Advanced Composites
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Lecture - 31
Generalized Solution for Semi-Infinite Plate
(Part-I)

Hello today is the start of the 6th week of this course and till so far what we have done is, we have developed differential equations which govern the mechanics of laminated composite plates. So, starting this week and for next couple of weeks, we will use these equations to solve some practical problems. So, that way you will have an understanding, you will develop some understanding as to how to use these equations for real life problems. Today we will start with the case of a Semi-Infinite Plate. Now, this is an idealized case of a plate which is extremely long in one direction and not that long in the other direction. So, this is the geometry of the plate.

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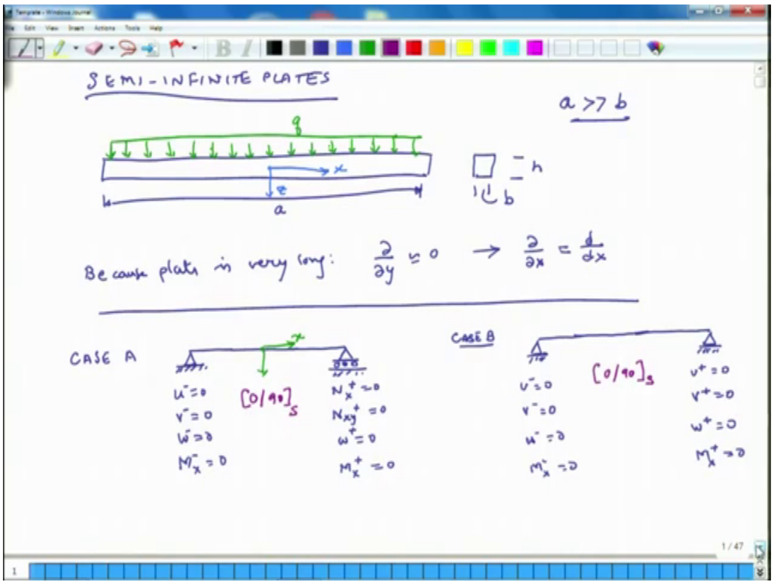

So, we are going to discuss semi infinite plates. Now, theoretically its length should be infinite, but we will that is an extreme case, but a plate which is very long in one direction and not that long in the other direction, we will call it something close to a semi infinite plate. Now, this plate we could have some external load on it and we will assume that the external normal load on the plate is q Newton's per meter; so, that the load intensity, this is uniformly distributed load. If I look at this plate from the other end, it will look like this. So, this distance, this dimension is width. The width of the plate is b, the length of the plate is a, and the height of the plate let say it is h.

So, when we say that this plate is semi-infinite in its length, it implies that a is extremely long compared to b, is extremely long compared to b and let us set our axis system. So, this is the midpoint of the plate. So, my x axis is here and that is my z axis and y axis goes normal to the direction of the board. Now, if that is the case, consider that there is a plate which is very long and if I bend it, it will only develop curvature in the x direction. The curvature in the y direction, it will not be that much significant. So, because of that we can say that because the plate is very long, we can say because of the geometry we can say that all the differentials in the y direction are approximately equal to 0 and because of this, so if all the differential in the y direction partial derivative is in y direction are approximately 0.

What that means is that the derivative in the x direction, we can approximated as the total derivative because there is no derivative in the y direction. So, this is the implication of a semi infinite plate. Now, we want to see that when we subject this plate to an external load of q, how does this plate bend or deform and as we discussed earlier, we also need to know the boundary conditions for such a problem. So, we will have for different problems which we will try to solve. So, what are these 4 problems. So, I will just draw a schematic. So, first there is case A. In case A, let say this is the plate and it is simply supported on both sides, but the nature of simple support is different. So, it is the simple support is not allowed to roll on a base at one end, but on the other end it can move.

So, if that is the case, then what is the boundary condition here because it is simply supported u I 0 again it is pinned. So, v is 0, w is 0 and because it is simply supported moment is 0 at this end and because my axis is from here. So, this is the negative edge of the plate. So, I will say u minus v minus w minus and M x minus these parameters are 0. Now, on the other end what is the condition? So, because this simple support is free to roll, it is free to role and anything can be free to roll only if it is not subjected to any external force. So, instead of u being 0 are at this end, we will say that N x plus is 0 because it is free to all there is no external force acting on it.

Similarly, instead of v being equal to 0, at this end we will say that n xy plus is 0, that is the force in the y direction force per unit length in the y direction is 0. The third thing is that it is simply supported. So, there is restriction of w. So, w is 0 and the force is M x plus is 0. The other thing is what is the lamination sequence? The lamination sequence is 0 90 symmetric. So, this is the definition of case A.

Next let us look at case B. We will do 4 cases, ok. So, in case B it is similar geometry, but the plate is pinned and fixed on both the ends. So, what are the boundary cases? Boundary conditions u minus equals 0, v minus equals 0, w minus equals 0 and M x minus equals 0 and on the positive side u plus equals 0, v plus equals 0, w plus equals 0 and M x plus equals 0 and the lamination sequence is still the same 0 90 symmetric. So, it has 4 layers 0 90 90 and 0. So, that is our case B.

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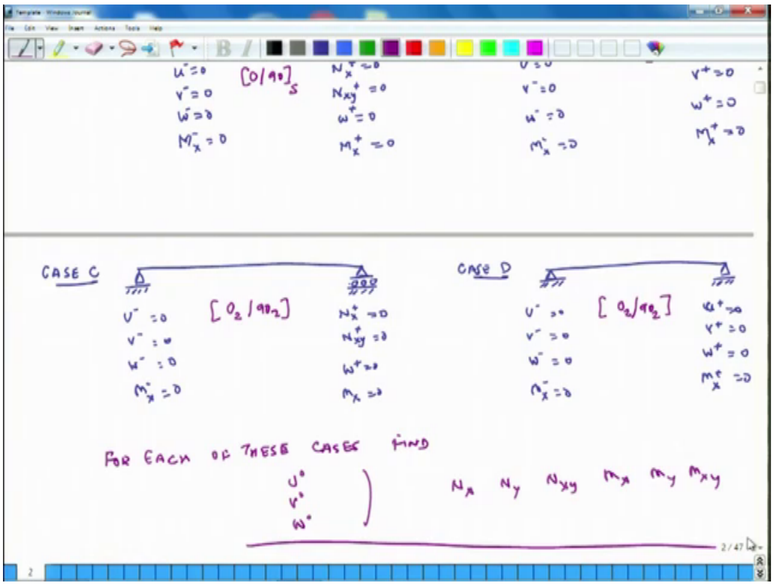

Then, we will do 2 more cases. case C again same type of plate and the boundary conditions in this case are similar to that in case A, ok. So, U minus V minus W minus M x minus all these are 0 and what is the boundary condition other side N x plus N xy plus W plus and M x plus they are 0. So, you see that case C and case A, they have same boundary conditions, but the difference is that here the lamination sequence we will say that it is not symmetric and let us see what happens when the lamination sequence is not symmetric, ok. So, that is case C and then, finally we have the 4th case, case D and the boundary conditions for case D are similar to case B.

So, U minus V minus W minus M x minus all these are 0 and the only difference between D and B is that the lamination sequence for D is not symmetric. So, both all the cases have 4 layers, but in two cases we have symmetric lamination sequence and in the other two layer cases, case C and case D, the lamination sequence is not symmetric. So, what is the question? The question is for each of these cases find U V W. We should find: what is the value of U V and W, and then, if we know U V and W, then we can also find N x N y N xy M x M y M xy. So, we have to find for all these cases, all these parameters. So, that is the problem.

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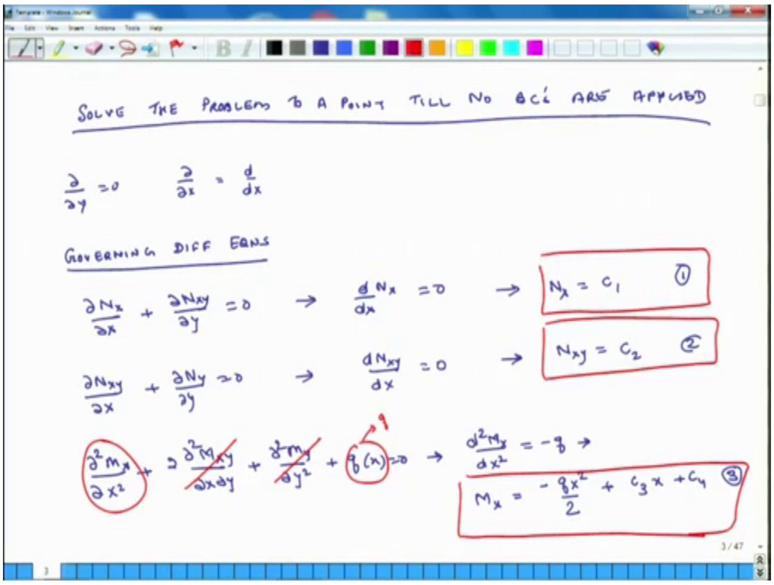

So, actually before we start with case a we will first, so first what we will do is, we will solve the problem to a point till no boundary conditions are applied. First we will develop, we will start solving this problem till we reach a stage that no boundary conditions is applied. So, till that point the solution will be common and then, the movement we start applying boundary conditions and the values of abd, then that solution starts diverging, ok.

So, first we will develop a common solution and then, we start, then we will apply the conditions for a matrix, b matrix, d matrix and also boundary conditions and then, that we will develop 4 different solutions. So, we know that del over del y equals 0 and del over del x equals d over dx, ok. So, if that is the case, then our governing equations, governing differential equations let us see how that changes or gets simplified. So, our first governing differential equation was del N x over del x plus del N xy over del y equals 0 and this simplifies to d over d x N x equals 0, right and if I integrate this, I get N x is equal toC1.

So, till so far we have not applied any boundary condition and we have not applied any stuff related to lamination sequence. So, this is valid for all the cases. What is the 2nd governing equation? It is del N x y over del x plus del N y over del y equals0 and once again del over del y is very small and del over del x is same as d over d x. So, what we get is, d N xy over d x is equal to 0 and that gives us N xy is equal to C 2.

Again this equation, this value, so what it means is that in all the forces, the 4 cases N x will be constant throughout the length and N x will y also will be constant throughout the length of the plate and then, what the 3rd equation is. Second derivative of M x plus 2 del 2 M xy over del x del y plus del 2 M y over del y square plus q. It is a function of x. Now, we know that anything which is partial derivative of y is 0. So, this is gone and this is gone and this thing it becomes a total derivative and also we know that qx in our case is constant. So, it is q in our case, we are applying the load uniformly on the entire plate

So, what I get is, so this gives me second derivative of M x is equal to minus q which is the constant and this gives me M x is equal to minus qx square. Actually I will write it below, so that it is cleaner. So, it gives me M x. If I integrate it twice, I get M x is equal to minus qx square by 2 plus C 3 x plus C 4. So, that is the 3rd equation. So, these three equations; 1 2 and 3, these are common to all the four different cases. So, these are common to all the four different cases. So, now what we will do is, we will now solve case A, ok.

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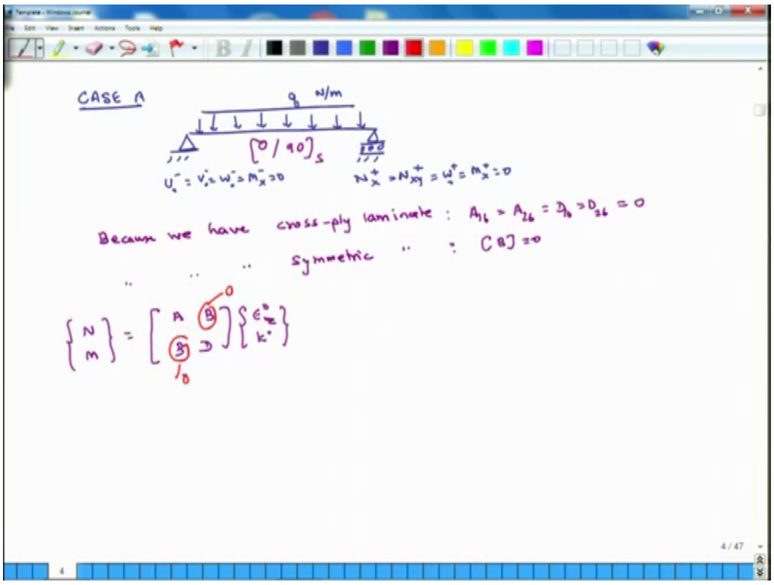

We will solve for case A. So, what is case A? Case A is this thing and of course, this constant uniformly distributed load here and its intensity is q Newton's per meter, q Newton's per meter. So, the boundary condition here is U minus equals V minus equals W minus equals M x minus equals 0 and in the other case, it is N x plus is equal to N xy plus is equal to W plus equals. So, these are all mid plane displacements is equal to M x plus equals to 0 and the lamination sequence is 0 2 92 symmetric. That is the lamination sequence. So, what we will do is because we have cross ply laminate.

Student: So, it contain minimize.

Student: There was 0 0 90 symmetric.

Yes. So, this is not 0 2, it is 0 and 90 symmetric, yeah. So, because we have a cross ply laminate and what does the cross ply laminate means that all the layers are either 0 degrees and 90 degrees. Just because of that we can say we have learned this earlier that A 16 is equal to A 26 is equal to D 16 is equal to D 26. This is 0 and also, the laminate is not only cross ply, it is also symmetric because we have symmetric laminate. We can say that the entire d matrix is 0, ok.

We know that N and N if we have to, so relate with strange, then this is equal to A B B and D times. Mid plane is mid plane strains and mid plane curvatures, ok. Now, it just turns out that this B matrix is 0.

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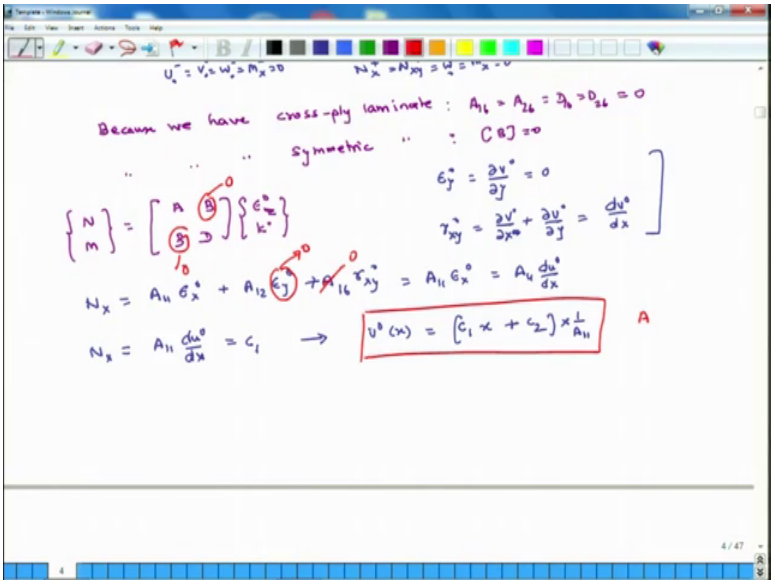

So, what we get is that N x is equal to A 11 epsilon x naught plus A 12 epsilon y naught plus A 66 gamma xy naught. I am sorry not A 66, A 16 A 16 gamma x y naught, but we know that A 16 is 0 because it is a cross ply laminate and also, we know for this thing. So, what is epsilon y? Epsilon y naught is del v naught over del y and because it is a partial derivative of y, this is also 0, ok. Similarly gamma xy naught is del v over del x naught plus del v naught plus del u naught over del y. So, this is again if I just ensure that partial derivative with respect to y is 0, then that means it is equal to d v naught over d x.

So, anyway this is there. So, in this relation for N x A 16 is 0 and also epsilon y is 0. So, N x becomes A 11 epsilon x naught and what epsilon x naught is. It is equal to A 11 times d u naught over d x. It should have been partial derivative, but because it is d over d x. So, partial derivative because of the semi infinite plate is a total derivative. So, it just simplifies to this thing. So, we can write this N x is equal to A 11 d u naught over d x and we have seen that from equation 1, this is a constant N x is equal to C 1 from equation 1, ok. So, we can say that this is equal to C 1. So, this gives me if I integrate it, I get U naught x is equal to C 1 x plus C 2 into 1 over A 11. So, this is the 4th equation. So, let us call this yeah we will call this equation A.

So, we have calculated now we do not know what these values of C 1 and C 2 are. We will find that later, but what we are seeing is that u in this case is linearly varying over the length of the plate. So, what we will do in the next class is, we will continue this discussion and we will solve for the entire problem for this particular plate in the next class. So, that concludes our discussion for today and I look forward to see you tomorrow.

Thank you.