Advanced Composites
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Lecture – 29
Boundary Conditions Associated with Different Edges of Composite Plate (Part-I)

Hello, welcome to Advanced Composites. Today is the 5th day of the ongoing week. And till so far over this week we have accomplished in terms of developing 5 different differential equations which represent force and moment equilibrium of a laminated composite plate.

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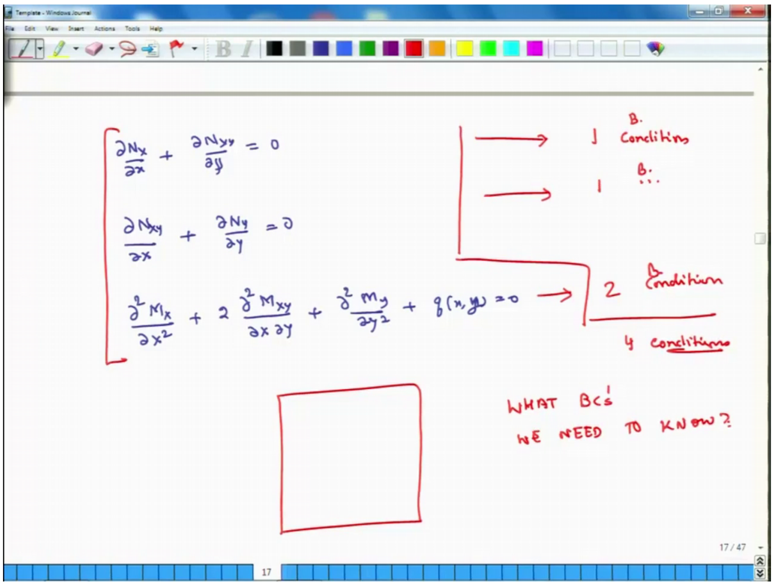

So, once again I will very quickly write down these equations del N x over del x plus del N xy over del y is equal to 0 del N xy over del x plus del N y over del y equals 0 and then, the 3rd. So, either we can write down 5 equations or if I combine the last 3 equations, I get a 2nd order differential equation in moment resultants and that is del square M x 2nd derivative of M x with respect to x plus twice of cross derivative of M xy with respect to x and y plus 2nd derivative of M y with respect to y square plus q xy equals 0.

Now, if I integrate 1st equation to find the integration function, I need one condition. Similarly, I need another condition from the 2nd equation. And because the 3rd equation is a 2nd order differential equation, partial differential I need two conditions to you know to find these things related to the 3rd equation. So, overall our need is for 4 conditions, 4 boundary conditions. So, these are boundary conditions, ok. So, then the question is there. If, suppose there is a plate. What kind of conditions we need to know, so that we can calculate the integration functions which emanate out of these 4 equations, out of these 4 differential equations, what kind of BC's we need to know.

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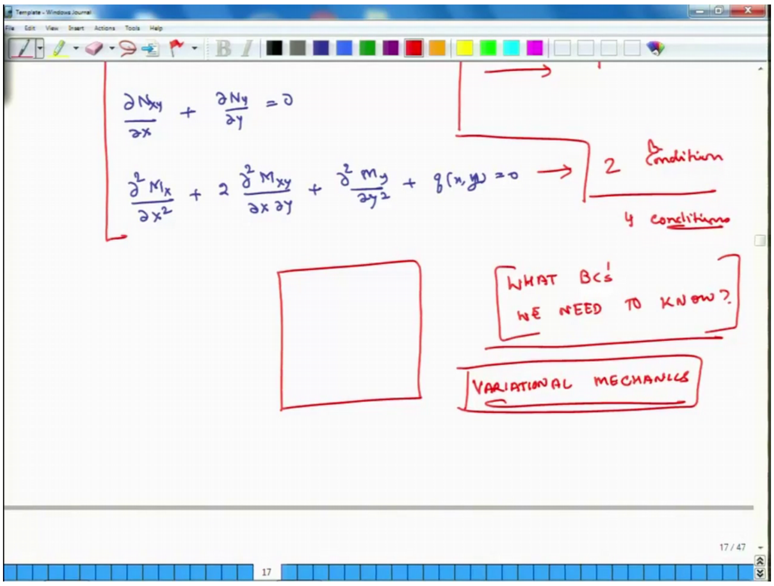

To answer this question using the laws of equilibrium which we have discussed till so far is a little tricky, but if we employ principles of variational mechanics, then these boundary conditions come logically out of variational principles. In this class, we will not discuss this process, but we will directly share with you the results that is what kind of boundary conditions which we need to know to solve these equations successfully, but if you are interested in how we came to these boundary conditions or the definition of these boundary conditions.

I will strongly recommend that you familiarize yourself with these variational mechanics principles. And then, use those variational mechanics principles to again derive equilibrium equation for this plate and then, you will see that these conditions which I will cite now, they will naturally flow out of the process. So, let us look at what are the boundary conditions.

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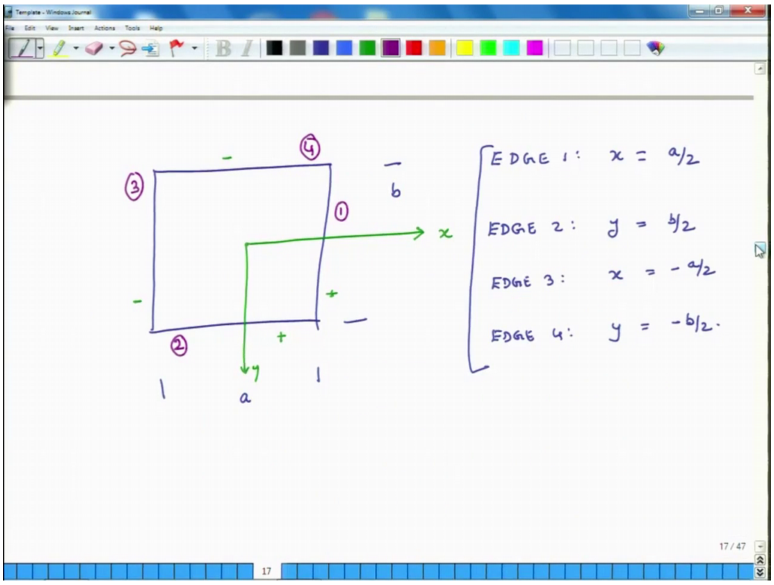

So, before we talk about this, we will have a reference picture, ok. So, this is edge 1, this is edge 2, this is edge 3 and this is edge 4 of them. And the way we have always specified the origin is located at the centre of the plate and this is my y axis. Excuse me the y axis is in this direction.

So, we can say that edge 1, it is specified by a line and let us say the dimensions of this plate are a and this dimension is b, then edge 1 is specified by x is equal to a over 2. The 2nd edge, edge 2 is specified by y equals b over 2. The 3rd edge it is specified by x is equal to minus a over 2 and the 4th edge is specified by y equals minus b over 2. So, at all these edges 4 edges, we will see what kind of boundary conditions we need to know to solve these differential equations. So, we will go by S by edge. So, edge 1.

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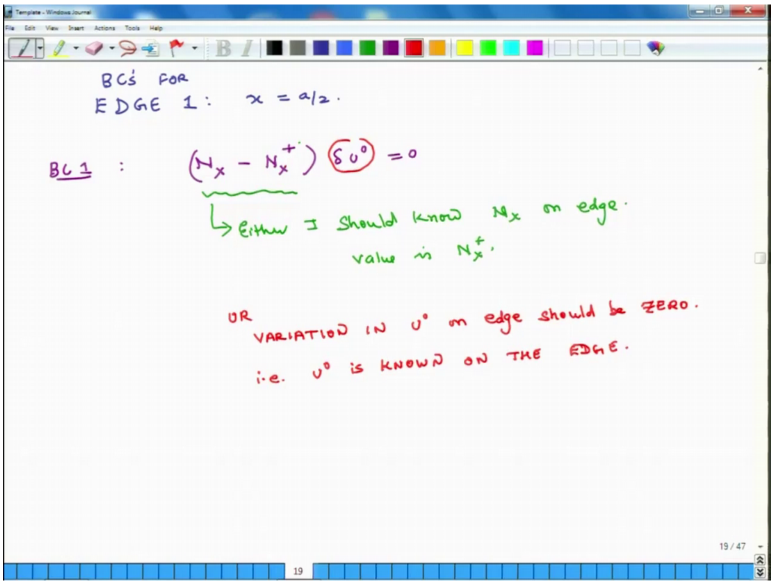

So, this is boundary conditions for each one and what if this edge 1 x is equal to a over 2, and as we said at each edge we have to know how many boundary conditions. It is 4 boundary conditions. So, we list those 4 boundary conditions, ok. So, BC number 1 boundary condition 1; and first I will write it and then, I will explain because this involves some slightly different terminology N x minus N x positive times delta u naught equals to 0.

So, this is the 1st boundary condition, ok? What this means is that either we should know N x on the plate either on that edge, we should know the value of N x. So, if this is a plate and if this is x is equal to a over 2, I should know externally how much force I am applying on this plate, on this edge. Suppose, this is a plate and this is x, this edge is represented by x is equal to a over 2, then either I should know N x on this edge. How is N x varying or how is force varying on this plate, if I know force. Then I can calculate an x, right.

So, how is? So, I should know N x on this edge that is. So, either I should know N x on the edge and if and what is the value of that N x. So, value is N x positive, ok. Why is it positive? Because this is the positive edge of the plate; the other edge of the plate is x is equal to minus a over 2. So, that is the negative edge over the plate, ok. This is just terminology, ok.

So, again we will go back to this picture. This is the positive edge positive plus x of the plate, this is negative x edge of the plate, this is positive y edge of the plate and this is negative y edge of the plate. So, that is why in the boundary condition we have this positive here, this positive means that it is on the positive x edge of the plate. So, either I should know this or this is the 2nd condition.

So, either I know this or variation in U naught on edge should be 0 and this is again some mathematical terminology variation does not mean that we should know how it varies. What it means is that; that is U naught is known on the edge U naught is known on the edge. So, an example suppose this is the positive x edge of the plate, then on this edge I should know at all the points on the mid plane either the values of N x and there what will that value I will designate as N x plus or I should know U naught on this edge. I cannot specify both, because I will never know both; either I should know N x on this edge and I will specify, I will call that N x as N x plus or I should know U naught on this edge, ok. This is so either I know N x or I know U naught.

If I know U naught, then U naught will not vary because why have we have prescribed it. So, that is why we say its variation is 0. So, if I say U naught is 5 millimeters on this edge, it does not have to be 0. If U naught is 5 millimeters, then I have specified U naught which means that the variation of U naught on that edge is 0, ok. So, this is the 1st boundary condition.

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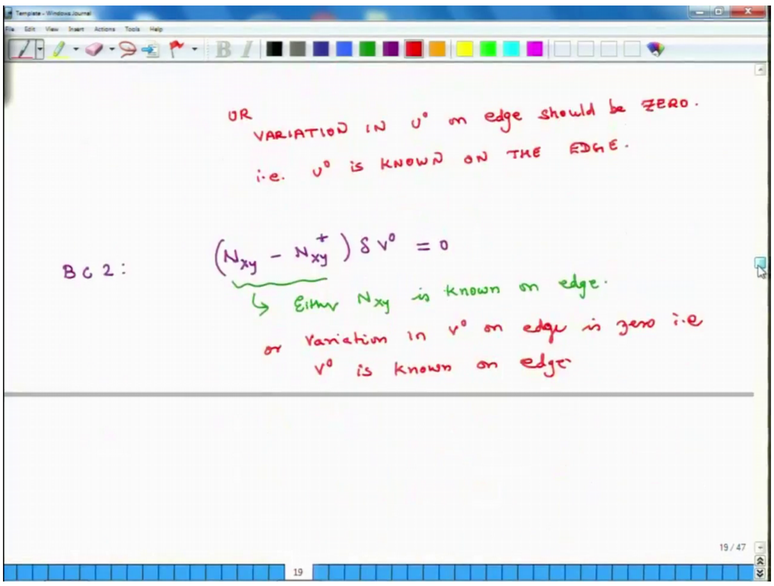

The 2nd boundary condition BC 2 is N xy minus N xy plus times delta V naught is equal to 0. What does this mean? Either N xy is known on edge or variation in mid plane displacement, V naught on edge is 0 that is V naught is known on edge, ok. So, we are still talking about the 2nd edge, we are still talking about the 1st edge which is x is equal to a over 2. On each edge, we left for boundary conditions. This is the 1st boundary condition.

The 1st boundary condition says N x minus N x plus times delta U naught our variation of U should be 0. The 2nd boundary condition we have to know is that either we should know what Nxy on this edge is on this, what he value of Nxy is. If we know the value of N xy, then this thing in the bracket will become 0, because N xy will be N xy plus. So, N xy plus minus N xy plus, it will be 0 or I should know the value of V on this edge. If I say V is on these edges 5 millimeters, then variation in that V will be 0. So, this boundary condition will be satisfied, ok. So, this is the 2nd boundary condition.

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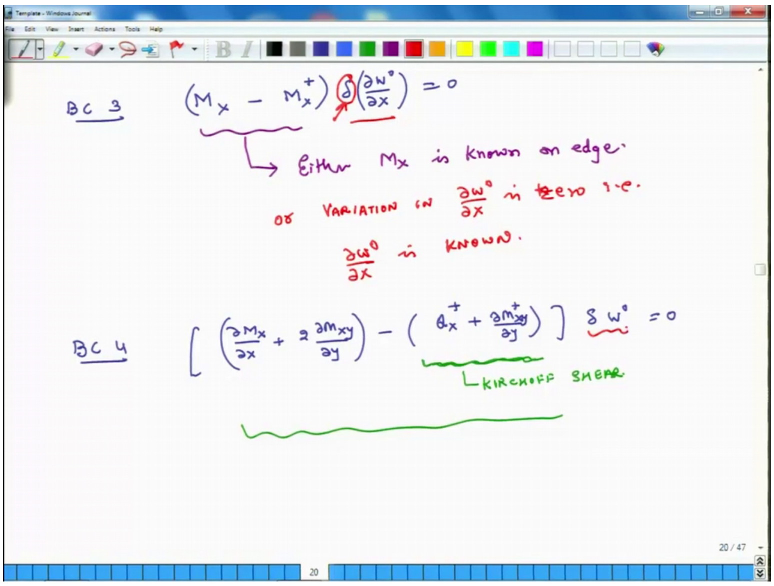

3rd boundary condition, boundary condition number 3 M x minus M x plus times variation of 0 variation of slope in dw, dx is 0. What does that mean? Iy means either M x is known on edge or so. This symbol designates variation, or variation in del w naught over del x is 0, that is del w naught over del x is known, ok. So, again 3rd boundary conditions either on this edge I should know what the moment in x direction is. Moment in x direction is this moment either I should know; what is the bending moment in this direction or I should know; what is the slope of this plate at this edge in x direction. So, this is like in a beam when you talk about the beam, either you specify at the end at a point either the moment or the slope. The same conditions are coming out here and the 4th condition and that is a little different and also, a little bit tricky 4th condition. So, this is a little long condition del M x over del x plus 2 del M xy over del y minus Q x plus del M xy over del y times del W naught equals 0 also and this is plus and this is plus. Now, what this means is.

So, first I will give a name to this entire term in this bracket is called Kirschoff Shear. So, what this boundary condition says is; what does this boundary condition say; so the 1st term and the 2nd term.

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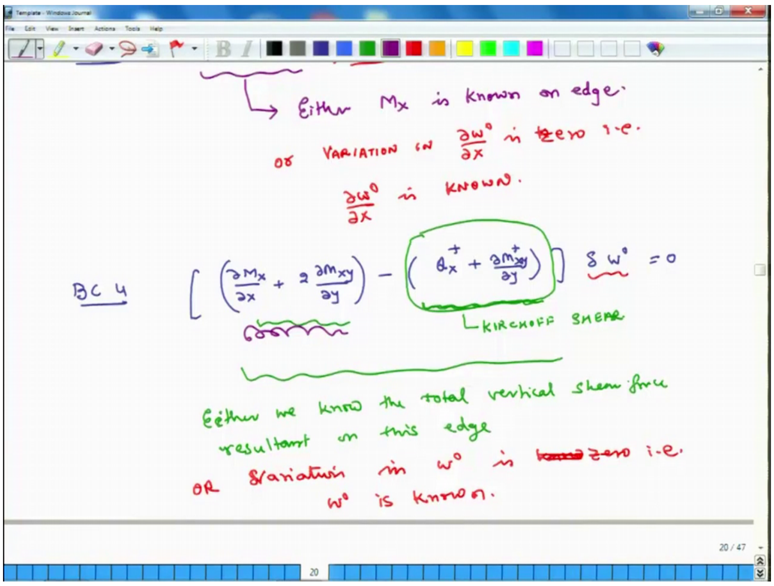

So, the 1st one is either we know the total vertical shear force, shear force resultant on this edge or variation in W naught is known as 0, that is W naught is known ok. Now, I will explain this. So, the 4th boundary condition says, so 1st thing is that this entire term is called Kirchhoff Shear: Kirchhoff because it is given in name of Kirchhoff. It has 2 components Q x plus and also del M xy plus with respect to del M y, the sum total of these.

So, suppose on this edge I apply a vertical force 100 Newtons which is distributed along this edge, that vertical force on this edge it will be resisted by two things: one will be Q x and the other thing which will resist is the moment which will be the moment, and that component is del M xy plus with respect to del y, ok. So, two things will resist is one is the vertical Q x which we have defined and the other thing is del M xy with respect to del M xy with respect to del y, ok.

So, I do not have to specify Q x plus individually or I do not have to express if I del M xy individually. If, I know the total shear force acting on this edge, then I specify that total shear force and that total shear force is this term, this term. So, if I specify this, so suppose that is 10 Newton's then, then that 10 Newton's equals Q x plus del M xy over del y. So, this term in the bracket will become 0 or if I do not know this shear force on the edge, then I should specify what the displacement is in z direction which is w, ok.

The moment I specify displacement in the in z direction, I say that the variation of w in z direction is 0, ok. So, these are the 4 boundary conditions associated with one edge. Likewise we have similar boundary conditions for edge 2, edge 3 and edge 4. And we will again go over those boundary conditions explicitly, so that it gets into our minds very clearly, but today we have run out of time.

So, we will just discuss these 4 boundary conditions and then, tomorrow when we meet again, we will discuss the boundary conditions associated with the other 3 edges of the plate, 2nd edge, 3rd edge and 4th edge.

So, that concludes our discussion and I look forward to seeing you tomorrow.

Thank you.