Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 20
Maximum Stress Theory

Hello; welcome to Advanced Composites. Today is the second day of the fourth week of this course and from today onwards we will start discussing failure of composites. Now it turns out that when we talk about failure of composite materials that is laminated composites, where each of these layers is not necessarily isotropic in nature then the theories of failure for isotropic materials, they do not necessarily work in context of anisotropic materials such as composites.

And the reason for this is that in isotropic materials the way we try to predict the failure of a material is first we find out what are the principal stresses in principle strains in the system at a point and then we start comparing these principal stresses and principal strains to the either to the tensile strength of the material or compressive strength of the material or shear strength of the material or we use some other more complex energy related theories; such as one misses stress theory or principle stress theory and so on and so forth here.

But in case of composites, the reason this does not work is, because the material is strongest in the L direction, because that is the direction of the fiber. It is not that strong in that transverse direction, because if I try to pull a single layer of an orthotropic composite in transfer direction it will fail and when we compute principal stresses, they may be not necessarily aligned to these two axis and also the strength of the material just keeps on changing with respect to theta. In case of isotropic systems it does not change with respect to theta the strength of the material. So, the strength of material in the direction of principal stresses and the direction of applied stresses is the same, but in case of these orthotropic composites or orthotropic materials the strength of the material keeps on changing as i keep on changing. So, if I compute the value of principal stresses we do not know the strength of the composite in that direction. So, then it starts between meaningless, the calculation of principal stresses to find out whether the thing is going to fail or not. In context of composite material it becomes a meaningless exercise.

So, what do we do? So, broadly speaking we will discuss three different theories of failure for composite materials. now earlier we had discussed that an orthotropic composite, it has five different types of strengths; one is the strength in the longitudinal tensile, strength in the longitudinal direction, compressive strength in the longitudinal direction, tensile strength in transverse direction, compressive strength in transverse direction and shear strength when we refer to the LT or longitudinal and transverse axis system.

So, using these five different parameters we try to figure out whether the material is going to fail or not fail. The first theory which we will discuss is known as the maximum stress theory and this maximum stress theory in context of composites is different than the maximum stress theory which we have discussed earlier in our other solid mechanics courses related to isotropic materials.

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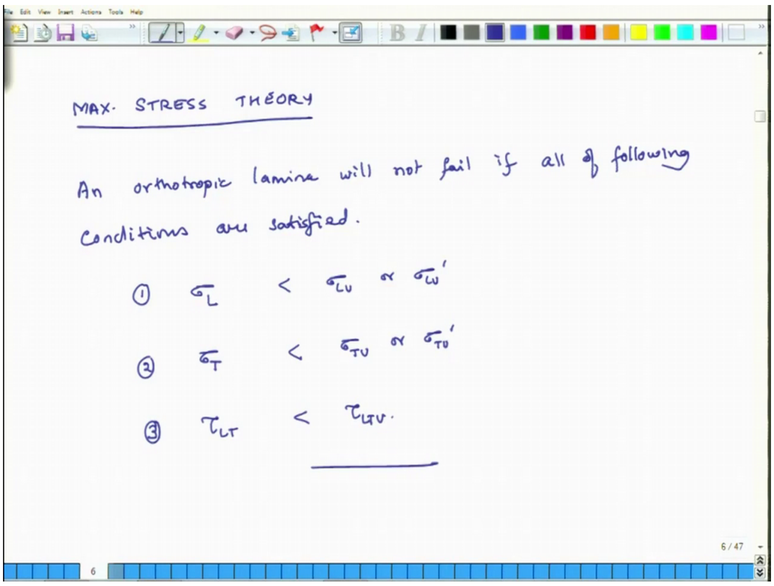

So, the first theory we will discuss is Maximum Stress Theory. So, what does it say? It says that a composite, an orthotropic lamina will not fail if all of the following conditions are satisfied. So, what are these conditions? So, if you have an orthotropic lamina which is loaded the first thing you have to do is, you have to find out what are the values of sigma L, sigma T and tau L T and then you look at these conditions.

So, sigma L should be less than sigma L U or if sigma L is compressive then it should be sigma L U prime, where sigma L U is the ultimate tensile strength of the composite in longitudinal direction and sigma L U prime is the ultimate tensile ultimate compressive strength of the composite in the longitudinal direction. So, if sigma L is tensile then you compare it with sigma L U, if it is compressive then you compare it with sigma L U prime. So, this is condition 1 condition 2 is sigma 2 T transverse stress.

This should also be less than sigma TU or sigma T U prime, whichever is appropriate and finally, tau LT should be tau L T U ok. So, this is what the maximum stress theory says. So, let us consider an example.

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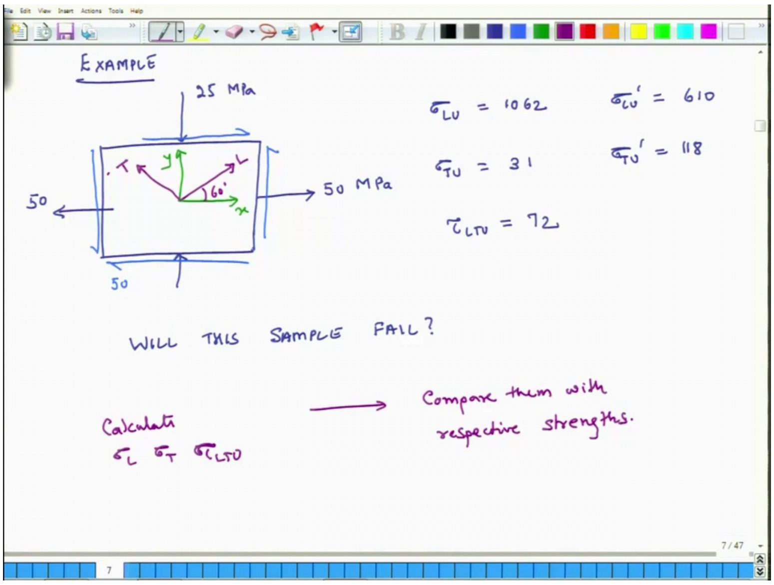

So, consider an example where we have a piece of composite, actually it is just a single layer of orthotropic material and the layer is such. So, this is my x axis, this is y and then my fibers are in this direction.

So, this is the L axis and this is the T axis and this theta is 60 degrees. Now I am not pulling this composite in the LT plane, but rather I am pulling it in the x y plane with respect to x y axis. So, it is experiencing a tensile stress of 50 MP a and I am also exerting an compressive stress on it and this is 25 MP a and then finally, I am subjecting it to shear stress and what is the value of the shear stress? This is 50 MP a ok.

And we are told that sigma L U equals 1062 MP a, compressive strength in L direction is equal to 610 MP a. Transverse tensile strength is 31 and transverse compressive strength is 118 and the shear strength is 72. So, the question is will the composite, will this sample fail, this is what we are supposed to figure out

So, how do we do this? So, we follow this following method. So, calculate sigma L sigma T and tau L TU and then compare them with respective strengths ok, compare them with respective strengths.

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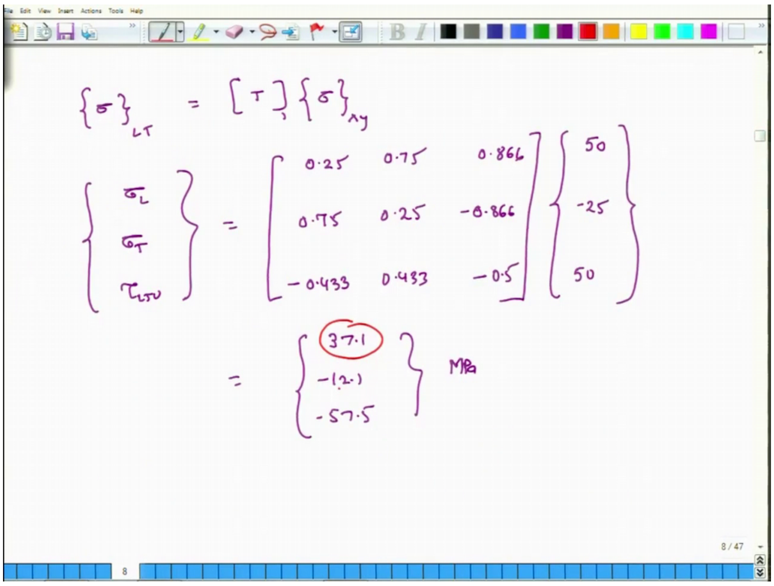

So, we know that the stress in x y plane can be transformed. So, stress in LT plane can be expressed in terms of stress in x y plane through our transformation matrix T 1 and we had discussed this matrix earlier. So, using this we will just directly calculate.

So, sigma L sigma T tau L TU equals and this matrix depends on this theta right and theta is 60 degrees. So, here I have sigma x and y so, sigma x is 50 MP a, sigma y is compressive, so it is minus 25 and the shear stress is positive. So, it is 50 and what are the values of these transformation matrices? So, the first one is cos square theta. So, that is 0.25 because cosine theta is half, second one is sine square theta, so that is 0.75 because root 3 by 2, the whole thing squared equals to 0.75.

The third one is what twice of sine theta cosine theta or sine 2 theta. So, this is 0.866 so, this is also 0.25 0.75, this is 0.866.

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And this is half of that thing and negative and this is also 0.433 and this is minus half cosine square of twice theta. So, you compute these and you it, these things come out to be 37.1 minus 12.1 and minus 57.5 MP a.

So, now you compare 37.1 is the stress in L direction tensile and it is significantly smaller than sigma L U. So, we are on that and then the next one is negative 12.1 and this is significantly smaller than transverse strength in compression 118 and the last number is minus 57.5 and this is also smaller than 72. So, we can say that composite will not fail; this is our conclusion

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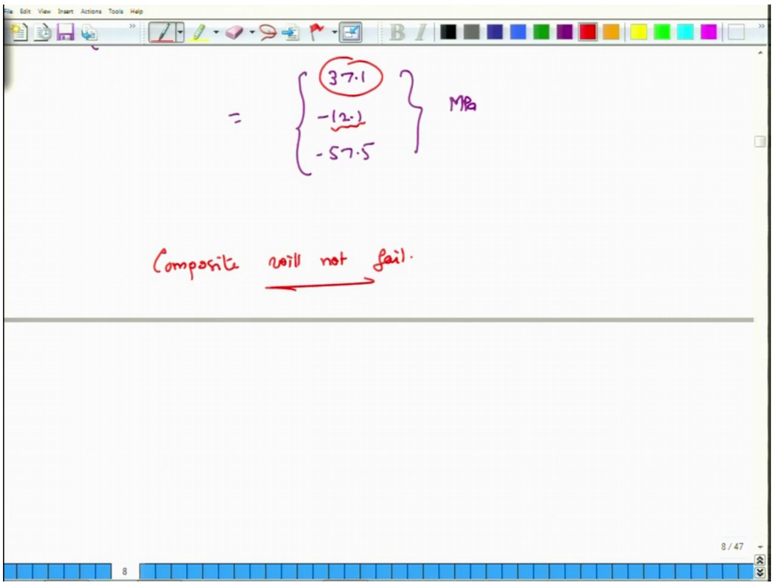

So, this brings us to the closure of today's lecture. Tomorrow we will continue this discussion and we will introduce another failure criteria known as the maximum strain criteria. So, we will also discuss that and then there will also be a third criteria known as related to energy, maximum work energy theory and using all these methods we will try to figure out how the system behaves for different situations. So, that is the closure for today's discussion and I look forward to seeing you tomorrow.

Thank you