Advanced Composites

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Lecture – 27
Moment Equilibrium Equations

Hello, welcome to Advanced Composites. Today is the 3rd day of the ongoing week which is the 5th week of this class yesterday. And day before yesterday, we have developed 3 equilibrium equations related to sum of forces equation in Y-direction and in the Z-direction, while developing the equation of equilibrium for force equilibrium in the Z-direction. We had also introduced two additional terms Q x and Q y. These are force resultants associated with shear stresses tau z x and tau z y. And the reason these additional force resultants had to be generated is to ensure that there is equilibrium in Z-direction.

Today we will continue this discussion and we will start discussing about Moment Equilibrium Equations.

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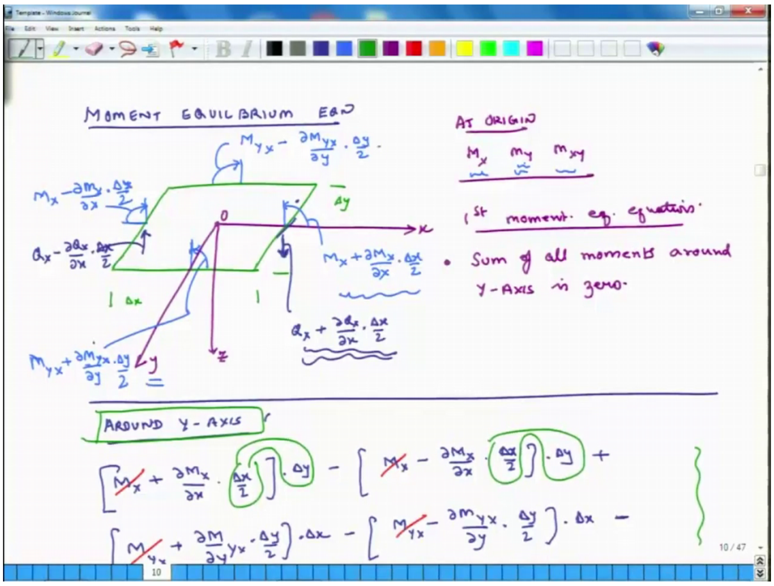

So, we will discuss about moment equilibrium equation. So, once again we will draw a plate. The size of this plate is very small. So, this dimension is delta x and this dimension is delta, that is my x axis, y axis and z axis. Add the origin which is 0. We will list down all the moments which are present. So, at origin we have M x, M y, M xy. So, these are the moments which are present at the origin. So, the first thing we will do is, there are 3 equations. We have to, 3 equations we have to generate. So, first moment equilibrium equation and this will be sum of all moments and whenever we have to specify moment, we have to say around which axis is it trying to, it is trying to rotate. So, we will say sum of all the moments around y axis is 0. So, let us look at what these moments are.

So, the first moment, we will in this picture we will draw only those moments which are going to cause this plate to spin around y axis. So, this is my y axis. So, one moment which will cause this plate to rotate more around y axis will be this thing, ok. What is this? This is called M x. So, remember even though we call it M x, it is causing the moment to rotate around y axis and this M x we call it M x, because it is acting on this plane which is normal to this plane is the x axis. That is why, ok. So, if at the center this moment is M x, then at this edge it will be M x plus del M x over del x times del x by 2. And remember this is not the moment, this is moment resultant. So, to find the moment, we have to multiply it by the length delta y.

Similarly, on the other side we will have similar moment acting in the other direction. So, this is M x minus del M x over del x times delta x over 2. So, this is one moment which is causing it to rotate around y axis. Then, what about M y is not causing rotation around x axis? It is causing rotation around x axis. So, we will not draw this M y in this picture. The other moment which causes it to rotate around y axis is, so I will draw it is M yx. So, this is M yx. This M yx is caused because of sigma yx or tau yx. So, this value is, so this is M yx.

So, this value is this thing. So, it is M yx plus delta M yx over del y times delta y over 2 because at the origin we have my x and similarly, on the other edge of the plate, we have M yx minus del M yx over del y times delta y over 2. So, these are all the moments which are acting on this small plate element which are causing it to rotate around y axis, but there may be forces also which can generate moments, there can be forces also and these forces could be n x and y. We do not have to worry about those, because nx ny nx y, they are also already incorporated in M xn M yn M xy.

So, we do not have to, but we have also introduced two additional force resultants Q x and Q i. So, Q x acts in this direction, Q x acts in this direction, right and Q i acts in. So, Q x also acts in Z-direction and Q i also acts in Z-direction. So, Q x and Q y will also cause moments. So, which of these two is going to generate moment around y axis? It will be Q x, because Q x is acting on this edge. So, Q x is also going to generate moment.

So, what is the value of Q x on this edge? First let us list on the value of Q x on this edge. It is Q x plus del Q x over del x times delta x over 2 and on the other edge, it will act in the other direction Q x minus del Q x over del x times delta x over 2. So, Q x will create a moment, but then this is not directly a moment. So, we have to multiply it by half length of the plate, ok.

So, now we write down all the moments around y axis. So, around y axis we write down M x plus del M x over del x times delta x over 2 times. The length of the side of the plate which is delta y minus M x minus del M x over del x times delta x over 2 times del y plus M xy plus del M xy. Actually I should specifically if I am correct then I should use M yx.

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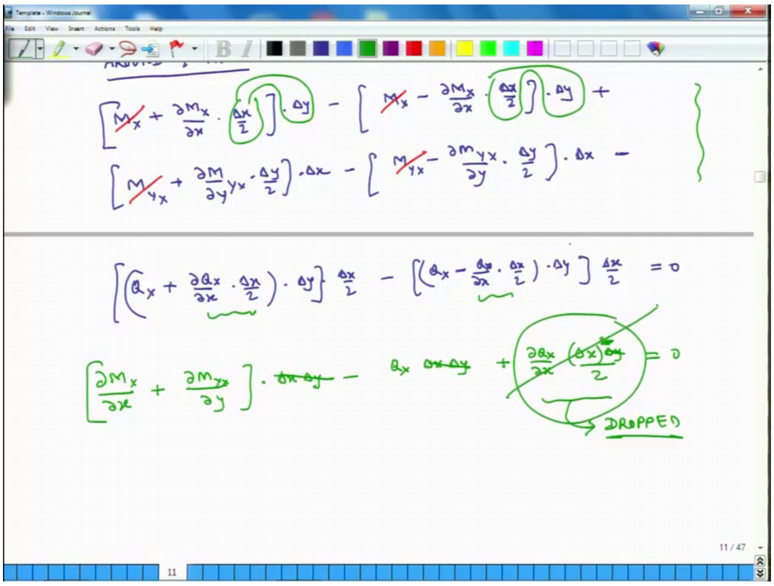

So, M yx M yx over del x, no del y times delta y over 2 times del x minus M yx minus del M yx over del y times delta y over 2 times del x. So, these are moments and then, the moment created, because of force Q x will be Q x plus del Q x over del x times delta x over 2. So, this times delta y is what this is just the force. This times is this is a force resultant. First I have to multiply it by delta y to create a force, right. So, this is a force and then, I multiply it by delta x by 2 delta x by 2. So, that is the moment created by force resultant Q x on the positive side of the plate and similarly, so now just consider this carefully, ok. This force resultant will create it as acting downwards. So, the moment it will create will be opposite of this guy, right. So, I have to put a negative sign here. So, we have to be careful about the signs. So, you have to put a negative sign here, right because this is a negative and then, this is the value of this force.

Then, the next thing we will do is this force is acting upwards. So, it will aid it, ok. So, this is Q x plus delta Q x over del x. Excuse me delta. What do I do is, delta x over 2 times delta y times delta x by 2 and this is equal to 0.

Yes, there has to be a negative sign here. So, if you simplify all this, this all these guys go away. Then, the other thing you are left with is this gives you delta x times delta y divided by 2 and this term and this term add up, ok. So, if you add up the first two rows, what you get is del M x over del x plus del M xy over del y times delta x delta y, ok. This is what you get from these two lines.

Everything else cancels out del M yx del M by x, and then, from this line you get minus Q x minus Q x delta x delta y and plus del Q x over del x and what you get here delta x delta y and delta x is squared and this entire thing is divided by 2, right. See these terms. This is this term and this term, they will add up. So, this comes to be 0, ok. Now, you can cancel out delta x y delta x y and this goes away and this 2 also goes away.

So, the only thing you are left with this delta x. And if I make this size of my element extremely small, I can make it extremely small 1 nanometer. Then, this can be dropped if the size of the element is extremely small. So, this entire thing can be dropped.

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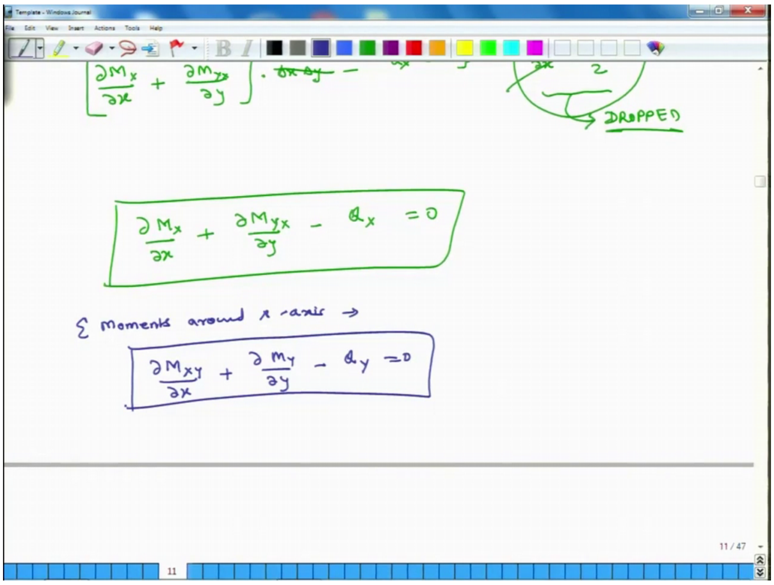

What I get is del M x over del x plus del M xy 0 del M yx over del y minus Q x is equal to 0, likewise. So, this is for the moment equilibrium equation around for all adding if you add up all the moments around y axis. So, if you add up all the moments around y axis, this is what you get. Similarly if you have sum of all the moments around x axis, we get del M xy over del x plus del M y over del y minus Q y equals 0.

So, now we have developed 5 equations; sum of forces in X-direction equals 0, sum of force in Y-directions equals 0, sum of force in Z-direction equals 0. And then, we have also developed two additional moment equilibrium equations that is, sum of moments around y axis and sum of moments around x axis. They are equal to 0 and the related equilibrium equations are given here.

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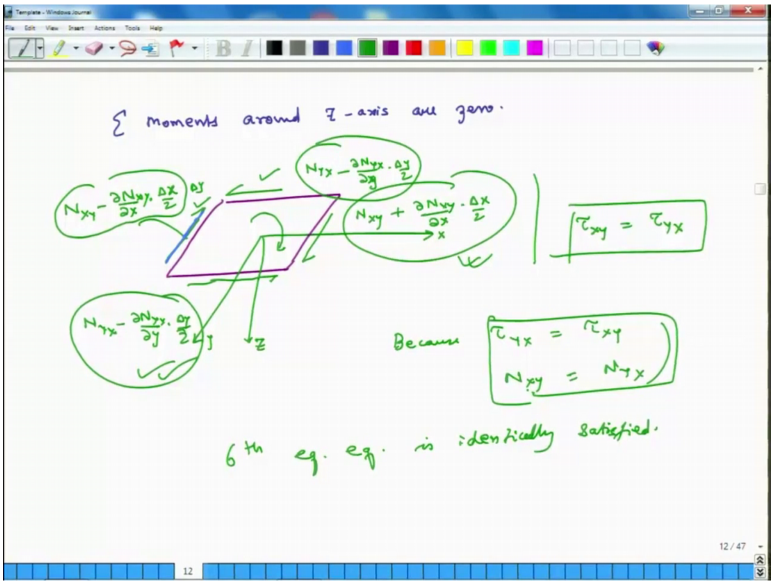

Now, we will develop the last equation and that is sum of moments around z axis or 0, ok. So, let us again make a picture. So, this is x, this is y and this is N yz axis. So, which kind of a force will cause this plate to rotate around z axis? What kind of forces will it call to rotate around this x axis? First thing is that there is no moment, external moment which is acting it to rotate like this. There is no moment which is being applied like this.

So, now the only question is which kind of forces will cause it to rotate? So, one force which will cause it to rotate is this guy and y x and then, N xy. This will also cause it to rotate. Similarly N yx and of course, this is del N yx over del x del y times delta y over 2 and this is this shear resultant is N yx minus del N yx over del y times delta y over 2. And then, on this side we have N xy plus del N xy over del x times delta x over 2 and on this side, we have N x plus del and x over del x times delta x over 2 and this should be negative.

Now, when we were develop relations for a b c d. We said that tau xy is always equal to tau yx, sorry. So, this is, so we will change the direction. What you are saying is right. So, this is like this, this is like this and this is like this, and there is a negative here that does not. So, if you do all this basically what it tells us is see this component. So, this is N xy. So, this component if you multiply it by delta y that will be the force you know that will be the force and then, you multiply it by that distance. So, essentially what I am trying to say is this component and this component, they have to be identically equal. So, this guy and this guy, they have to be identically equal because tau xy is equal to tau yx.

Similarly, this guy should be identically equal to this guy because tau xy is equal to tau yx. If that is the case, then we do not have to create a new equation for equilibrium because if these forces are equal, then the moments which they will generate will be also equal because the distance is the same. So, because tau yx is equal to tau xy, N xy equals n yx and thus, the s6th equilibrium equation is identically satisfied.

We do not have to create 6 equilibrium equation. It gets identically satisfied because of these conditions because the equal equivalence of tau yx equals tau xy and then, if that is true, then N xy will be same as N by x, ok. So, effectively we have only 5 equilibrium equations 3 for force and 2 for moment. So, that concludes our discussion for today.

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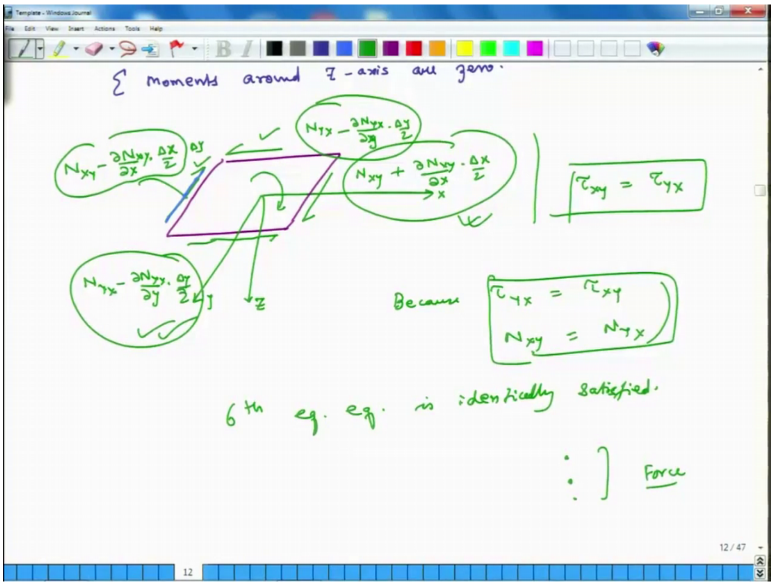

Tomorrow we will extend this discussion. And we will actually, we have these 5 equations; 3 for force and the other 2 for moment and we will combine these two equations and will actually shrink the number of these equations to 3. So, that is something we will do tomorrow, and then we will also try to solve these equations subsequently.

So, that concludes our discussion and then, I look forward to seeing you tomorrow.

Thank you.