Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

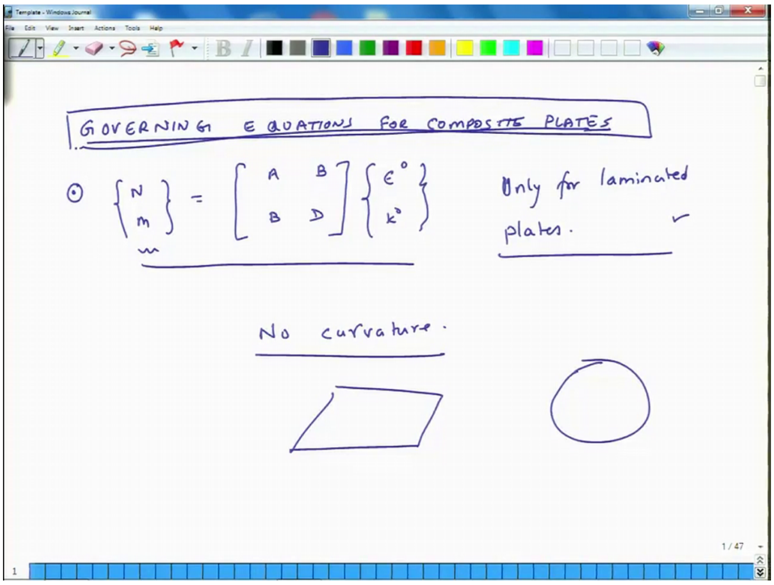
Lecture - 25
Governing Equations for Composite Plates

Hello. Welcome to Advanced Composites. Today is the start of the fifth week of this course. And starting this week we will start developing some new knowledge in the area of composites, specifically first we will develop governing equations which determine the mechanics of advanced composites specifically laminated composite plates and then we will also learn as couple of tricks to solve these equations.

Till so far, what we have done is wavelength how to predict the engineering constants of single layer composites a specifically EL ET NU-LT GLT and so on and so forth. Then we learnt how to compute q bar matrix using these values of engineering constants for a single layer. And then finally, what we have learnt till so far is how to compute A B and D matrices and these matrices, how do they link the strains and curvatures in laminated composite plates to force and moment resultants.

And lastly we also learnt a couple of failure theories about individual layers. And then we extended that knowledge to failure of composite plates and in that context we also came across the idea of progressive failure of composite laminated plates. So, that is what we have covered.

(Refer Slide Time: 01:57)



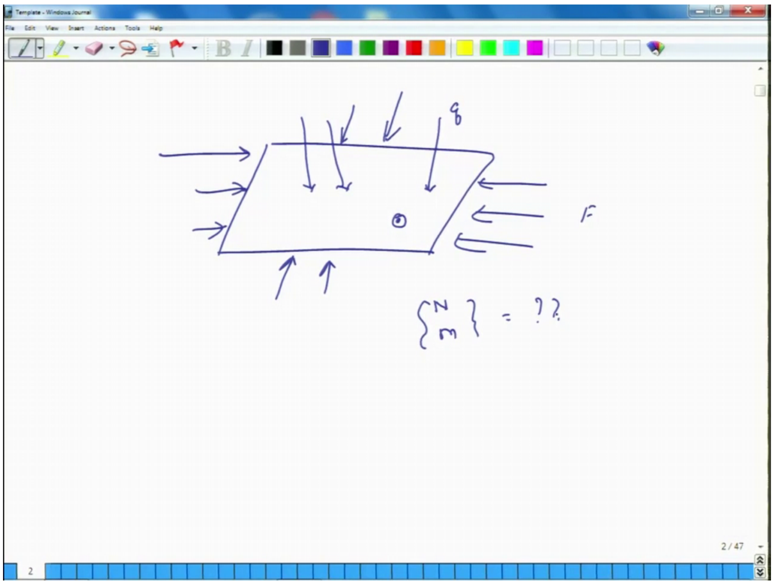
So, the next thing what we will do is Governing Equations for Composites and specifically we will talk about composite plates. So, I like to make couple of points in this context. First is that we had learnt the force and moment resultants; they are related to mid plane strains and mid plane curvatures through this relation. So, these are mid plane curvatures and mid plane strains and mid plane curvatures. One thing I would like to retreat is that these relations are good only if the plate for; so, these are good only for plates.

So, what is a plate? A plate is you know a stack up of composite materials which has when it is not loaded; it has no curvature no curvature. So, these laminate these equations may not be necessarily true for curved systems for shells, for cylinders, for spheres and things like that, but if we have a plate in use.

So, it can be of this shape or it can be of any shape. But as long as the curvature of the plate is not there, it we call it a plate. So, from extraction standpoint that is a plate and these equations are valid only if we are considering plates. So, this is one important thing to consider.

The second thing is that these relations help us compute. So, if we know an NM, then we can compute epsilon and K or if we know epsilon and K, then we can compute N and M.

(Refer Slide Time: 04:31)



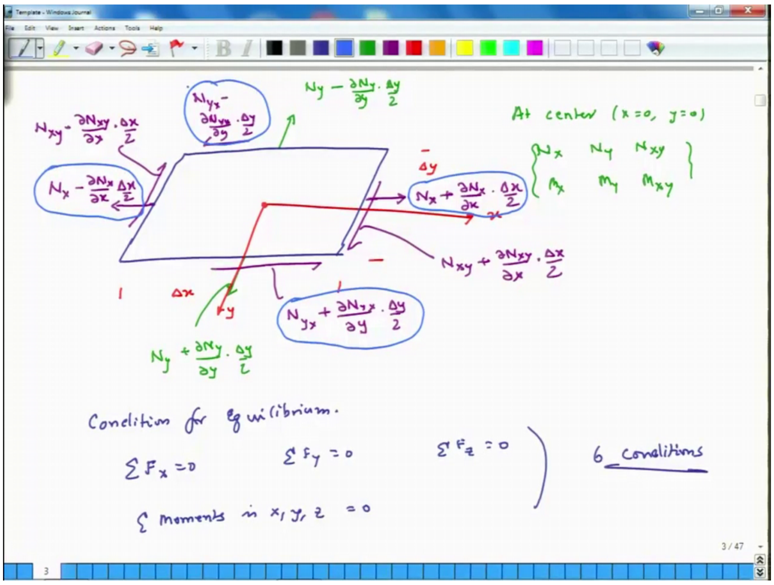
But most of the times, the problem is not whether we know N and M or epsilon K, but we may have some force on a plate this force. It may not be necessarily uniformly distributed and then it may the plate may also be loaded in the transfers direction q ok. So, in that case, we for at any point let us say x and y, we do not necessarily know what is the value of N and M.

So, this is unknown we do not know what is N or M and if we do not know what is N or M, then we cannot compute anything. When we cannot compute mid plane strains, we cannot compute mid planes curvatures and because of that we do not know what are the stresses in the individual area. So, we cannot even predict the deflection or stresses or failure of the plate.

So, the question is that if we know the overall situation that how the plate is loaded on its exteriors. We know how it is being acted upon by different forces and moments and also it is being loaded in the transverse direction by this load q Newton's per square meter, then how do we compute N and M? That is what is the basic question.

Because once we know N and M, then we can compute mid plane strains, mid plane curvatures and then the ply level strains, ply level curvatures, ply level stresses and we can deflect find out the failure and so on and so forth. So, that is what we plan to start thinking about and that is what I imply, by that we are going to develop governing equations for composite plates.

(Refer Slide Time: 06:32)



So, whenever we are talking about composite plates and if we talk about mid plane I am sorry about force and moment resultants, then these force and moment resultants by definition they act on the mid plane of the plate. They may act on the mid plane of the plate. So, let us say this is the mid plane of a plate. So, I am not drawing anything. It is the mid plane of the plate and I am positioning my access system; let us say in this direction this is the access system x and y.

And as the mid plane; so, let us say this is a small; this is a very small rectangular (Refer Time: 07:15). This length is delta x and this length is delta y ok. So, it is a mid plane, small piece of plate represented by its miss plane, length is delta x, width is delta y. And let us say at the center of the plate, the values of N s and M s are so, at center that is at x is equal to 0 y is equal to 0. Value of N x is N x, then we have N y N x y M x M y and M xy ok. So, these force and moment resultants N x N y N xy, M x M y M xy they exist at the center of the plate.

So, once we know that by Taylor series expansion, we can say that in this direction on the edge of this small component. What is the value of N x? It will be a N x plus del N x over del x times delta x over 2. This is from Taylor series expansion. There are higher order terms also, but we are ignoring those terms. Similarly on the other side, we have N x minus del N x over del x times delta x over 2

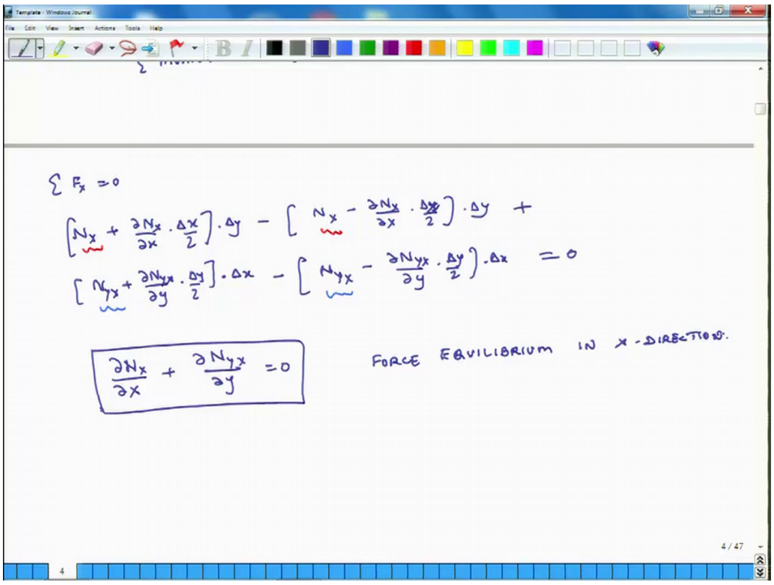
Then let us look at shears. So, on this side, we have this shear will be N x y plus del N x y divided by del times delta y over 2. And on this other surface, so it will be N x by minus del N x y over del y times delta y over 2 ok. So, this is the story for N x likewise we also will have N y s. So, on the positive side, this will be N y and this will be N y plus del N y over del y times delta y over 2. And on the other side we have N y minus del N y over del y times delta y over 2 and lastly we have to talk about N y x. So, actually I made a small mistake and I have to be consistent. So, N y x is on the y plane mathematically, they are identical N y x and N x y are same.

So, mathematically it does not make a difference, but we have to be consistent. So, I will write it correctly. So, that is there. So now, I put an x y. So, this is N x y and this is N xy plus del N x y over del x times delta x over 2 and on the other surface, the other N xy resultant is its value is N xy minus del N xy over del x times delta x over 2. So, what we have put in this figure are all the force resultants on all the four edges of the small plate element. We have not put moment resultants on this plate element, we will put that in another picture, but this is the representation.

Now, if the plate is in equilibrium, what is the condition for equilibrium? What is the condition for equilibrium? The condition for equilibrium is that sum of forces and x direction should be 0 sum of forces in y direction it should be 0 and sum of forces and z direction it should be 0. This is one and then because this is a non this is not a point object, it is a object with finite non zero size. So, we also should consider moments

. So, some of moments associated with x direction should be 0 and y should be 0 and what is that ok. So, actually just to not create confusion because we have M x M y M x y, we are calling them as force resultants. So, I will not use M x y, but essentially some of moments in x y z direction they should be individually 0. We will not use M x M y M x y terms because they are for moment resultant which is not savage moment. So, what we will do is we will implement these 6 conditions is a; 6 conditions and we will come up with 6 different equations of equilibrium which will ensure that this plate is in equilibrium. And we assume that the plate is in static equilibrium. So, there is no acceleration term, there is no motion happening plate is in static equilibrium. So, that is what we will ensure.

(Refer Slide Time: 13:54)

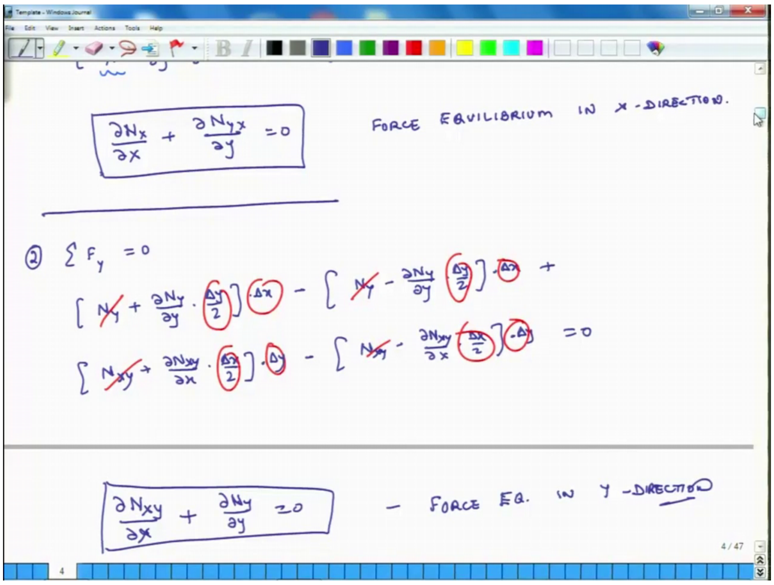


We will start with the x direction some of forces in the x direction is 0 ok. So, what do we do? Let us identify all the forces which are in the x direction. So, this is one force in the x direction. So, this not force, this is a force resultant. So, I cannot sum this up directly, I have to multiply by this y the length delta y. This is another force resultant in x direction which will generate force in x. The third x direction force related resultant is this and this is the fourth one.

So, if we do this we get Nx plus del Nx over del x times delta x over 2 and this is times how much delta y minus N x minus del N x over del x times delta y delta x over 2 times delta y plus. So, this is N y x plus del N y x over l y times delta y over 2 times delta x minus N y x minus del N y x over del y times delta y over 2 times del x. And all these guys should add up to 0 is the system is in equilibrium.

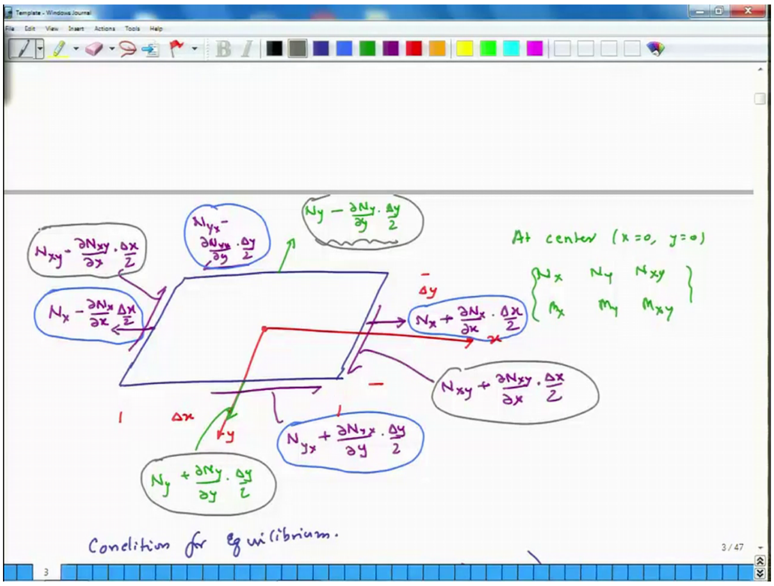
So, what you find is that this guy and this guy, they cancel out because all the terms are being multiplied by delta x times delta y over 2; also N y x and N y x they cancel out. So, if we do all this simplification ultimately what we are left with is del N x over del x plus del N y x over del y is equal to 0. This is the equilibrium equation for x direction related to forces ok. So, this is the force equilibrium equation force equilibrium in x direction.

(Refer Slide Time: 16:44)



So, the next step: so, this is an one equation that the first equation. The next equation is sum of forces in y direction should be 0. So, this is the second condition for equilibrium.

(Refer Slide Time: 17:06)



So, now let us look at that picture again and what are all the forces in the y direction. So, in the y direction, this is one force, this is another force what else. These N x y related terms are another, this will also generate a force again we have to multiply this by the length element and this is the fourth piece of the puzzle. If we add up all these forces, what do we get? y plus del N y over del y times delta y over 2 times delta x minus N y minus del N y over del y times delta y over 2 times del x plus N x y plus del N x y over del x times delta x over 2 times delta y minus N xy minus del N x y over del x times delta x over 2 times delta y is equal to 0.

So, again these terms gets cancelled out and also these terms go away because they are common to all the terms and eventually what we are left with is the second equilibrium equation which is del N xy over del y del x plus del N y over del y equals 0. So, these are the 2 equilibrium equations. So, this is force equilibrium in y direction

So, the next one is we will explore force equilibrium in the z direction and that is a little bit tricky. And we will start that discussion tomorrow, but today what we have covered, we have just started developing the equilibrium equations for composite laminated plates and we have developed these 2 equations del N x over del x plus del N y x over del y equals 0 and partial derivative of N x y with respect to x plus partial derivative of N y with respect to 0. These are the 2 equations of equilibrium related to force equilibrium sin x and y directions respectively.

So, that concludes our discussion for today. Tomorrow, we will continue this discussion and we will start developing the equilibrium for the z direction force equilibrium for the z direction. Till then have a great day, and I look forward to seeing you tomorrow. Bye.