Advanced Composites
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Lecture – 24
Progressive Failure of Laminae in A Laminate

Hello, welcome to Advanced Composites. Today's is the last day of the ongoing week which is fourth week of the class. And what we have covered over this entire week is essentially different types of failure theories, and in terms of how they predict the failure of composites at lamina level. And then yesterday we just started discussing how composites fail at the laminate level, and how to predict the failure of the first layer. And because failure in composites in composite laminates specifically it is a progressive thing. So, we had discussed the overall framework of how to predict the failure of the first layer.

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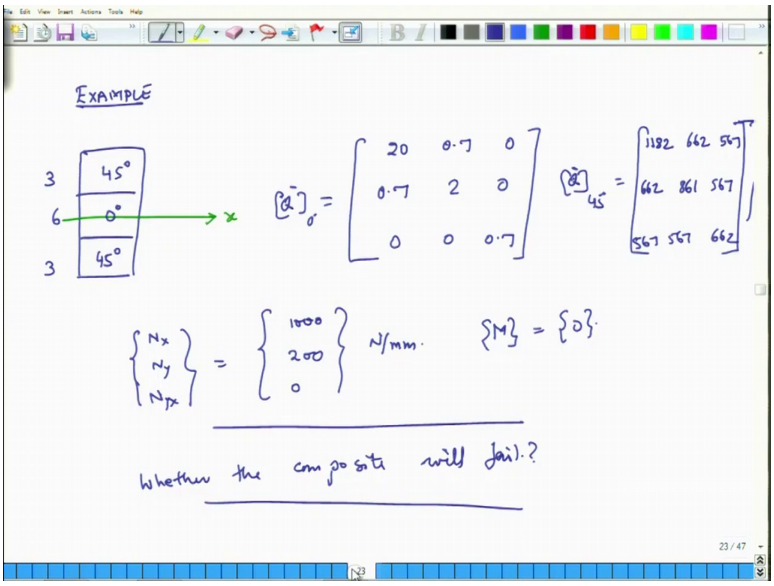

And then we had started discussing this particular example. And the example is of a three ply laminate the top ply and the bottom ply are 3 millimeters thick, the middle ply is 6 millimeters thick the laminate is 45, 0, minus, 45, 0 and 45. So, it is a symmetric laminate. We know the value of Q bar Q bar matrices for both these layers. And we also know that external force resultants and moment resultants we know their values. So, then we have to figure out that whether this composite fail, fail or not and. If it fails, then which layer is going to fail first?

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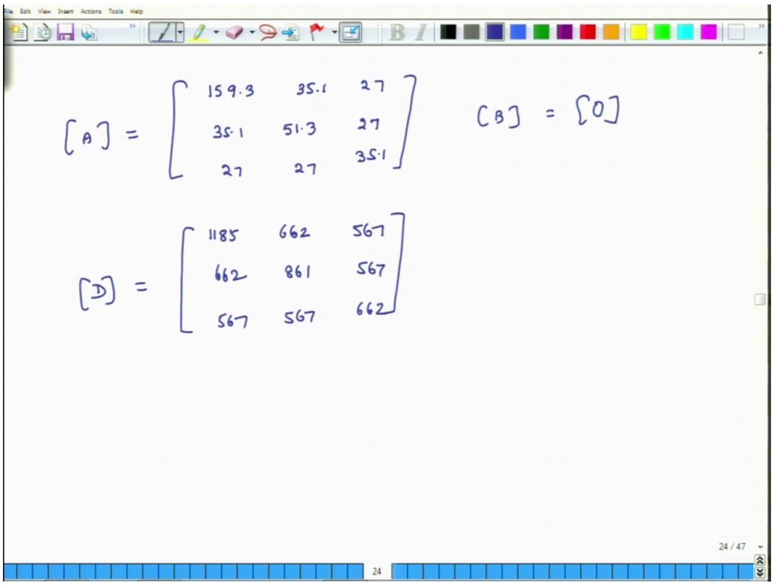

So, as we had discussed earlier the first step in this is we compute A, B and D matrices. So, you already know how to compute A matrix. We have all the values of Q and thicknesses. So, I am just going to directly write the value of A, so it is 159.3, 35.1, 27; 35.1, 51.3 and 35.1; 27, 27, 35.1. Because this laminate is symmetric about the midplane, it is symmetric about the mid plane. Therefore, its B matrix is 0 it has no elements in the B matrix. And the D matrix, if we compute the numbers for its elements look like this 1185 662 567; 662 861 567; 567 567 and 662. So, so we have computed A, B and D matrices.

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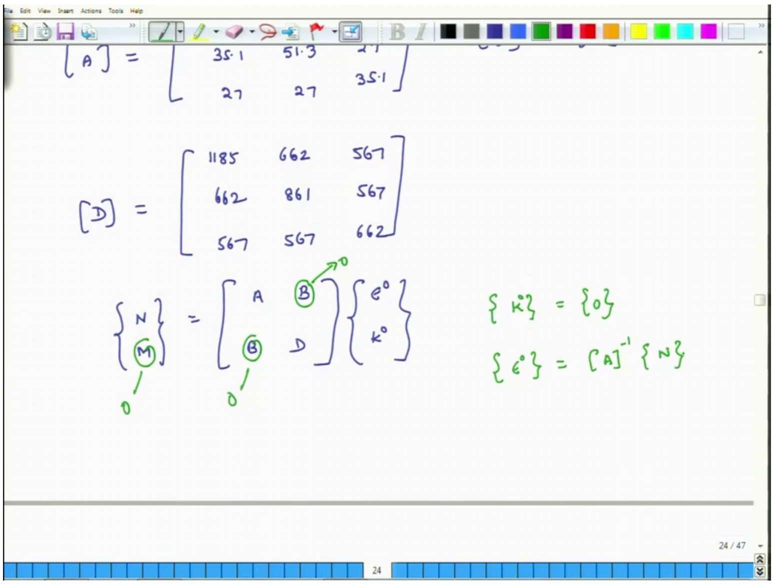

The next thing is we have to compute mid plane strains and mid plane curvatures. So, we know that N M equals A B B D, these sub matrices multiplied by the vector epsilon naught and k naught ok. Now, we know that M is 0; we also know that B is 0. So, in this case because the system is symmetric and also because there are no external moment resultants, we do not have to do all detailed calculations we can directly write that the mid plane curvatures will be 0 ok. There is no external bending moment, so the plate is not going to bend and it does not also have a bending coupling matrix. So, where this bending extension coupling matrix, so B is 0; M is 0 which directly means that k is going to be 0. And the mid plane strain vector will be nothing but in this case inverse of A times N ok.

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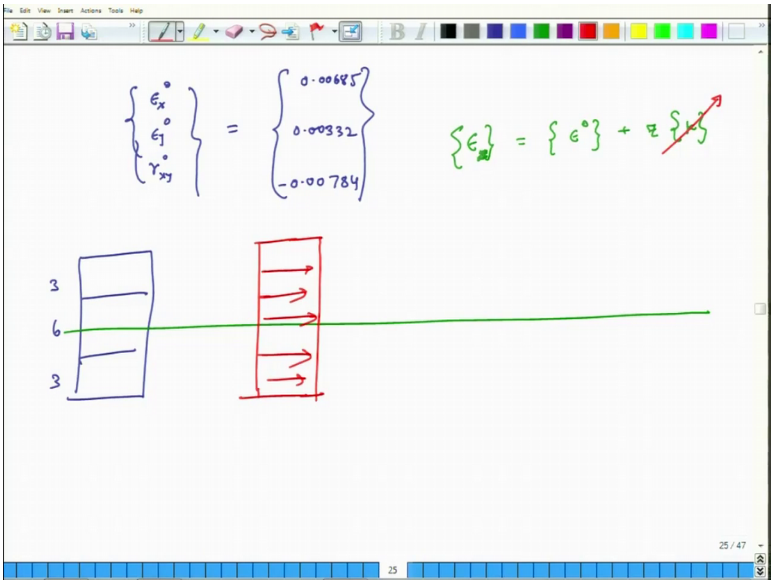

So, if we do the math we get epsilon x, epsilon y and gamma xy. And these are all mid plane strains. So, we computed them and their values are 0.00865, 0.00332 and minus 0.00784. So, what does this mean, what this means is that we have this laminate and this is 33. So, this is 3 millimeters, 6 millimeters, 3 millimeters right. So, these are mid plane strains. And the strains at other locations are what, mid plane strains plus z times curvature. Now, since curvature is 0, it means that the strains in the laminate are constant through the thickness. So, it strains are same in all the layers.

So, if I plot for strain, it is same in all the layers, its values same. So, in all the layers, epsilon x naught epsilon x is 00685, epsilon y is 00332 and gamma xy is 00784 negative of that. This will not be the case if there is bending ok, but because curvatures are 0, then strains are uniform.

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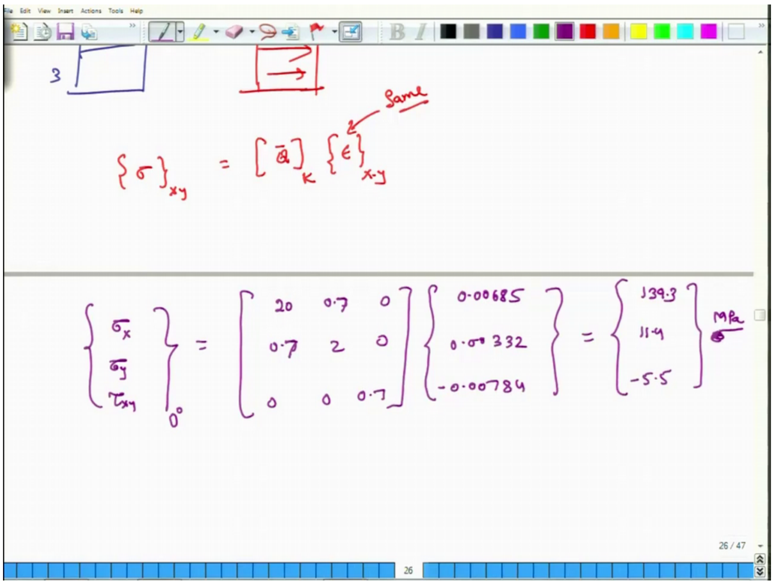

Now, once this is done, then what we do is we compute sigma x, sigma y and tau xy for each layer. So, sigma in xy plane for each layer, so let us say for kth layer is Q k times epsilon with respect to x y plane. Now, this is same for all the layers in this problem. So, we find that sigma x, sigma y, tau xy equals, so for the 0 degree layer first we will compute for the 0 degree layer, so this is equal to Q bar for the 0 degree layer. And Q bar we have already said it is 20, 0.7, 0, 0.7, 2 0 excuse me and this is 0.7 times strains 0.00865 0.00332 and minus 0.00784. And if you do the computation, this comes out to be 139.3 11.4 and minus 5.5 into, so this is mpa, this is mpa.

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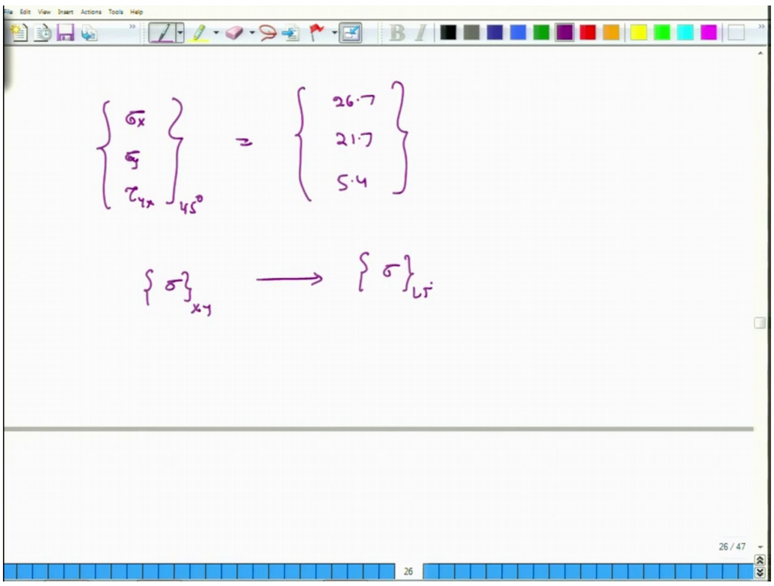

And similarly sigma x, sigma y, tau xy for 45 degree layer is basically you again multiply this vector with the Q bar matrix for 45 degree layer, and it is going to be the same tougher whether it is for top 45 degrees or bottom 45 degrees because curvature is 0. So, this comes out to be 26.7 21.7 and 5.4. So, now, we can do the transformations. So, now, we can compute sigma from xy system to sigma 2 LT coordinate system by using appropriate transformations.

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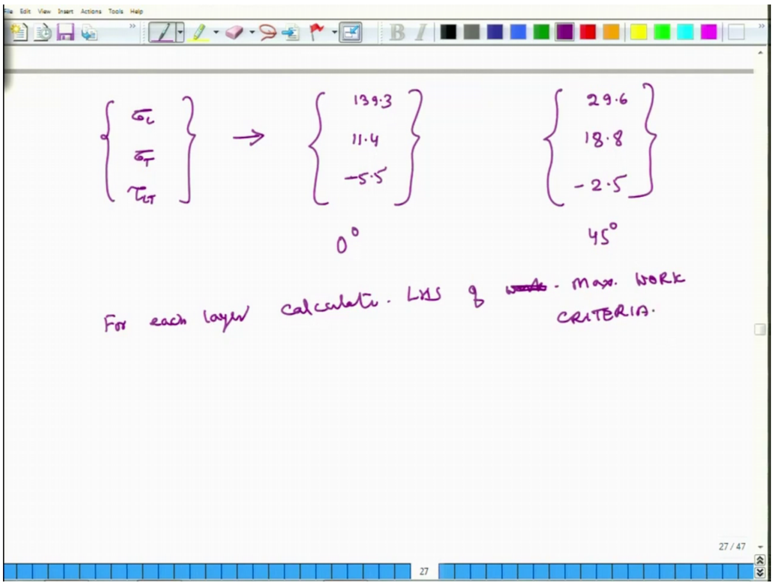

And finally, what we get is sigma L, sigma T, tau LT. So, what is this is equal to 139.3 11.4 minus 5.5. So, this is for 0 degrees. And for 45 degrees this is 29.6 18.8 and minus 2.5. So, this is for 45 degree ply. And once again the stress in both the 45 degree plies same because there is no curvature, there is no curvature.

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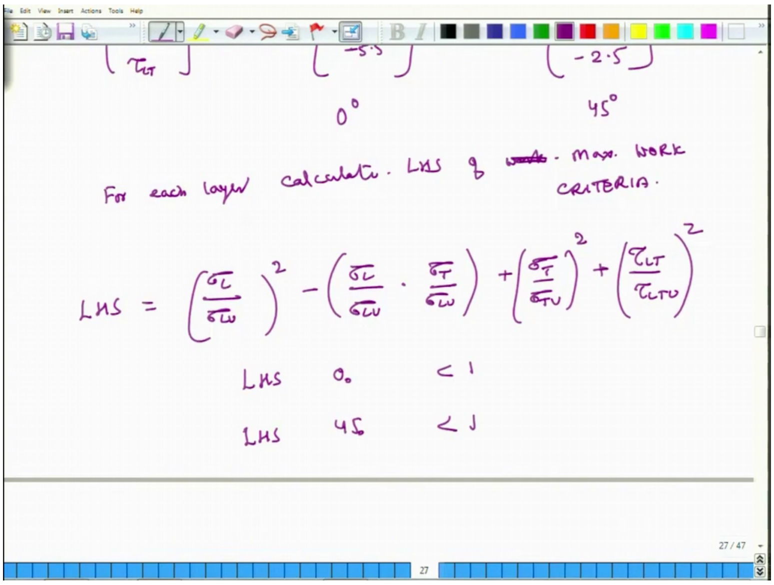

So, once this done, then for each layer calculate the LHS of work maximum work criteria. So, what is that LHS, LHS of that is what sigma L over sigma LU square minus sigma L over sigma LU times sigma T over sigma LU plus sigma T over sigma TU square plus tau LT divided by tau LTU square. And you compute this ok. Now, if LHS for 0 degrees is less than 1; and LHS for 45 degrees is less than 1, then the material will survive. If either of these exceeds 1, then the material will fail; and that particular layer will fail first, it will fail first. Then what happens to the other layer, we have to discuss that again, but whichever layer fails first, it will have the highest value of this LHS and this value will be more than 1.

The other thing I wanted to discuss here is the fact what happens if I increase my stress. So, suppose this is N and suppose I make this N twice, what does it mean I make instead of thousand I put 2000 this I make 400 and this again 0. So, everything goes up by a factor of 2. What will happen in that case A does not change, B does not change, D does not change, but the strains will double ok, because this is a linear elastic material. When strains double, stresses will double. Then stress doubles all these calculations of stress will double ok, but this factor LHS what will happen to this it will go up by a factor of 4 ok. So, LHS is quadratically related to changes in N and Ms. So, if you increase the value of N by a factor of x, LHS goes up by a factor of x square this is something important to understand, so that is there. So, in this way we can calculate which layer fails first.

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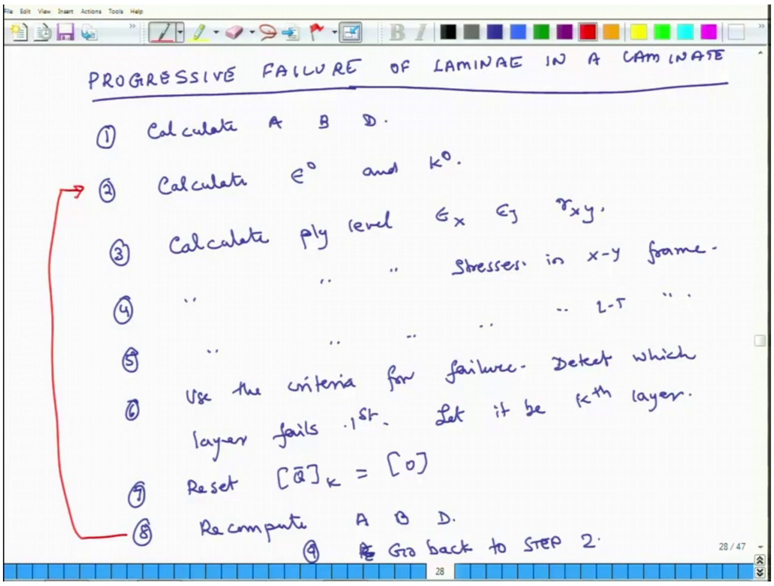

Now, we will discuss the second part of this puzzle and that is progressive failure. And here I will first write an algorithm. So, progressive failure of laminae in a laminate. So, we will just do the steps. First step calculate A, B, D for the laminate which is not broken which is not it which is a fresh thing. Second, calculate epsilon naught and k naught. Third, calculate ply level epsilon x, epsilon y, gamma xy ok; at each ply we calculate. Fourth, calculate ply level stresses in x y frame. Fifth, calculate ply level stresses in LT frame. Sixth, use the criteria for failure and detect which layer fails first, so let it be k th layer.

So, suppose k th layer fails, then what happens then you do then that load will get transferred to other layers right that layer will not take any load. So, how do you account for that? Reset cube r matrix for k th layer to be 0, because it is not taking it has 0 stiffness. Eight, re compute, re compute A, B, D. Ninth go back to step 2. So, we keep on doing this ok. So, now, we again do all this stuff and then we will find that another layer has failed. So, again you set the cube our matrix for that other layer also to 0, and you keep on doing this.

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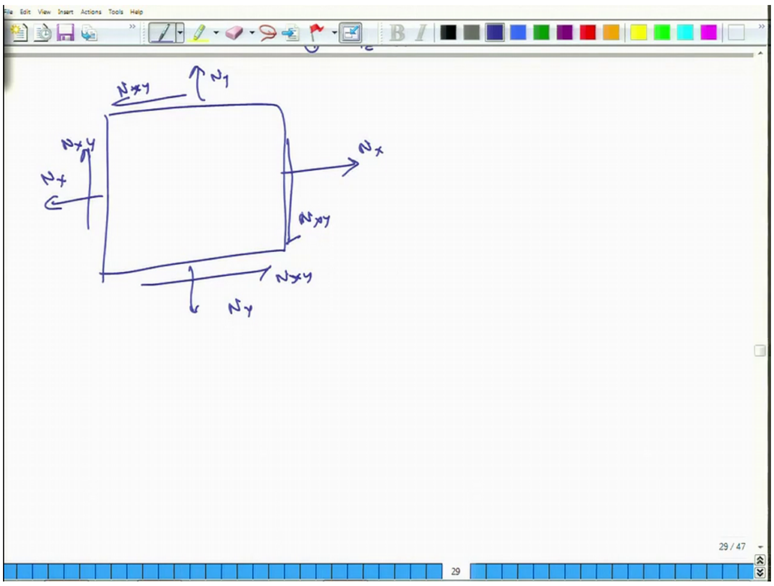

Suppose you have a laminate and it is being subjected to N y, N x and then it is also having N x y and so on and so forth. It may also have Ms, it does not really matter. But what you are doing is you are having applying all these forces, initially the force is zero and then you are simultaneously and proportionally increasing all these forces and moments more and more. You are increasing it more and more. What will happen?

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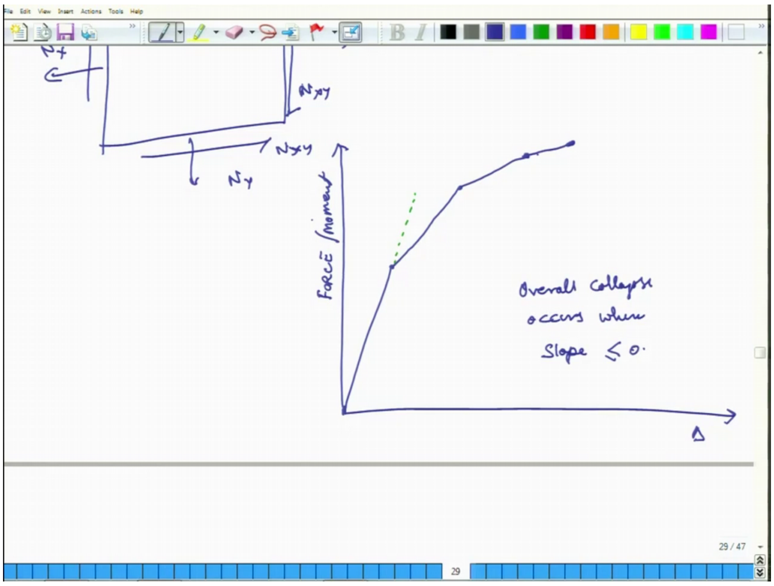

If I plot on the x-axis, let us say I am plotting deflection. Suppose, I am also applying some moments on it and all sorts of things I am doing, but all the external forces are proportionally increasing initially let us say all the forces are 1 1 2, then after some time they become 2 2 4, so everything is going up proportionally. And if I am observing some deflection in the plate, and on the y-axis I am saying that this is force or you can also have moment does not matter.

Then initially when there is zero load deflection will be 0. And as I increase the load, it will keep on increasing. And then as I keep on increasing the load there will be one particular load at which failure will start happening. When that failure happens, what will what does that mean one of the Q bar matrix has become 0.

So, when that Q bar matrix becomes 0, the overall A of the system, B of the system and D will come down right.

So, now if I load this a composite further, it will not go in this direction. This will not be a continuous line the slope of the line will change. It has become less stiff. So, if it becomes less stiff, it does this. And then another failure happens. So, then it does this and then another failure happens. Then it does this, so this is how the stiffness of the plate keeps on reducing. And this trend will continue as long as this slope remains positive.

The moment this slope becomes 0 or negative, this whole structure will collapse because now it cannot take any further load. So, overall collapse occurs when slope becomes equal to or less than 0. It will never become less than 0, because but it will become when in the moment it becomes 0, then overall thing will collapse ok. So, this is what I wanted to talk about in this week. Next week, we will start discussing another set of subjects.

And till then, have a great weekend. Bye.