Advanced Composites

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Lecture - 32
Generalized Solution for Semi-Infinite Plate (Part-II)

Hello, welcome to Advanced Composites. Today is the second day of the ongoing week which is the second week of this course which is the 6th week; I am sorry 6th week of this particular course. And yesterday, we just started solving these 3 important governing equations for laminated composite plates in context of semi-infinite plates.

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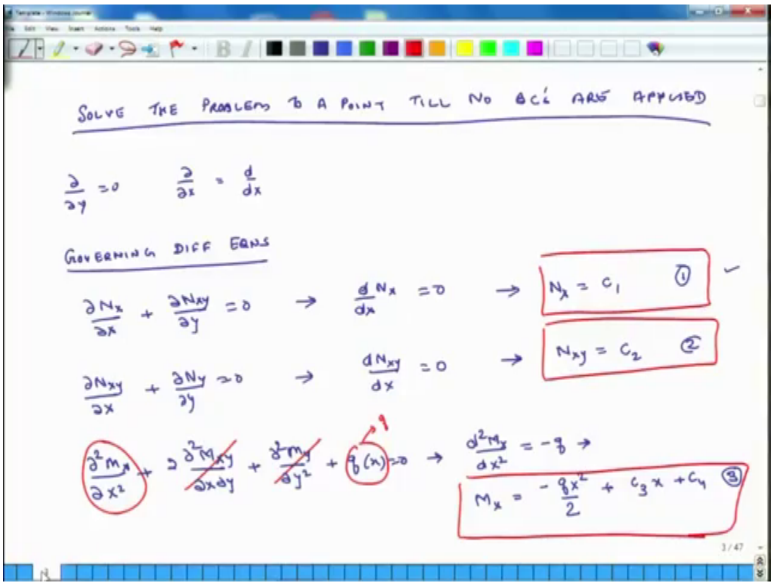

And what we had developed yesterday is first we developed 3 relations for N x N x y and M x and what we saw is that N x equals C 1 and x y equals C 2 and M x equals minus q x square over 2 plus C 3 x plus C 4. And these 3 equations are valid for all the 4 cases which we had discussed earlier.

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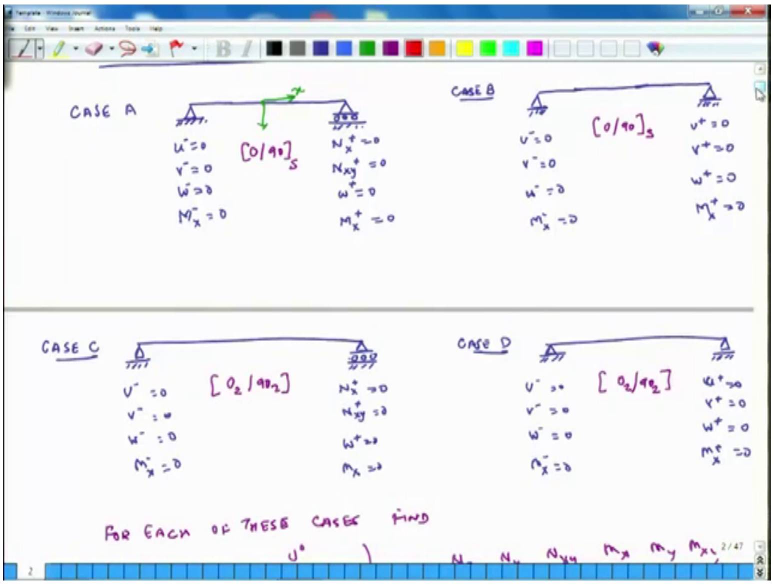

So, these are the 24 cases which we had explained earlier in case A and case B, we have a symmetric laminate; in case C and case D, we have an un symmetric laminate and case A and case C have similar boundary conditions actually same boundary conditions that is pinned and fixed on one end and fixed and roller on the other end while case B and case D have pinned and fixed and so on both the ends. So, these are the 4 different cases we would like to solve. So, the first step was that we solved for N x N x y and M x and these solutions are same for all the 4 cases.

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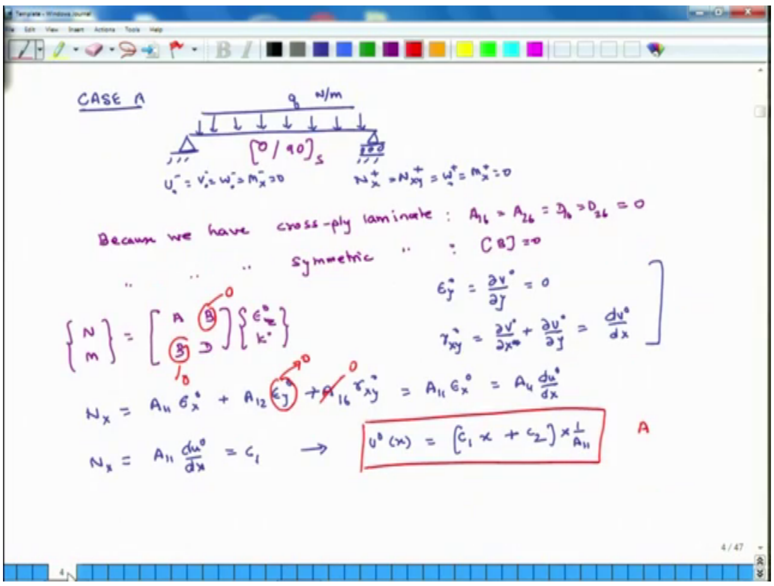

And then, we started applying the lamination sequence. So, by virtue of lamination sequence, so, we still have not yet applied the boundary conditions by virtue of lamination sequence. We have seen that epsilon y equals 0 gamma x y equals d v naught over d x. And when we do all the math , we also find out that U x U which is the displacement in the x direction is linearly varying and it is value is C 1 x plus C 2 divided by A 1 1.

So, this is what we had accomplished till that point of time and what we will do is actually we will name this equation as equation 4. So, this is 4 equation number 4. So, we will proceed and we will continue to solve for U V and W. So now, we will try to find out the value of V. So, how do we do that?

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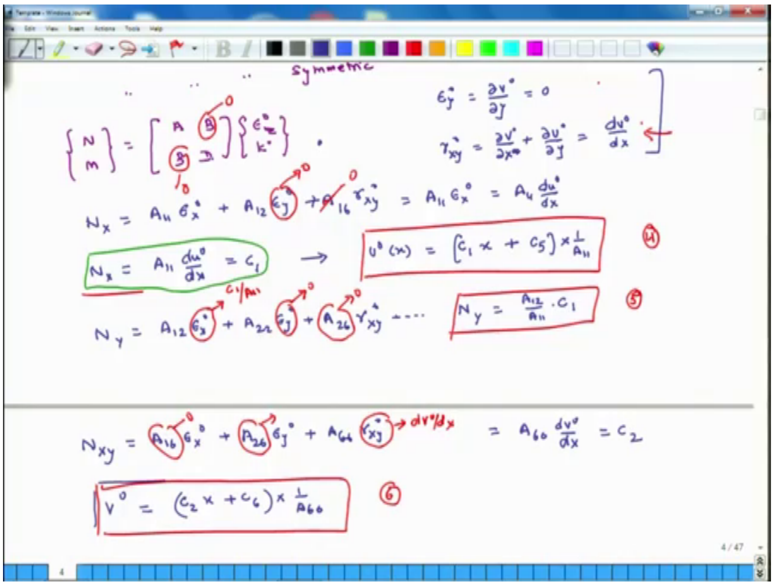

So, we know that N x y equals A 1 2 epsilon x naught plus A 2 2 epsilon y naught plus A 2 6 gamma x y naught ok. And the all the other terms are 0 which involved the B matrix. So, I have only considered A 1 2 A 2 2 and A 2 6 terms. I have not included B 1 1 B 1 2 B 2 2 and B 2 6 terms because, they are identically 0 because the matrix is symmetric. And also, we know that epsilon y this is 0 and gamma x y is again 0; oh I am sorry, not gamma x y. This A 2 6 is 0 because the laminate is cross ply and symmetric and because it is cross ply, then A 2 6 will be 0.

So, the second 2 later 2 terms in this whole equation becomes 0 and epsilon x is already there in place. So, this epsilon x naught when we see it is equal to C 1 over A 1 1 how do we get C 1 over A 1 1 from this equation ok. Here, d u naught over d x is epsilon x naught. So, epsilon x naught is equal to C 1 over A 1 1 ok.

So, what this gives us is that N x y is equal to A 1 2 over A 1 1 times C 1. So, this is another relation we get and that is 5.

Student: Sir, can you put at the place of (Refer Time: 03:49) 4 (Refer Time: 03:50) C 1 x plus S 2 and you already use (Refer Time: 03:54) another constant (Refer Time: 03:56) 4.

[FL] 4 or 5.

Student: 4

Oh [FL].

Student: (Refer Time: 04:02) [FL] (Refer Time: 04:03) N x y (Refer Time: 04:05).

Oh yeah, yeah yes. So, one thing we did earlier is that we have used this constant C 2, but C 2 has already been used earlier. So, instead of C 2, we will use a new constant. We will call it C 5 C 5 ok. This relation for N x y I mentioned is incorrect. So, this should be for N y ok. This should be for N y N y is equal to A 1 2 epsilon x plus A 2 2 epsilon y and so on and so, forth. And so, likewise this relation gets changed and corrected and this is actually for normally the third relation is.

So, we have N y N x N y and the third relation is for N x y, third relation is for N x y and what it tells us is that this is equal to A 1 6 epsilon x naught plus A 2 6 epsilon y naught plus A 6 6 gamma x y naught gamma x y naught. Now, once again A 1 6 is 0 A 2 6 is 0 and gamma x y naught is d V naught over d x in our case which we have developed here ok. So, if that is the case, so, this simplifies to A 6 6 d V naught over d x and we have seen that N x y from equation 2 is equal to C 2. From this equation, it is equal to C 2.

So, we will use that here. So, what I get is and I when I integrate it I get V naught is equal to C 2 x plus C 6 into 1 over A 6 6. So, this is I will call it equation 6. So, we have developed an expression for U we have developed an expression for V and what is remaining is an expression for W.

So, for W, let us look at it further M x using the A B D relation. So, when we see from this equation M is equal to B times epsilon naught plus D times K naught, but B is 0. So, the only terms which matter are terms related to D in this thing.

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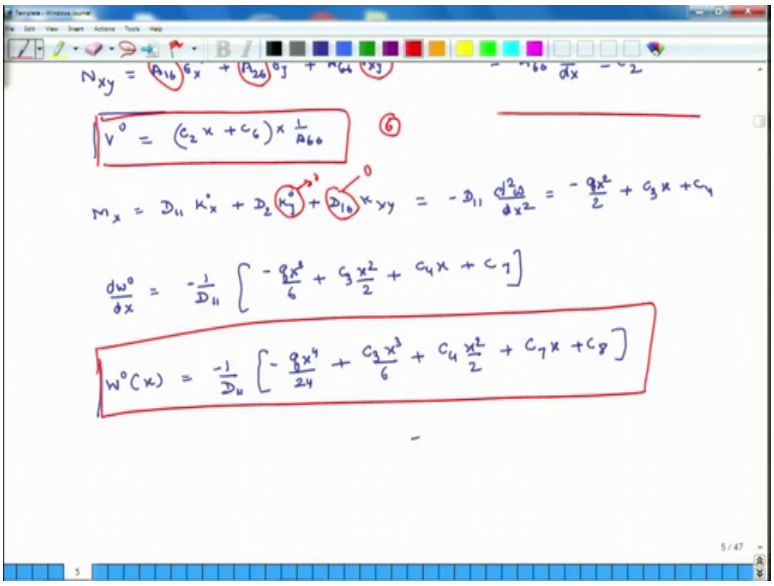

So, M x is equal to D 1 1 K x plus D 1 2 K y. So, these are all mid plane strains mid plane curvatures plus D 1 6 K x y. Because we have the laminate which is having all cross ply layer 0 degrees and 90 degrees D 1 6 is 0 also, because K y is the second derivative of w with respect to y direction. So, this is also 0. So, the only thing which I am left with is D 1 1 times K x. So, this is equal to minus D 1 1 del 2 W naught over del x square and because it is a very long plate. All the second derivatives with respect to x are actually we can consider them as total derivatives. So, I call this as D 2 W over d x square ok.

And now, we have seen from equation 3 that M x is equal to minus q x square plus C 3 x plus C 4. So, I use bring that relation down here. So, this is equal to minus q x square over 2 plus C 3 x plus C 4. So now, what I do is I integrate this equation twice. So, I get d W naught over d x equals minus 1 over D 1 1 and I am integrating this relation for the first time. So, I get minus q x cube over 6 plus C 3 x square over 2 plus C 4 x plus C 7 and if I integrate it one more time, this is what I get minus q x 4 over 24 plus C 3 x cube over 6 plus C 4 x square over 2 plus C 8. So, this is my

Student: (Refer Time: 11:19).

Yes, plus C 7 x plus C 8. So, this is my relation 4 W, ok. So, we will do is we will summarize all these relations.

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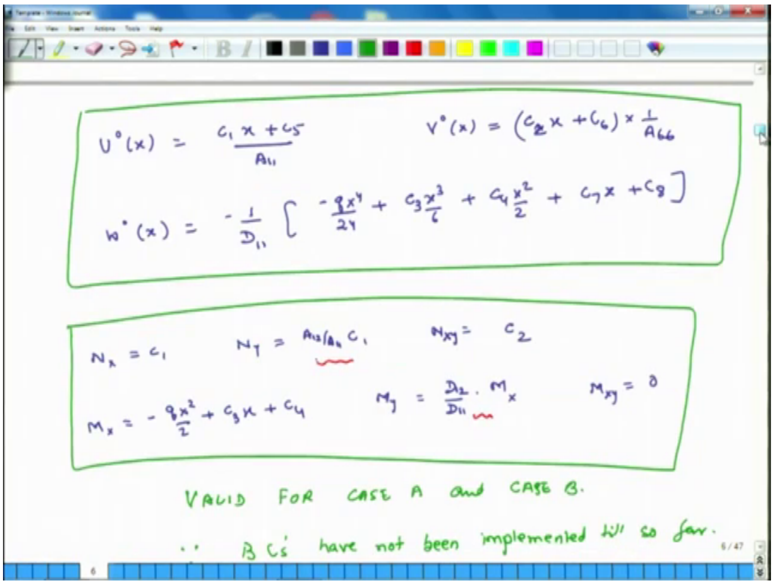

So, U naught x is equal to C 1 x plus C 5 over A 1 1, V naught x is equal to C 2 x plus C 6 into 1 over A 6 6 and W naught x equals minus 1 over D 1 1 minus q x 4 over 24 plus C 3 x cube over 6 plus C 4 x square by 2 plus C 7 x plus C 8. So, that is the relation for W and so, this is one block of equations which tell us about U V and W.

And then, we will also write down expressions for M S and N s. So, N x is equal to C 1 N y equals A 1 2 over A 1 1 C 1 N x y N x y equals C 2. M x is equal to minus q x square over 2 plus C 3 x plus C 4 and my now if you do the math, what is M y? M y is equal to D 1 2 K x plus d 2 2 K y plus D 2 6 K x y and if we do the math similarly in the way which we have explained earlier, what we find is that M y is nothing but D 1 2 over D 1 1 Times M x see this relation is very similar to this relation. So, it the mathematics is very similar. So, anyway; so, that is what is M y and M x y is equal to 0. So, this is what we find. So, this is the second set of relations.

So now, reveal let us review the first 3 relations which we developed here. These are valid for all the 4 cases because here we never applied any things related to the material. We started with these general differential equations and the only simplification we did in these simply equations was that del over del y is 0 and del over del x is D over dx and these simplifications are valid for all the 4 cases.

So, if these equations are valid for all the 4 cases, we can use these 4 3 equations for all the 4 cases, then the next thing we started doing was that we considered the laminate is symmetric. And when laminate is symmetric, then we said the D matrix is 0 and also we said that laminate is not only symmetric, but it has all the layers in 0 and 90 degrees direction. So, that means, A 1 6 A 2 6 D 1 6 d 2 6 are also 0.

So, using these 2 facts that B is 0 and A 1 6 A 2 6 D 1 6 d 2 6 are 0, we again calculated relations for displacements and N x and N x y and we came up with these equations. So, these equations are valid for case A and case B. Why they are valid for case A and case B? Because, we have not used boundary conditions till so far because B C S have not been implemented till so far.

We have not implemented the B C'sand because we have not implemented the B C's. They are valid for case A and case B because both these cases have the same lamination sequence which is 0 90 symmetric and for 0 90 symmetric or laminate which is symmetric and which has z. These lies in 0 and 90-degree direction, these equations are valid. So, these equations are valid for case A and case B.

The last thing I will like to show is that when we came up with these equations, how many constants? We have constants of integration we have ended up with 8 integration constants; C 1 C 2 C 3 till C 8 and we do not know the values of these 8 different integration constants. The way we are going to calculate or find out they are values is from the boundary condition.

Now, let us look at. So, we have 8 integration constants which we do not know, but then you also see that there are 8 different conditions for each case. So, we know 4 conditions at one end of the plate and 4 conditions at the other end of the plate and we will use these 8 different conditions to evaluate the values of these constants. And in that way, we will be able to solve for the actual solution for U V W and also for N x N y N x y M x my M x y.

So, that is how we are going to solve this thing. So, we have 8 integration constants which we will find out by implementing 8 boundary conditions. The value of these integration constants may be different for different set of boundary conditions. So, anyway, so, that is what we plan to do next and that is what we will start doing tomorrow. So, I look forward to seeing you tomorrow. Until then, have a great day.

Thank you.