Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 19
Quasi-Isotropic Laminates

Hello, welcome to Advanced Composites. Today is the start of the fourth week of this course. And, this week we can we will particularly focus on failure of composites, at the laminar level as well at the level of the entire composites track that is the laminate level.

But, before we start these two topics, I would like to come where one more lamination sequence because, in last week we had discovered discussed several types of lamination sequences which had some special properties. So, we had discussed angle ply laminates, cross ply laminates and so on and so forth. Today we will discuss one more lamination sequence known as Quasi-Isotropic Laminate.

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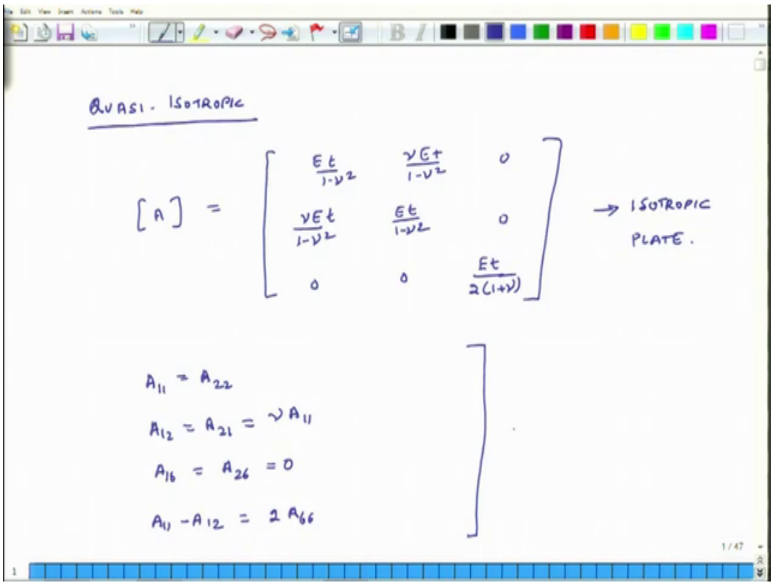

So, these quasi-isotropic laminates they are such that they are isotropic in the plane of the laminate. If you go out of the plane they may still be anisotropic, but within the plane wave behave as isotropic laminates; which means that there A matrix because, A matrix relates to stress stresses and strains within the plane of the plate.

So, their A matrix is very similar to that of isotropic plates. However, these laminates may or may not have B matrices which are 0 and they may not necessarily have a D matrix which is similar in nature to that of a isotropic material. So, that is why we call these types of laminates quasi-isotropic where isotropic in the plane of the plate, but not necessarily isotropic outside the plane of the plate. Now, if we look at an isotropic material let us say steel and then we try to develop its A matrix.

Then, for isotropic materials the A matrix looks something like this E t over 1 minus nu square nu E t divided by 1 minus Poisson's ratio square. Third element is 0, this is also 0, this is 0, this is 0, this is a symmetric matrix. So, nu E t over 1 minus nu square and this is E t over 1 minus nu square.

And finally, the last element is E t divided by 2 into 1 plus nu E t. So, this is the A matrix for a isotropic plate, it is the A matrix for an isotropic plate. And, if we look at this A matrix what we find is that A 1 1 equals A 1 2 this is one equivalence, the other thing we notice A 1 2 equals A 2 1 and that equals nu times A 1 1.

So, this should be A 2 2 and the third relation is that A 1 6 equals A 2 6 equals 0. And finally, we can also see that A 1 1 minus A 1 2 equals twice of A 6 6. So, these are the characteristics or relations between different alum A matrix elements for a plate which is made up of purely isotropic material. So, if we want to have a quasi-isotropic laminate we will it will be here, it will be quasi-isotropic if these relations are preserved for a quasi-isotropic laminate. Now, typically you can get a quasi-isotropic plate or a lamination sequence if you follow some rules, when you are arranging the different layers together.

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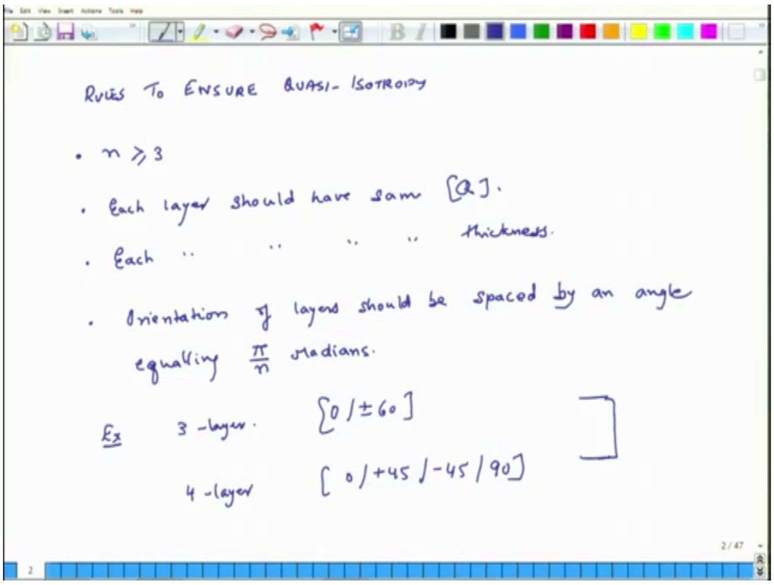

So, rules to ensure quasi-isotropy. So, so, the first thing is that the number of layers should be more than or equal to 3. You cannot have a quasi-isotropic system a laminate, if you have less than 3 layers so, minimum is 3 ok. The second is that each layer should have same Q matrix, I am not talking about the Q bar matrix, but Q matrix should be the same which means that the material system for each layer should be same.

The third-one is that each layer should have same thickness and the fourth-one is orientation of layers should be spaced by should be spaced by an angle equaling pi over n radiance. So, if you have a 3 layer system then they should be maybe, if the first layer is 0 degree, second layer is 60 degrees; third layer is 120 degree like this. If you have 4 layer system then 0, 45 minus 45 90 or things like that.

So, some examples so, for a 3 layer system we will have 0 and then plus minus 60 because, minus 60 is same as 120 degrees ok. For a 4 layer system so, by the way this does not mean that adjacent layers have to be off by pi over n because, what really matters is thickness times Q bar. So, it does not matter the location of the layer in the system is not that important. The location of the layer in the z direction becomes important when we are calculating elements of B matrix and D matrix, for A matrix it is the thickness which matters ok.

So, for a 4 layer system it could be 0 plus 45 minus 45 and 90. Now, it just turns out that both these lamination sequences are not symmetric in nature. So, they will have a B matrix. So, if we want to have a quasi-isotropic plate, which is also having 0 B matrix, which is it does not have a bending this coupling stiffness matrix then in that case we could have a lamination sequence of this type.

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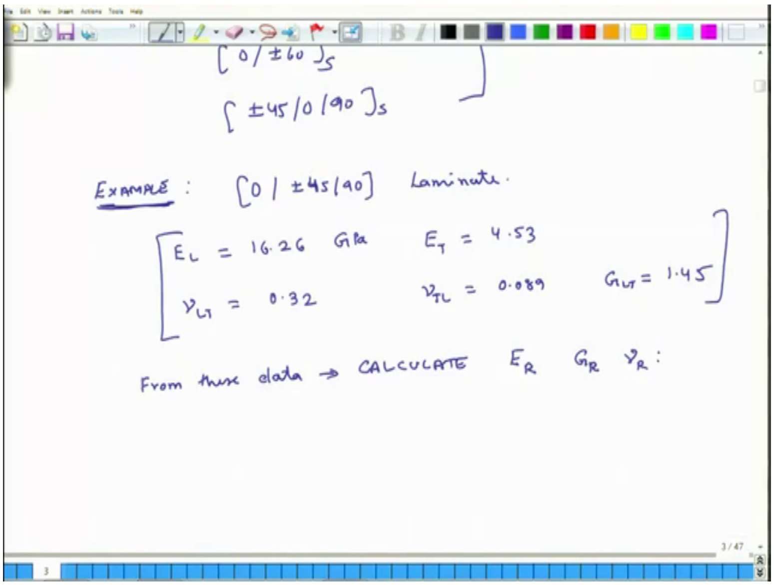

So, it is symmetric or another lamination sequence could be plus minus 45 0 90 symmetric and so on and so forth. So, these are also quasi-isotropic systems and the first set is also quasi isotropic in nature, but it just turns out that the first set does have a non-zero coupling stiffness matrix.

So, what we will do next is we will do an example and before we start doing the example so, these quasi-isotropic laminates can also be used to estimate the Young's modulus and G that is the shear modulus of single layer lamina or layer single layer composites where the fibers are randomly placed.

We can do that also and how we do that is less that is that is something we will learn in this example. So, what is this example all about? So, we say that let us say we have a 0 plus minus 45 90 laminate ok. Now, and we are making this laminate not necessarily from continuous fibers for, but from short fibers, we are making it from short fibers. So, we are making two laminates: one is this plus 0 plus minus 45 90.

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And, then we are also making another laminate a different laminate where the fibers are randomly placed and if they are randomly placed, then its properties in all the directions they will be the same. But, we do not know what is the value of Young's modulus of this randomly oriented fiber mat. So, we what we will do is we will somehow figure out a way to calculate that by using the theory whatever, we have learnt till so, far for 0 plus minus 45 90 quasi-isotropic laminate. So, that is what we are going to do.

So, this 0 plus minus 45 90 this is a laminate and it will and because so, it has 4 layers 0 degree layer, 45 minus 45 and 90 degree layer. And, all these layers are of the same material and what we do is; the first thing is that we should know what is the value of E L E T and all this stuff. So, E L either we can measure it or we can calculate it. So, E L is 16.26 GPa for this laminate, for one of these layers, E T is equal to 4.53 GPa, nu LT equals 0.32 and nu TL, we can actually calculate it from nu LT.

So, nu TL will be nu LT into E T divided by E L hm. So, this is the relation we already have discussed earlier. So, this works out to be 0.089 and then G LT is 1.45. So, these values we can either measure from a single 0 degree oriented layer or if we have the material properties of fiber and matrix and we can actually calculate these properties also. So, so this is the data. So, from these data we calculate E R.

So, what is E R? It is the Young's modulus of layer where all the fibers are randomly oriented ok. And, because they are randomly oriented so, its material properties in all the directions is the same. So, we have to calculate E R and then we have to calculate G Rand Poisson's ratio nu R. So, this is what we are supposed to do. So, here is what we will do, first we will compute the A matrix of this laminate.

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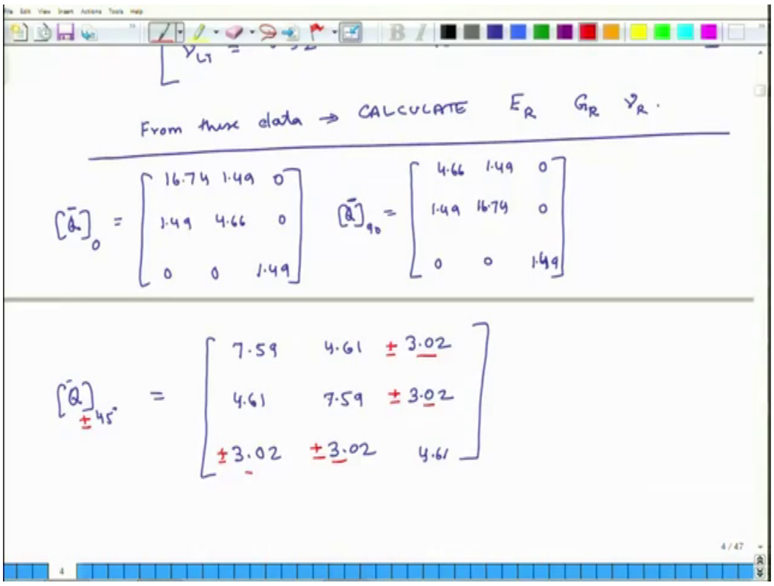

So, A matrix for calculating A matrix we have to first find the Q bar for each of these layers. So, Q based on whatever we have learnt till so, far Q bar for a 0 degree ply and I am not going to show all the calculations, but it I am just directly going to share the numbers. So, it is 16.74 1.49 and 0, 1.49 4.66 0 0 0 1.49. So, this is Q bar then, Q bar for 90 degrees because there is a 90 degree ply all also. So, it is actually the same thing, but E L this Q 1 1 becomes Q 2 2 and Q 2 2 becomes Q 1 1. So, 4.66 1.49 0 0 1.49 16.74 0 0 and 1.49 and then the next one is Q 45.

So, we develop find Q 40 Q bar matrix for 45 degree layer and this is equal to 7.59 4.61 3.02. 3.02 4.61 7.59 4.61 3.02 and 3.02 and the next one is Q bar matrix for minus 45 degrees. So, we know and we have discussed this that the only term which is sensitive to the value of sin are Q 1 6 and Q 2 6 because, they are even functions odd functions of theta.

So, for minus 45 degrees these numbers they just become negative, everything else remains the same. So, I will just put a plus minus here. So, if you are trying to find out the plus value then take the plus thing and if for negative 45 degree orientation use the negative 3.02 as the value.

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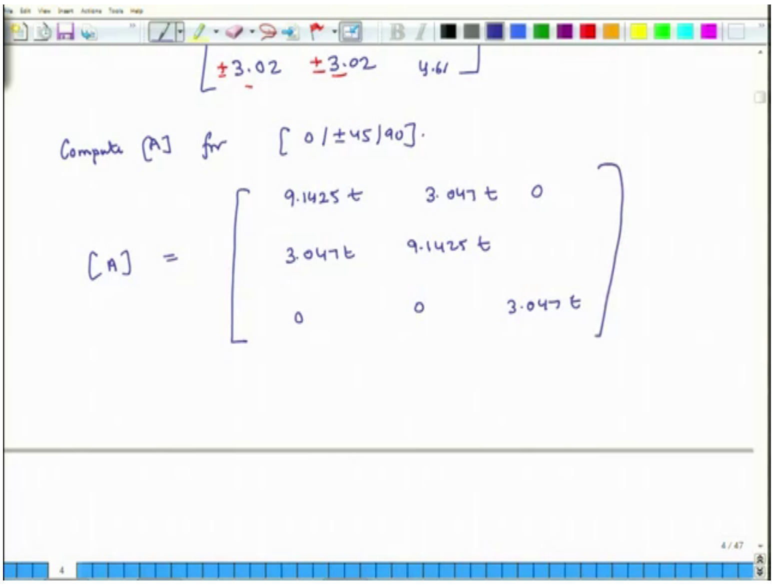

So, using this now, we do we compute A matrix for the quasi-isotropic laminate. We compute A matrix for quasi-isotropic laminate for this system. So, we know how to calculate it. So, I am just going to directly share the results, but before we talk about the A matrix we have to know the thickness of each layer.

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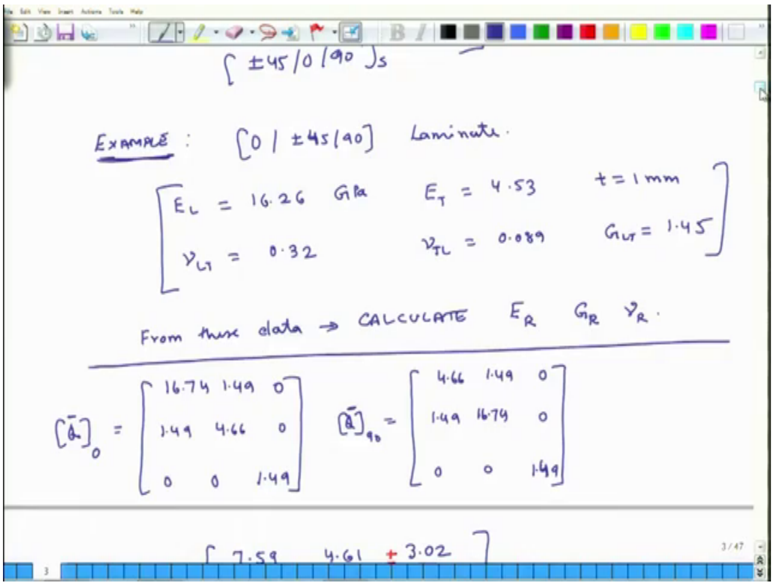

So, we assume that thickness of each layer is 1 millimeters or it does not matter we can even call it t, it does not matter. So, let us assume that it is t if t is so, it is 9.1425 t. We are assuming that the thickness of each layer is t millimeters, this one is 3.047 t and then this is 0 3.047 t 9.1425 t 0 0 3.047 t ok. Now so, this is the system for this is the A matrix for this quasi-isotropic laminate 0 minus 45 90 degrees system.

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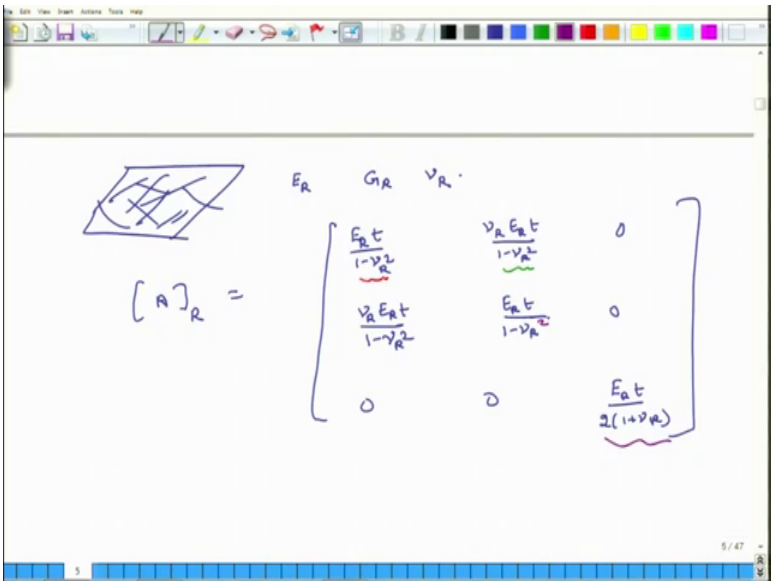

Now, consider another situation where we have a randomly placed fiber random. So, so in this case it will have three important material properties: E R G R and U R. And because, this is also having isotropic in the plane so, what will the mat A matrix for this randomly oriented thing will be; it will be E R times t divided by 1 minus mu square right. We are using the same relations for which we initially showed for an isotropic plate. This is nu R E R t by 1 minus nu R square. How are we getting these relations? We are getting this relation basically from this equation ok.

Because, this is good for isotropic systems this is 0 0 nu R E R t by 1 minus nu R square. And, this is E R t by 1 minus nu R t 0 0 and this is G or forget it, it is E R t by 2 into 1 plus nu R ok. So, what I do is next is I compare the numbers and by comparing numbers then I can calculate the values of E R. So, I can I compare see this number should be equal to this number, this number should be equal to this number and finally, this third number should be equal to this guy.

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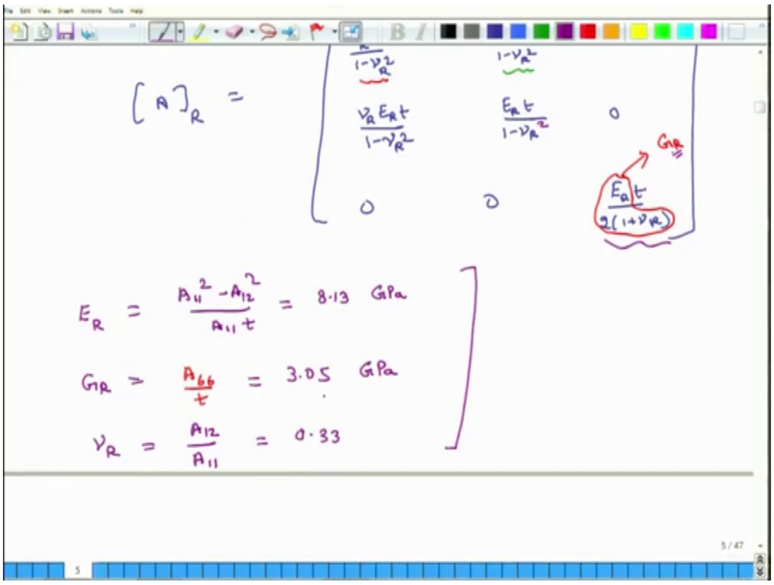

So, by making these comparisons so, by making this comparisons I can say that E R is equal to A 1 1 square minus A 1 2 square divided by A 1 1. So, if you do this math you will find that it actually comes out to be E R and this is works out to be 8.13 GPa ok, then you have GR. So, actually this entire thing E divided by 2 into 1 plus nu, this is actually same as GR.

So, GR is equal to A 6 6 by t right actually, I am sorry this is there is a small error here I should divide this by t also. So, this works out to be this A 6 6 divided by t, G R into t is same as A 6 6 ok. So, this works out to be 3.05 GPa and nu R is equal to A 1 2 divided by A 1 1 and that comes out to be 0.33.

So, you can say that a randomly oriented fiber mat which has the same volume fraction and the fiber should be of the same length ok, which has the same volume fraction fibers are of the same length as in the act oriented layer and the matrix is same, if everything else is same then using this approach you can predict that its Young's modulus will be 8.13 GPa, G R shear modulus will be 3.05 and Poisson's ratio will be 0.33. So, this is what I wanted to capture today and starting tomorrow we will start discussing failure of composites. So, that is all for today and I look forward to seeing you tomorrow.

Thank you.