Advanced Composites
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Lecture – 28
Equilibrium Equations for Composite Plates

Hello, welcome to Advanced Composites. Today is the 4th day of the ongoing week which is the 5th week of this course. And what we have discussed over this week is: we have started with force equilibrium equations and we developed these 3 of these equations for X, Y and Z-direction and then, yesterday we developed 3 moment equilibrium equations. And, what we found that while developing the moment equilibrium equation for 3rd axis that is z axis, we found the fact that tau xy equals tau yx, that means N xy equals N yx. And that implies directly that the 6th equilibrium equation will be identically satisfied.

So, effectively we have only 5 equilibrium equations; mutually independent equations and we will again rewrite them and process them further.

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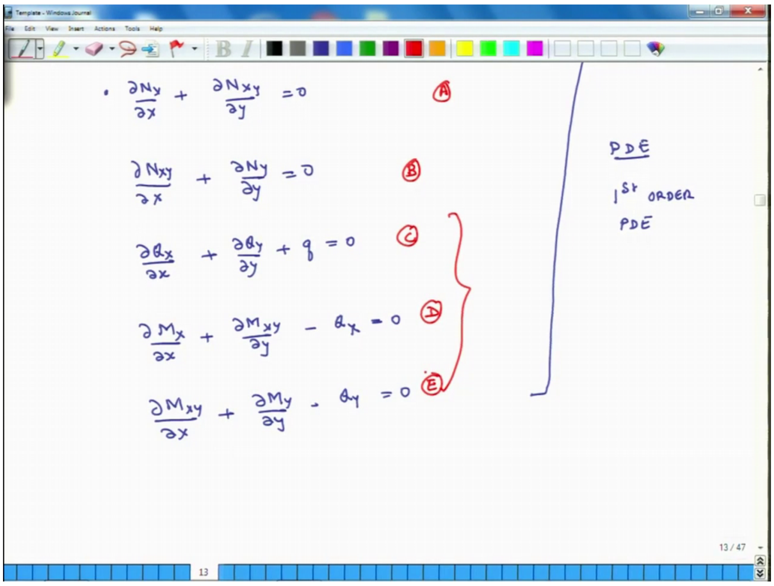

So, what are these equations? Del N x over del x plus del N xy over del y equals 0. This is for x equilibrium for forces del N x y. And now, because N xy is equal to N yx, I am just going to write it like that. Del x plus del N y over del y equals 0. This is the second force equilibrium equation.

The third equilibrium equation is del Q x over del x plus del Q y over del y plus Q equals 0. And then, we have the moment equilibrium equations del M x over del x plus del M xy over del y.

Student: Minus Q y.

Now, Q x equals to 0 and finally, we have del M xy over del x plus del M y over del y minus Q y equals 0. So, these are 5 independent equilibrium equations. All of them are partial differential equations in N and M moment resultant and force resultant. What is their order? Their order is they are first order PDEs, first order Partial Differential Equations. And if we solve these equations, we will get the solutions for N x N xy N y M x M y and Q x N Q i. So, these are the equations.

Now, what we will do is, we will combine the last 3 equations into one single equation because that makes things a little bit easier to handle mathematically. So, we will number these. So, this is A, B, C, D, E.

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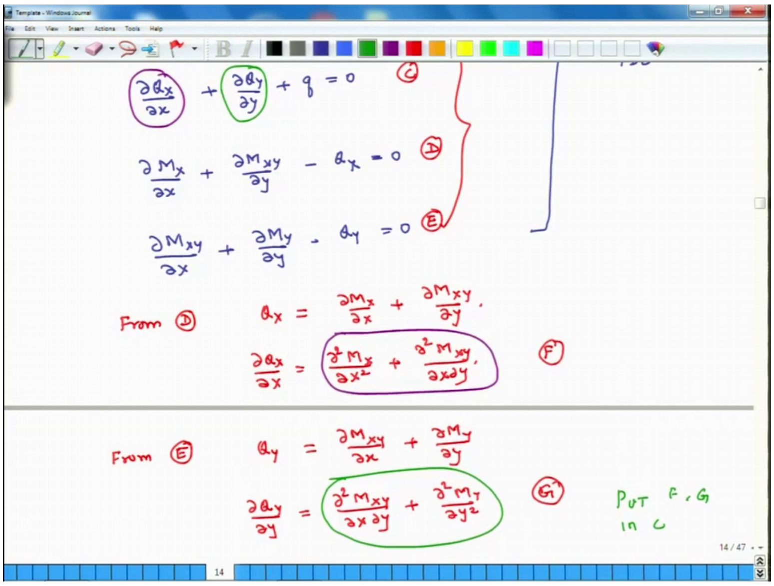

So, we know that from A, now from D we know that Q x equals del M x over del x plus del M xy over del y. So, if we partially differentiate this equation in x, we get del Q x over del x is equal to second derivative of M x with respect to x plus del 2 M xy over del x del y.

So, let us call this equation F and then, likewise from E, what do we have we know that Q y equals del M xy over del x plus del M y over del y. And if we partially differentiate this whole equation with respect to y, then we get del Q y over del y equals cross derivative of M xy with respect to x and y plus del 2 M y over del y square. So, we call this equation G, ok.

So, we see that this is del Q x over del x which is same as this and in equation C, the 2nd term is partial derivative of Q y with respect to y. And that is there in equation G. So, we put F, G in C.

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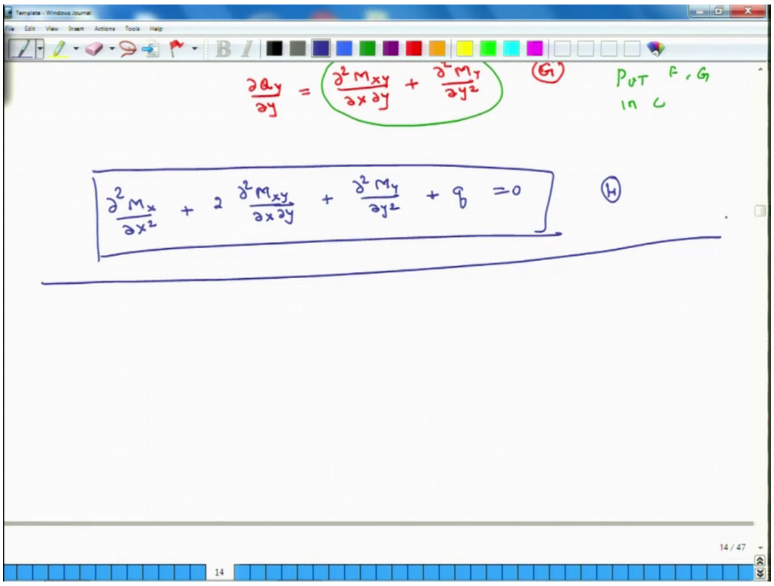

Finally, what we get is del 2 M x over del x square plus twice of del 2 M xy over del x del y plus del 2 M y over del y square plus Q equals 0. So, this is other equation. So, we have H. So, I can either use these 5 equations A B C D, any or I can use these 3 equations.

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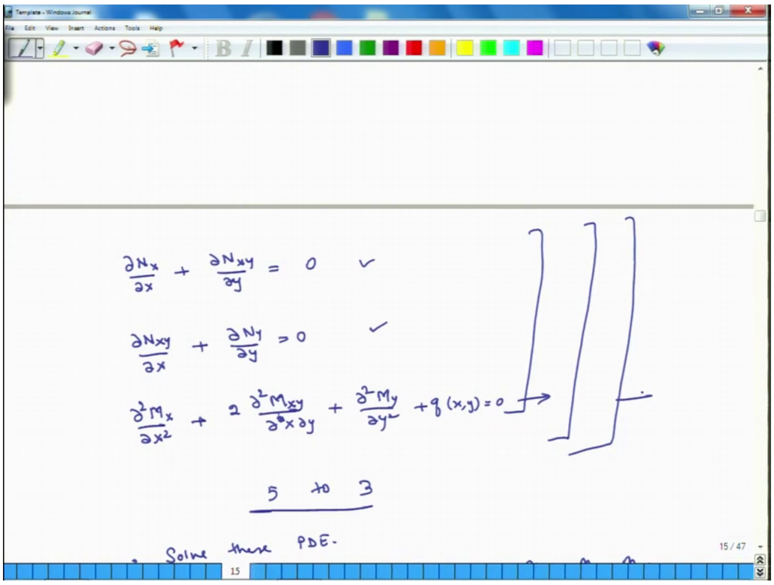

I am sorry is equal to Q. So, did I write Q in the other one? So, actually Q is on the left side, ok.

So, here what we have done is, we have reduced the number of differential equations from 5 to 3, but whenever you do such types of reductions, what ends up happening is that you raise the order. So, we have reduced the number of differential equations, but we have increased the order of 3rd differential equation. So, earlier it was just first order, now this is a second order differential equation ok.

So, if we solve these equations, we can find out what is so at every point ok. So, what is the point? So, the aim is to solve these partial differential equations. Once you solve these partial differential equations, we get values of N x, N y, N xy and what else M x M y M xy at all points in plate, ok. So, if we are able to solve these partial differential equations, then we will be able to calculate N xy, N y, N xy at all the points in the plate.

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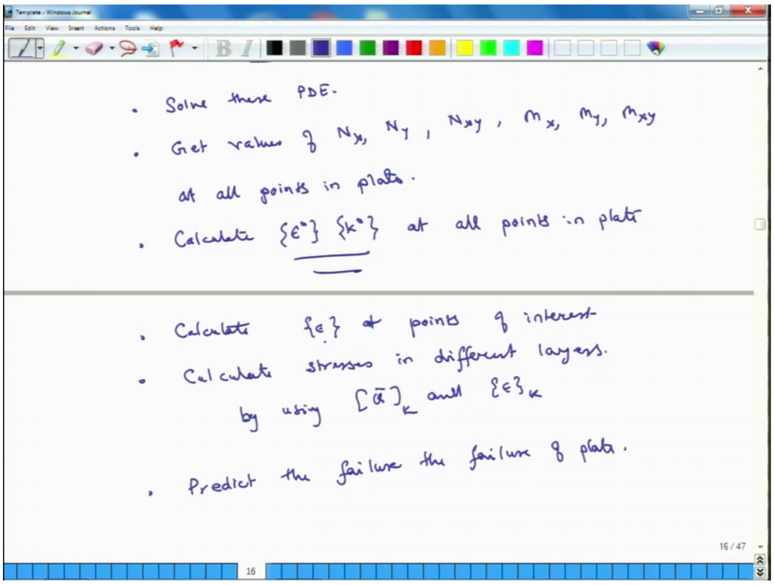

Once we do that, then as a next step what we can do is, calculate epsilon naught and k naught curvatures and mid plane strains and mid plane curvatures at all points in plate. Once we are able to do that, then what do we get out of it calculate the strains at points of interest. And once we know the strains at in different layers because, now if we know epsilon naught and k naught. Then using the relation between epsilon and epsilon naught and k naught, we can calculate strains at different layers at different points.

So, then we can calculate stresses in the plate, right. We can calculate stresses by using Q bar matrix and epsilon matrix for the case layer, and once we know that we can predict the failure of the plate. So, we can know whether our plate is strong enough or not. So, this is the overall process. You start with partial differential equations. Solve the partial differential equation, you get N x, N y, N xy, M x, M y, M xy.

Once you have knowledge about these moments and force resultants, using this force and moment resultant, you compute mid plane strains and mid plane curvatures at different points in the plate or actually at all the possible locations once you have that. Then, you can at every cross section you can calculate how strain is varying along the thickness of the plate and that will help you calculate the stresses in each layer of the plate using, because then you can use fine epsilon k and Q k Q bar k and you can calculate stresses in each layer of the plate.

Then, you can transform these stresses to LT planes, right. So, you can find out sigma L sigma T and tau LT and then, using the work energy, this maximum work criteria or maximum stress criteria or maximum strain criteria, you can compute whether the plate is going to fail and if it fails which particular layer is going to fail and so on and so forth. So, this is the overall process through which you can design composite laminated plates.

In this entire process, there is one thing which we have not discussed till so far that whenever we solve these equations, whenever a differential equation is solved. Then once we solve these equations we will essentially what does that mean when we solve a differential equation, this is a differential equation. We integrate it, right and whenever we integrate a partial differential equation, we get some functions.

So, when we solve this, we will get a function from here, another function from 2nd equation and 3rd equation is a second order differential equation. So, we have to integrate it twice. So, we will get 2 functions from 3rd equation, and these functions are unknown. So, we will have to figure out their values and the way we figure out their values is that if we have a plate, then on each edge there will be some boundary conditions.

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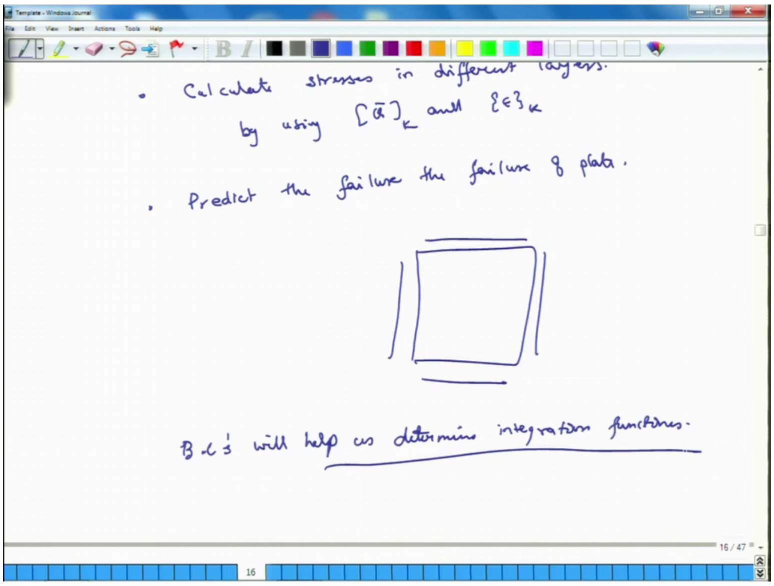

If we know these boundary conditions, then because if we know these boundary conditions, then in terms of these boundary conditions we can figure out or calculate the integration called constants or integration functions which are forgotten because of integrating these things. So, the boundary conditions will help us determine integration functions. So, we have to know what kind of boundary conditions are existing on every single edge of the plate.

So, in the next class and this is, this requires some detailed discussion. So, in the next class, we will actually discuss what kind of boundary conditions are required, what kind of boundary conditions we should know and we have to specify, so that we can determine these integration functions. So, that is what we plan to discuss tomorrow.

So, that is pretty much all for today and I look forward to seeing you tomorrow at the same time.

Thank you.