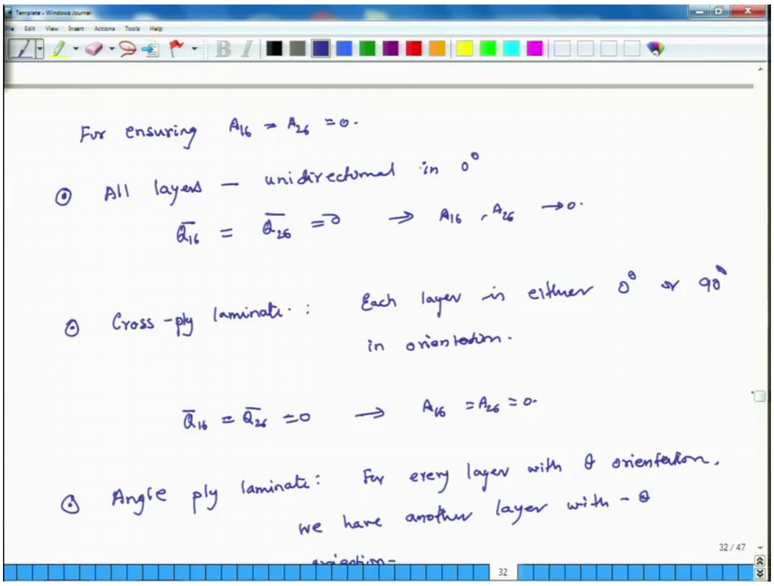

So, for ensuring A 1 6 is equal to A 2 6 is equal to 0 we can do several things option one we use all layers should be unidirectional in the 0 degree ok. If all the layers are unidirectional and are in zero degree direction what happens?

Then for each layer Q 1 6 bar is equal to Q 2 6 bar is equal to zero and as a consequence A 1 6 and A 2 6 are 0. This is one option. The 2nd option is we have a cross ply laminate. So, in a cross ply laminate each layer is either 0 degrees or 90 degrees in orientation and if that is the case then once again for each layer Q 1 6 bar equals Q 2 6 bar equals 0. And as a consequence A 1 6 is equal to A 2 6 is equal to 0.

The 3rd option is we use an angle ply laminate and which is a more realistic option and we will discuss this.

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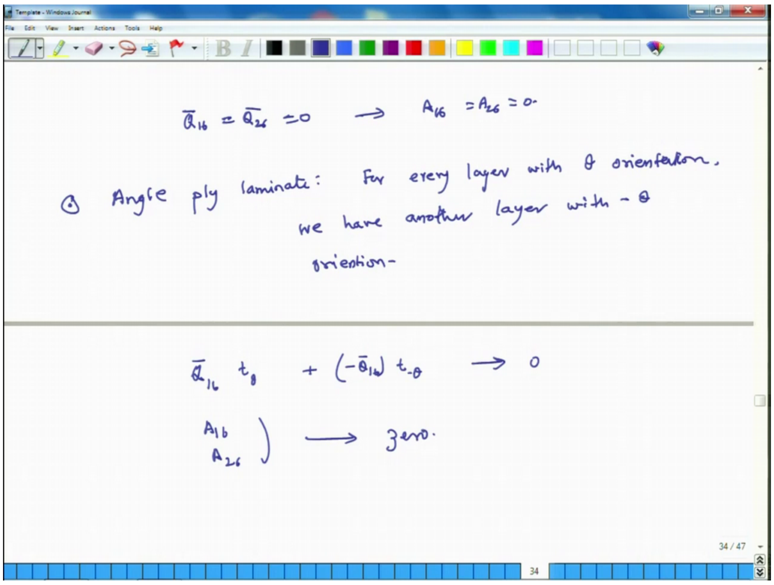

What is the condition for having an angle ply laminate that for every layer with theta orientation we have another layer with minus theta orientation.

In this case what happens is that, let us say there is a layer which has theta orientation and because of that it has a value of Q 1 6 which is this Q 1 6 bar. Let us say its thickness is t and it is having theta orientation let us say we call it t theta. Then there is another layer which has negative theta orientation and last week we had discussed that Q 1 6 bar is an odd function of theta.

So, the contribution from this negative theta layer will be minus 1 6 bar and t minus theta and because the thicknesses are same these contributions add to 0. And as a consequence A 1 6 and A 2 6 for angle ply laminates it they become there 0. So, we can design laminates in such a way that the overall value of A 1 6 and A 2 6 for such laminate becomes 0 is a by having unidirectional laminates, which is not a realistic proposition or by having cross ply laminates which is again not so much of a realistic thing.

But we can also use angle ply laminates which are practical approaches to ensure that there is no coupling between shear and extension.

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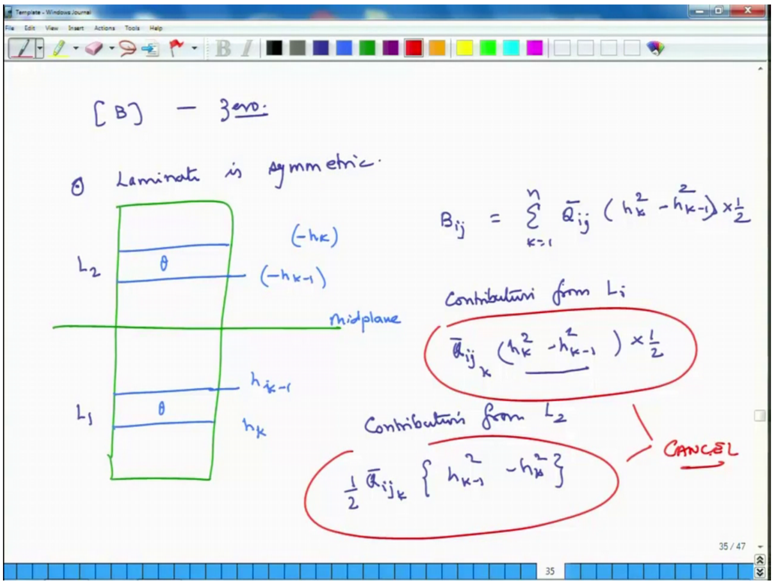

The next thing we discuss is to ensure that the B matrix is0 the B matrix is 0 and to ensure that what we have to make sure is that the laminate is symmetric. If the laminate is symmetric then all values in the B matrix they will come out to be 0 why does this happen?

So, consider a laminate and let us say it has a large number of layers and asymmetric laminate is such that whatever is the sequence of layers above the mid plane the same sequence is reflected below the mid plane ok. So, suppose there is a layer here and there is a layer here, both these layers are equidistant from each other because the laminate is symmetric and the orientation is also same ok.

So, because and this is the mid plane where z is equal to 0. So, how do I show that the laminate which is symmetric in nature its B is going to be 0. So, suppose this thing coordinate is h i or let us say h k and this coordinate is h no h k minus 1 and this is h k and because it is lamina asymmetrically located the value of this coordinate will be minus h k minus 1 and the value of this will be minus h k these are the coordinates.

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Is this is not a minus this is just a line ok. So, we know that B i j is what is the sum of Q i j times h k square minus h k minus 1 square and we sum it up from k is equal to 1 to n and that multiplied by half. So, contribution from let us call this layer L 1 and let us call this layer n L 2. So, contribution from L 1 will be Q i j bar.

For the kth layer times h k square minus h k minus 1 square into 1 by 2 and contribution from layer l two is the same orientation same material everything. So, Q i j will be same. So, it is same Q i j Q i j k and times. So, of course, there is half vector, but the thing in the parenthesis will be minus h k minus 1 whole square

So, it will be h k minus one square minus h k square. So, what you see is that this is negative of this and they all cancel out and because they cancel the B matrix eventually turns out to be 0 because there is a layer, which cancel out the effect of another layer which is equidistantly placed away from the mid plane on the other side. So, this is the way we can have the B matrix as 0 and then the last one is a little tricky.

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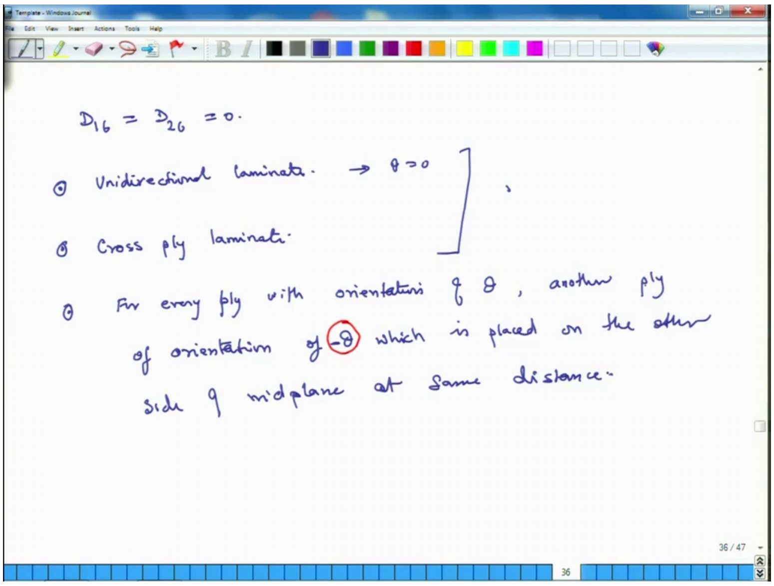

So, we want D 1 6 and D 2 6 to be 0 ok. Now we can do it again in several ways. So, one is we have unidirectional lamina so you have unidirectional laminate and everything is 0 and theta is 0 then that will ensure that D 1 6 and D 2 6 will be zero because for a 0 degree direction laminate D 1 6 and D 2 6 Q 1 6 Q 2 6 are 0.

The other one is cross ply laminate. So, this again in this case also D 1 6 and D 2 6 will be identically 0, but again these two lamination sequences are not practical because they create other problems when we actually use such laminates. So, these two are in this case

D 1 6 and D 2 6 are exactly 0, but then what do we do to make sure what is the other option. So, the other option is that for every ply with orientation of theta.

We put another ply of orientation, orientation of theta which minus theta this is minus theta which is placed on the other side of mid plane at same distance at same distance. So, what does that mean?

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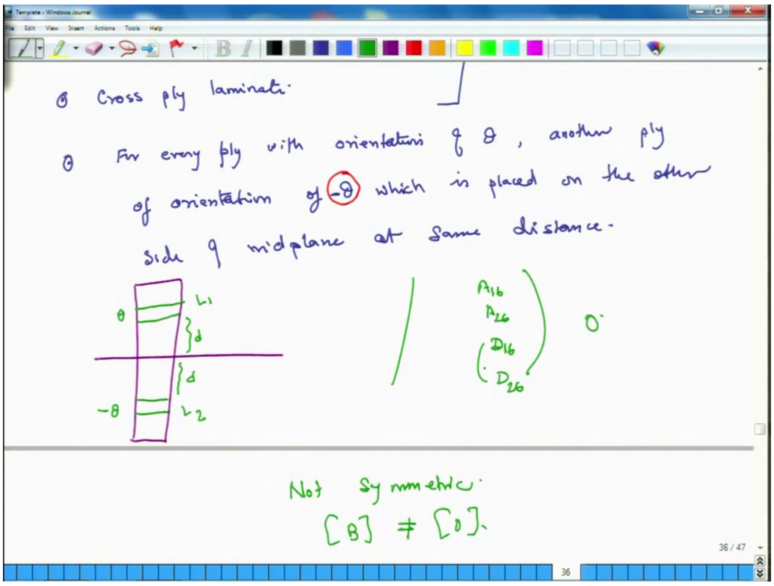

So, what it means is that suppose I have a laminate this is the mid plane and this is a ply of orientation theta and let us say this distance is d.

Then I would another ply away from the mid plane on the other side distance is still d and its orientation should be minus theta. If that is the case then the contribution from layer L 1 and contribution of layer L 2 it will exactly cancel out ok. So, it will cancel out. So, in such a case A 1 6 A 2 6 D 1 6 and D 2 6 all will be 0 and that is good, but then this creates a problem because of this situation the laminate is not symmetric and thus the B matrix is not 0.

So, we are able to solve the problem of problem associated with A 1 6 A 2 6 D 1 6 D 2 6, but we have introduced the problem of B and the problem of B is much more important than the problem of D 1 6 and D 2 6. So, we do not like this solution. So, what we do is we go for some approximate solutions where D 1 6 and D 2 6 are not exactly 0, but they are fairly small. So, what do we do in such cases in such a cases?

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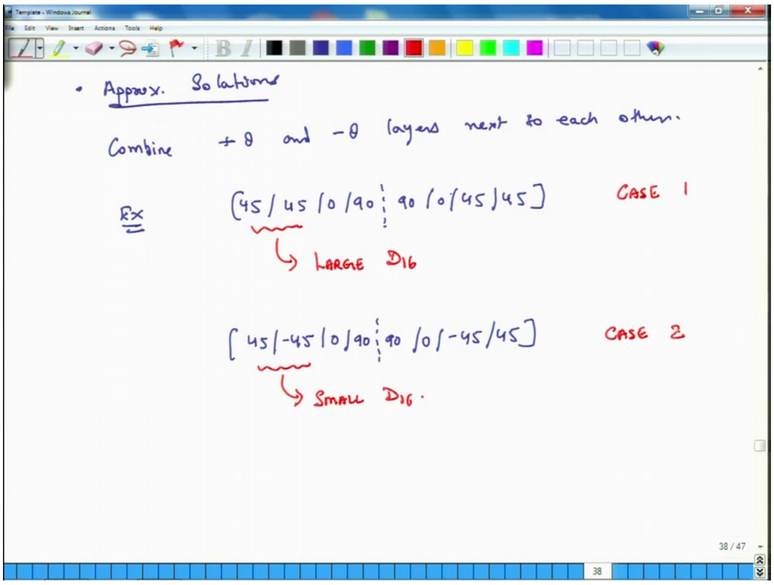

So, approximate solutions. So, in such a case what we do is, we combine plus theta and minus theta layers next to each other next to each other. So, example would be. So, we can have this could be a lamination sequence 45 45 0 90 this is the mid plane and the 90 0 45 and 45.

Here these two are 45 and 45 degrees and they generate Large D 1 6 Large D 1 6, but if we want to reduce D 1 6 what do we do we have another configuration 45 minus 45 because it says combine theta and minus theta placed next to each other. So, I have put them next to each other 0 90.

This is the mid plane 90 0 minus 45 and 45 and then you see if you calculate the contribution from these two values 45 and minus 45 you will find that it is Small D 1 6 Small D 1 6. So, let us very quickly show this and then we close the discussion. So, we will call this CASE 1 and we will call this CASE 2.

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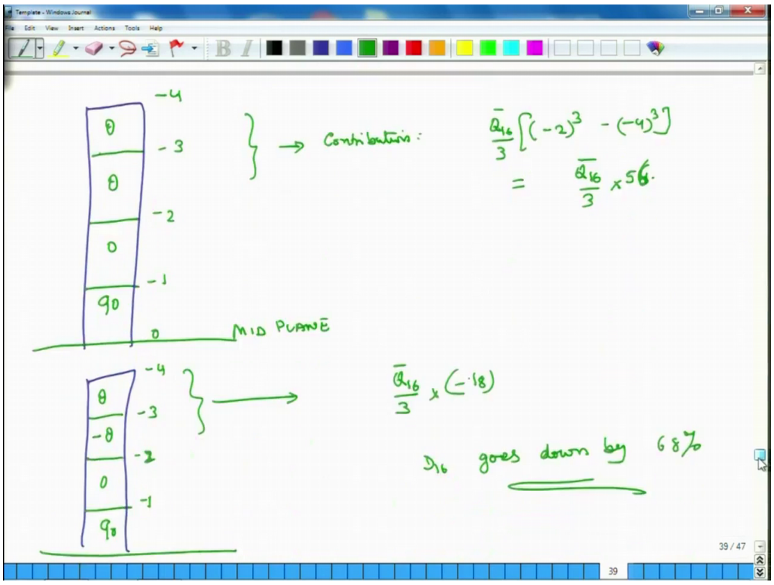

So, let us have. So, this is the mid plane, and let us say this is theta, theta 0 and 90 and let us give these some coordinates. So, this is minus 4 minus 3 minus 2 minus 1 and 0. So, these top two layers have theta configuration and their total contribution. If you calculate it comes out to be Q 16 bar by 3 into minus 2 cube minus, minus 4 cube and that works out to be Q 1 6 bar by 3 into 54 oh actually 56 its 56 and then we have the other configuration.

So, this is my mid plane. So, this is theta and I have putting minus theta here 0 and 90 and this is minus 4 these are the coordinates minus 3 minus 2 and minus 1 and if we calculate the contributions of these two guys we will find that the contribution to D 16 is Q 16 bar by 3 into minus 18.

So, if you twit the absolute magnitude here it is 56 by three and here it is 18 by 3 if you take the absolute magnitude D 16 goes down by roughly 68 percent. So, in this way and if the laminate asymmetric B will still be 0 because there is a theta and there is negative theta. A 16 A 26 are 0 and D 16 and D 26 are get minimized. So, in this way we are able to address this problem in an approximate way.

So, that it becomes manageable. So, this concludes our discussion for today and starting next week we will open a new chapter in this course and that will be related to failure of composites. So, that is what we plan to start from the next week until then have a great weekend.

Thank you.