Advanced Composites
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Lecture - 30
Boundary conditions Associated with Different Edges of Composite Plate (Part - II)

Hello, welcome to Advanced Composites. Today is the last day of this week, which is the fifth week of this course. Yesterday we just started discussing about different boundary conditions we need to know, associated with edges of a rectangular plate.

And in context of that; what we found was that they are 4 different boundary conditions related to each edge, and we had described boundary conditions associated with the first edge is specifically which corresponds to the line x is equal to plus a over 2.

Today we will specify boundary conditions associated with the other 3 edges is specifically x is equal to minus a over 2 y is equal to positive b over 2 and y equals minus b over 2. So, that is what we planned to accomplish today. And also as we do this we will learn a few things more about these boundary conditions. So, that is what we planned to do and let us see how this works out.

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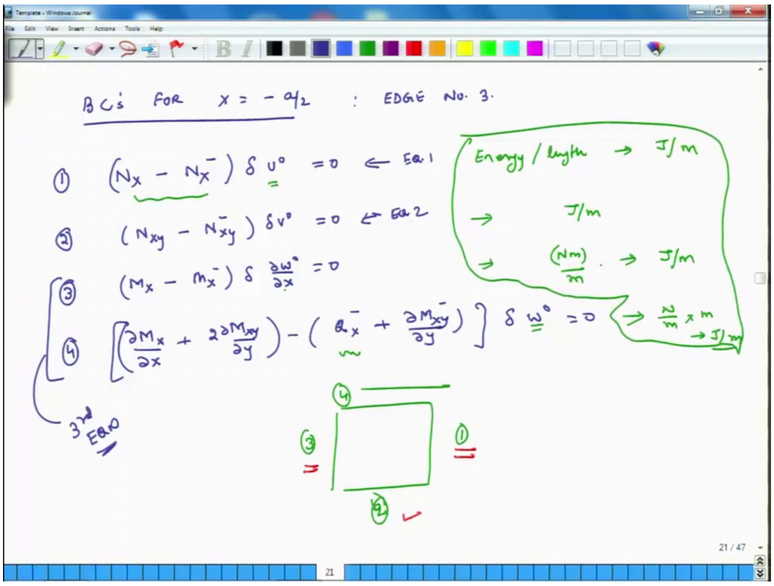

So, today we will discuss B Cs for x is equal to minus a over 2 ok. So, that is edge number 3 is that 3.

So, we will write these conditions, the first condition is N x minus N x minus del u naught equals 0. So, the only thing which is different in this is that instead of N x plus, I have specified N x minus because it corresponds to the negative edge of the plate. The second one is N x y minus N x y minus del v equals 0. The third is M x minus M x minus variation of del w naught over del x equals 0. And the 4th one is del M x over del x plus twice of M x y over del y minus Q x negative plus del M x y negative over del y equals 0.

So, these 4 boundary conditions are; the equations are almost identical the only thing we have changed in these 4 boundary conditions is we have replace the super script plus with a positive sign to a negative sign, but I wanted to say some couple of more points about this boundary conditions. And these points are also applicable to the boundary conditions which we discussed earlier.

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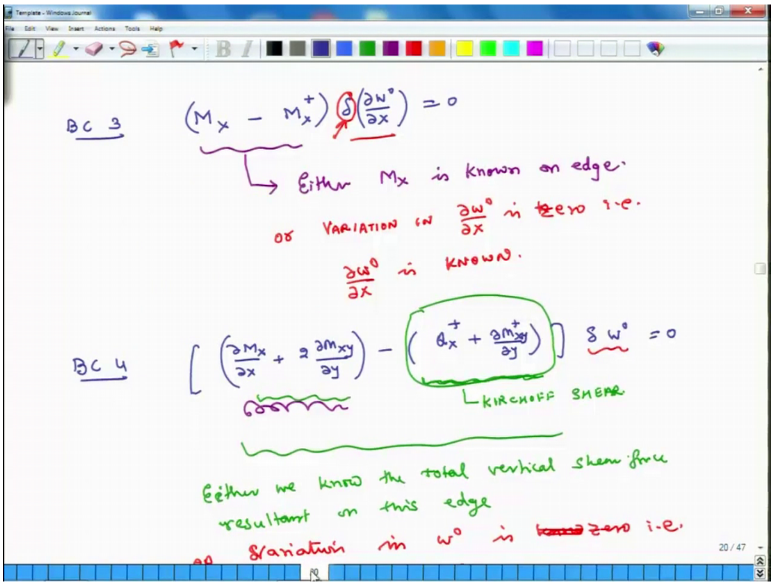

See as I said earlier these boundary conditions come out from principles of variational mechanics.

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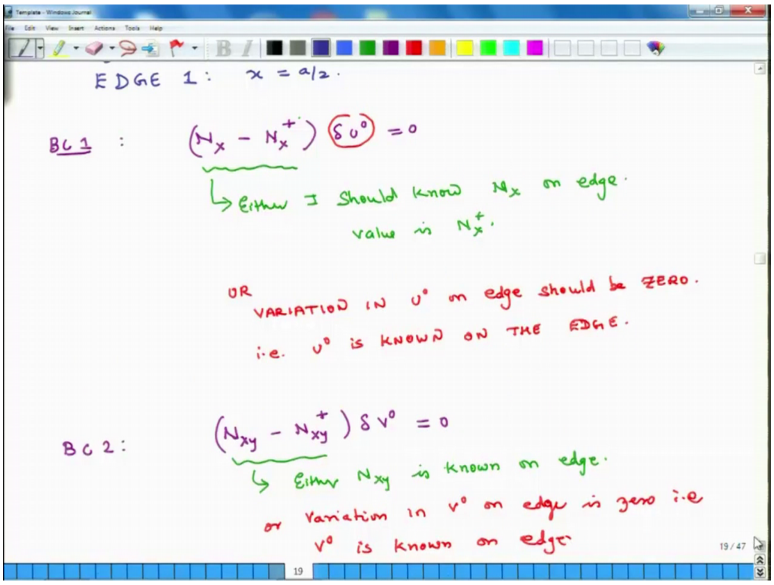

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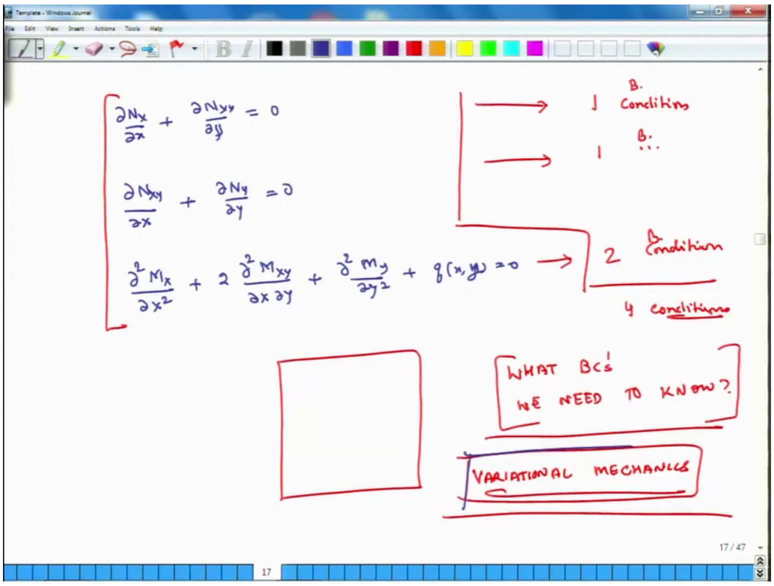

So, we said they come out from principles of variational mechanics. And in variational mechanics essentially what we do is; we the step one like, when we were developing equilibrium equations we had added up all the forces on an element and we said sigma f x is equal to 0, so we get one equation sigma f y is equal to 0 we get another equation. In principles of variational mechanics we do not use Newton's laws at all. What we do is; we add up energy of the entire system.

So, we add a potential energy of the system, if there is kinetic energy be add up all their things. And we say that the total energy of the system it is it has to have a minimum total potential energy and that happens when the variation of that total energy is 0, the first variation is 0. Now what first variation means it is beyond the scope of this course, so, we will not discuss it.

But from that principle we get these equilibrium equations. And from the same variational energy principle we get these boundary conditions. So, in these boundary conditions, you see things related somewhat to energy for instance, what is N x? N x is force per unit length.

So, force per unit length multiplied by variation in u, force times u is displacement force times u is displacement this; that means, this is energy per unit length right? So, the units are Joules per meter, what is the unit of N x? Newton's per meter multiplied by u; u is displacement, so it is joules per meter.

Look at the second equation you will see the same thing N x y is joules per unit meter times, no this is force per unit meter times displacement. So, again the unit is joules per meter ok, look at the third equation you will see the same thing. Movement is Newton meter, but this is moment per unit length. So, unit is Newton meter per meter, and what is d w over dx? Slope. So, slope is unit less. So Newton meter is again. So, unit comes out to be Joules per meter and then look at the 4th equation.

In the 4th equation you consider let us consider Q x, Q x is Newtons per meter times displacement w naught. So, again it comes to be joules per meter. So, the point what I am trying to show is that these variations are directly coming out from principles of variational mechanics. And if you develop interest in this area it will be very useful because these principles are very once you get it you can use these principles in any area of mechanics and you can develop a equilibrium equations and boundary conditions in much more easily.

So, this is these are the boundary conditions associated with the second edge, which is x is equal to minus a over 2 and the number of this edge is edge number 3. So, what we have done is; this is edge number 1, edge number 3, edge number 2 and edge number 4. And we have done BCs for these 2 edges.

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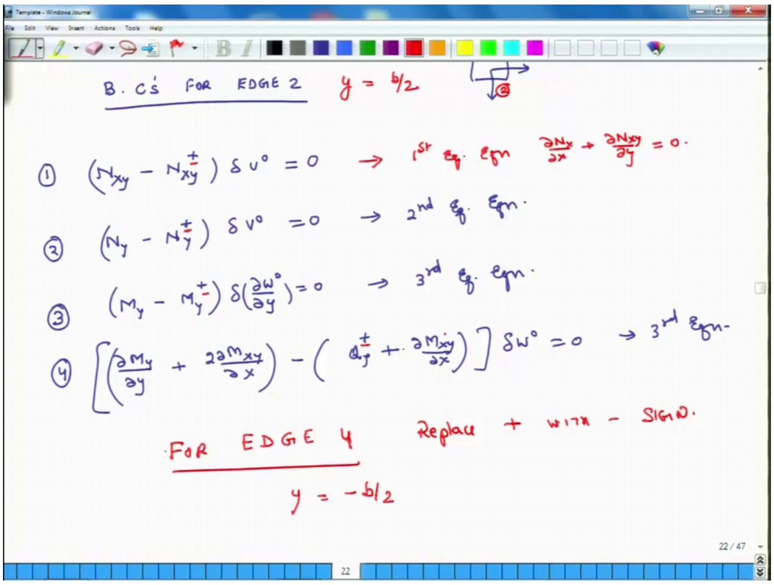

Next we will look at edge number 2 B Cs for edge number 2. So, what is this edge number 2 is what? This is the edge we are talking about; this is a edge number 2.

So, this is not x y equals B over 2. So, before I talk about that, one more thing I wanted to share on this one is that the first equation. So, all these are related to energy the second thing is this first this is the boundary condition which comes from first equation, which comes from first equation. Which is del N x over del x plus del N x y over del y equals 0.

So, this is from the first equation, this is from the second equation. This is from and these 2 equations BCs are from third equation, which is the second order partial differential equation ok. So, these come these are associated with third equation so ok.

So now we will go back to second edge and the boundary conditions are N x y minus N x y plus. So, this is first boundary condition times del u naught equals 0 ok. So now, see this here for edge 3 N x y minus N x y minus. It was multiplied by variation v, but in this case it is multiplied by variation in u naught and this is associated with first equilibrium equation. And which says del N x over del x plus del N x y over del y equals 0 it comes from there ok. And I do not have to explain this now because essentially what it means is that on this edge either we know N x y or we should know u.

The second equation is; the boundary condition is N y minus N y plus times variation in v naught equals 0. So, this is associated with second equilibrium equation. The third boundary condition is M y minus M y plus times variation of del w over del y equals 0 ok. This is the third equilibrium equation and the 4th boundary condition is del M y over del y plus 2 del M x y over del x minus Q y plus plus del M x y over del x times variation of w is 0 so this is also from the third equation.

And finally, for edge 4 we replace the plus sign with negative sign. So, for edge 4 y is equal to minus b over 2. So, all we do is we put a negative sign here ok. So, we just put a negative sign at these locations, replace plus by negative and then we get the boundary conditions for the negative edge. So, this is what I wanted to discuss there are a couple of other important points I would like to talk about.

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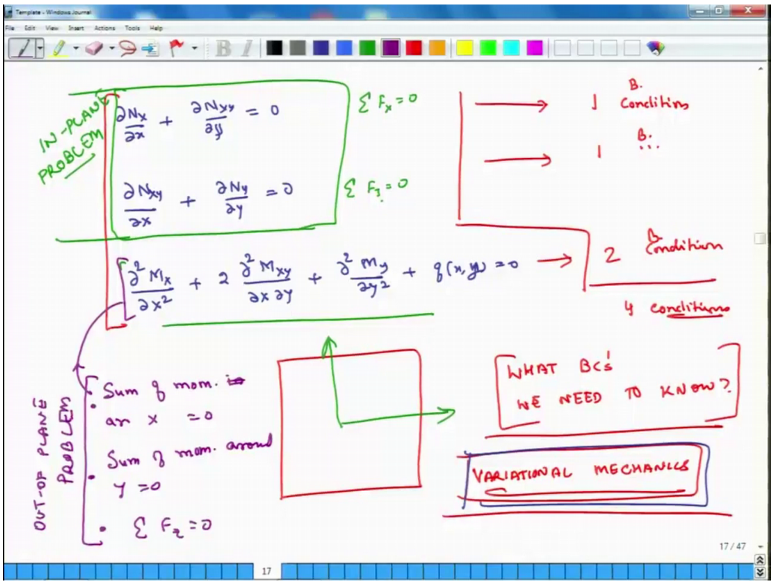

So, the first thing I had said is that this term Q x plus plus del M x y over del y. I had said that this is Kirchhoff shear. And this entire thing we will designate it as Q x plus effective. So, this is effective shear it has 2 components Q x plus and del M x y over del y, but the effective thing is the sum total of these 2 and that is what we are interested in, so we are not interested in finding this individual values of these 2 guys it does not matter.

So, when we are specifying boundary conditions we have to specify the sum of it we do not have to, so we need only one information not 2 pieces of information. And the sum of S if there is an external force which is acting on it then we use that to specify the sum of these 2. Similarly, Q y plus plus del M x y over del y so this is x plus is Kirchhoff shear for y direction for the y plane and this is Q y plus and this is again effective. So, these are effective out of plane shear resultants on the boundary.

Because that is why we have plus subscript or superscript ok, these are on the boundary. Last thing let us look at these equilibrium equations one more time. The first equation and the second equation, so this relates to sigma F x is equal to 0 this relates to sigma F y is equal to 0. And this equation, what does it represent? It represents some of moments in no around x equals 0, some of moments around y equals 0 and it also represents sigma F z is equal to 0, because we had compressed 3 individual partial differential equations into one single partial differential equations.

So, this represents 3 different equilibrium conditions these 2 equations represent only one equilibrium condition, equilibrium in x direction, equilibrium force equilibrium in x, force equilibrium in y, but the third equation represents a lot of equilibria. It represents summation of moments around x axis to be 0, summation of moments around y axis to be 0 and sum of forces in the z axis to be 0. Because so these first 2 equations relate to forces in the plane of the plate because F x is in the plane of the plate, F y is also in the plane of the plate.

That is why these 2 equations are help us understand in-plane problem. Because they relate to forces which are acting in the plane of the plate F x F x this is x axis.

So, F x is in the plane of the plate, F y is in the plate and x y is also in the plane of the plate N x, N y, N x y all these parameters are in the plane of the plate, but the third equation everything M x M y, M x y Q, Q x, Q y all of these are acting out of the plane. So, the third equation represents out of plane problem. Because when a plane plate experiences forces and moments it will experience displacements and distortions in the plane and it will experience pro deist bending and twisting and also w direction motion.

So, the out of the plane problem is reflected by the third equation and the first 2 equations tell us about in plane part of the problem. So, this is all what I wanted to discuss today, next week we will continue this discussion on differential equations which govern the equilibrium of laminated composite plate. So, and then we will actually start solving some initially some simple problems and then maybe slightly complex problems later in the part of this course.

So that we learn how to use these equations effectively, that is all what I wanted to discuss today. Thank you very much and I look forward to seeing you next week.

Thank you.