Advanced Composites
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Lecture - 33
Particular Solution for Semi-Infinite Plate
(Case A)

Welcome to Advanced Composites. Today is the 3rd day of the 6th week of this course. Yesterday we solved a part of the semi-infinite problem in the sense that what we have done till so far is that we have solved and integrated the partial differential equations. The solutions which emanate by integrating these partial differential equations are valid for all the 4 cases; cases A B C and D and then, we implemented the fact that the laminate is symmetric as well as cross ply and that led to the development of the solution as shown in this slide.

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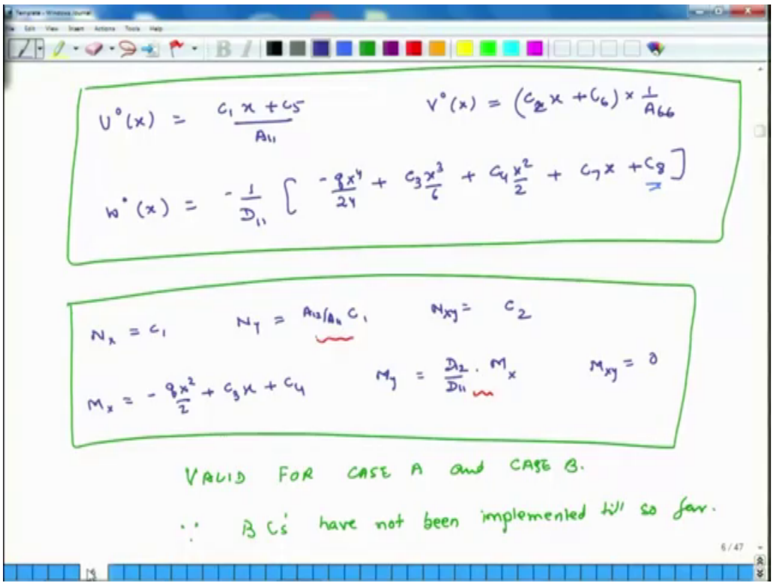

So, we have now relations for U V and W and also relations for N x N y N xy M x M y and M xy and in these equations, there are 8 integration constants C 1 to C 8 and now, it is up to us to figure out the values of these integration constants by implementing the boundary conditions for cases A and B separately. So, now we will start implementing the boundary conditions.

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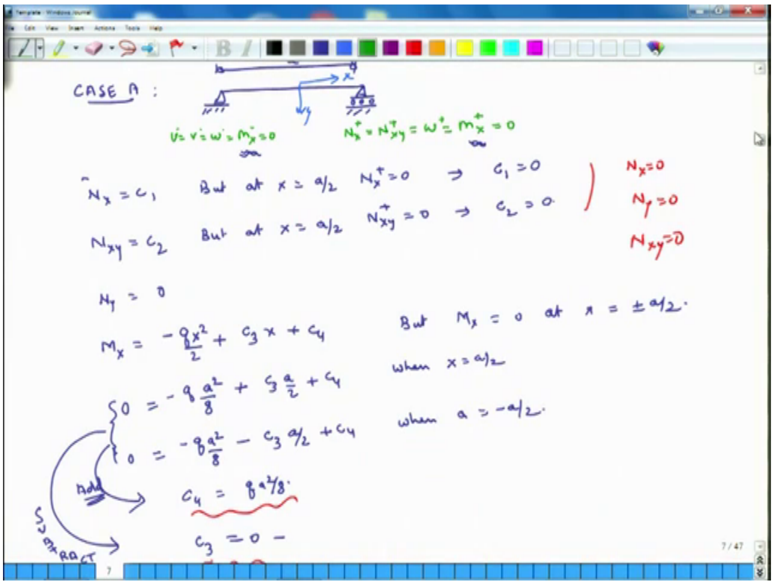

So, case A, so what is case A let us look at; so, it is a pinned, fixed pinned and roller pinned situation. So, this is my x, this is the y direction and the boundary conditions are U equals V equals W equals M x equals 0 as the negative end and at this end, we have N x is equal to N xy is equal to W equals M x at the positive end. This is equal to 0.

So, first we know that N x equals C 1, but at x is equal to a over 2. So, this is the length of the plate is a. So, at x is equal to A over 2 N x plus equals 0 which means C 1 equals 0, then we have N xy equals C 2 and we also know that, but at x equals A over 2 N xy plus equals 0. So, this gives us C 2 is equal to 0 and also we see that N y is equal to A 11 A 12 divided by A 11 into C 1 and C 1 is 0. So, because of that N is equal to 0.

Finally, we have M x is equal to 0. Now, M x, the relation is minus qx square over 2 plus 3 x plus C 4 C 3 x plus C 4. Now, we know that so, but M x is equal to 0 at x is equal to plus minus a by 2 is 0 at both ends, right. This is the boundary condition M x is 0 at both ends, at this end and at this end M x is 0. So, we put that thing.

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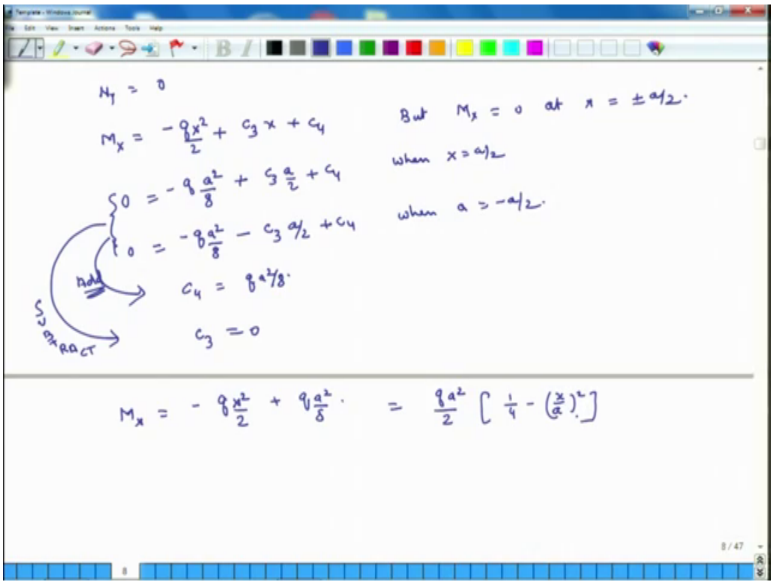

So, we first put for the case at plus a by 2. So, it is equal to q a square over 8 minus plus C 3 a over 2 plus C 4. So, this is the condition at when x is equal to a over 2, and when x is equal to negative a over 2, then I still get q a square over 8 minus C 3 a over 2 plus C 4. So, this is when a is equal to minus a over 2. So, if I add these up, if I add these two equations, what I get is that C 4 is equal to q a square by 8.

If I add these up and if I subtract these equations from other, when I subtract these equations from each other, I get C 3 equals 0. So, that way I have solved for C 3 and C 4. So, essentially what I get is M x is equal to minus qx square by 2 plus q a square by 8 or I can also explicit it somewhat another form q a square over 2. I take it out q a square over 2, I take out and I get 1 over 4 minus x over a square, ok.

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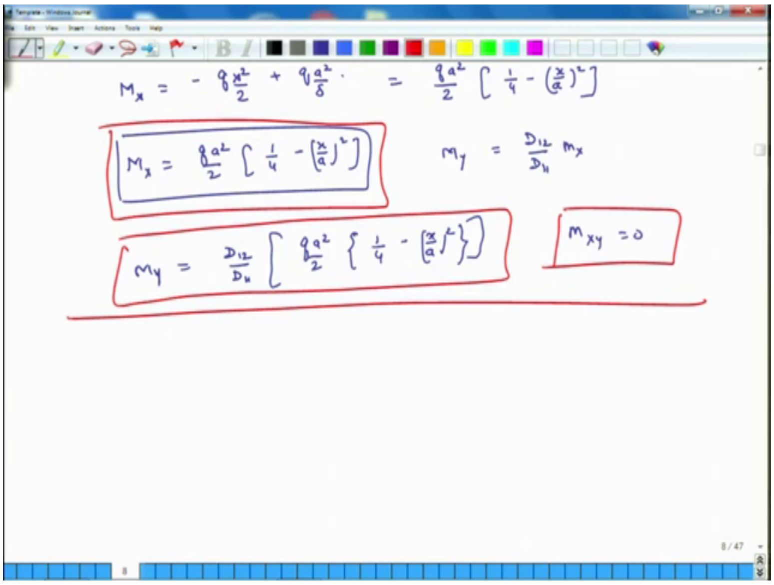

So, M x is equal to q a square over 2, 1 over 4 minus x over a whole square. So, that is M x and M y is equal to d 1 to over d 11 times M x. So, I can write it as my equals d 12 over d 11 q a square over 21 by 4 minus x over a square. So, this is my another addition, and then, of course M xy. So, this is my M xy is 0. So, what we have done till so far figured out N x is 0, N y is 0, N xy is 0. I mean this gives us N x is equal to 0, N y equals 0 and N xy equals 0 and M x is not 0.

It is q a square over 2 times one-fourth minus x over a whole square and M y is nothing, but M x times d 12 over d 11 and M xy is 0. The other thing we have also found is, so let us look at it. So, we have now figured out the values of N x and N y N xy M y M x and M xy, but we have not yet calculated the values of U V and W, U naught V naught mid-plane displacement.

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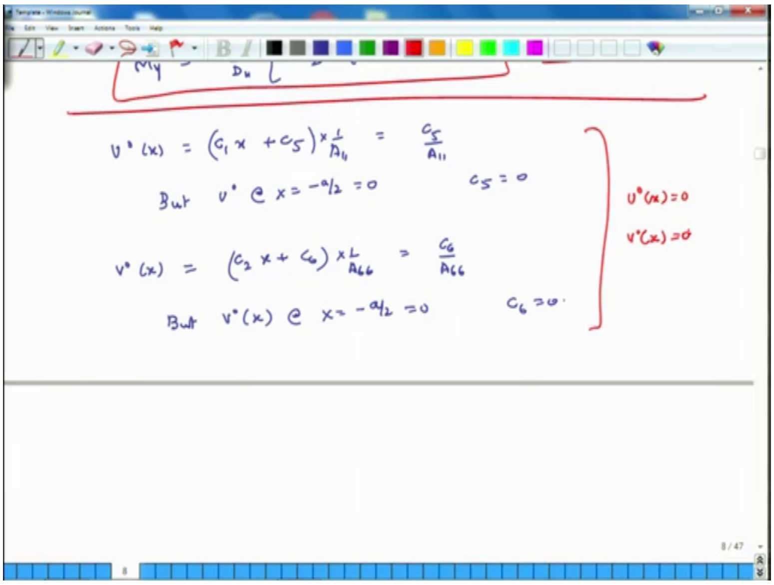

So, let us look at this U naught x is equal to C 1 x plus C 5 times 1 over A 11, but we have already found that C 1 is 0. So, is equal to, it is C 5 over A 11 and now, we implement the other boundary condition, but which is U naught at x is equal to minus a by 2 is 0. So, C 5 is 0 V naught x. What is V naught x? It is equal to C 2 x plus C 6 into 1 over A 66 into 1 over A 66 and we have already seen that C 2 is 0. So, this is equal to C 6 over A 66. It is equal to C 6 over A 66 and we have seen, but that V naught x at x is equal to minus a by 2 is equal to 0. So, C 6 is also 0, C 6 is 0. So, what these things give us is U naught x equals 0 and V naught x equals 0 and the last thing is about W naught.

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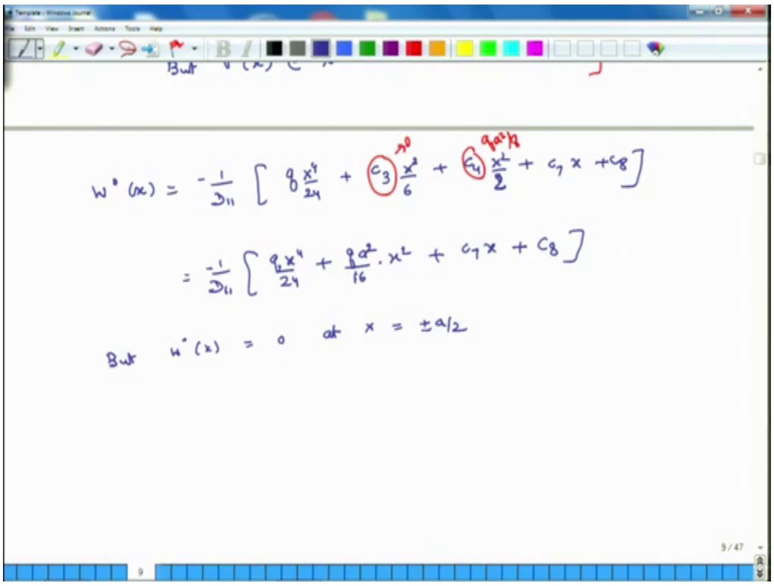

So, W naught x and the relation for W naught x, we have already developed is 1 over D

11 qx 4 by 24 plus C 3 x q by 6 plus C 4 x square over 2 plus C 7 x plus C 8. Now, we have already said C 3 is equal to 0 from here and C 4 is q a square over 8, right. So, we first use those conditions. So, this is 0 and C 4 is q a square over 8. So, we rewrite it after making these substitutions. So, we get qx 4 over 24 plus q a square over 16 times x square plus C 7 x plus C 8, ok.

Now, we know that that W naught x is equal to 0 at x is equal to plus minus a over 2 from the boundary conditions. These are the only two boundary conditions which have not been implemented.

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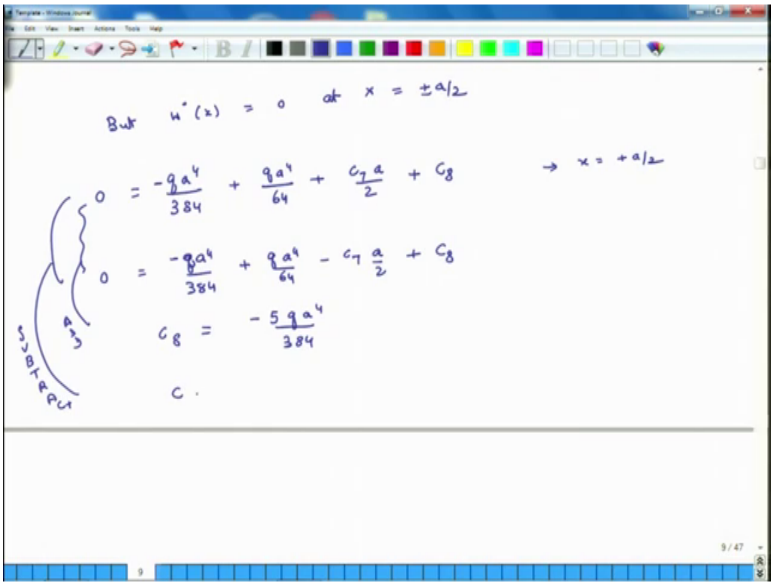

So, if that is the case, so let us write down. So, if we implement the first boundary condition at x is equal to plus a by 2, we get this is equal to and I can drop D 11 because the left side is 0. So, I get q a 4 over 24 times 16. So, that is 384. Instead of x, I am putting a by 2. So, I get q a 4 over 384 plus q a 4 over 16 times 4, 64 plus C 7 a over 2 plus C 8. So, this is the condition when x is equal to plus a over 2.

Student: As per it minus sign 24.

Yeah you are right. Here it should be here also yes and the other boundary condition is that W is 0 at x is equal to negative a over 2. So, this is equal to minus q a 4 over 384 plus q a 4 over 64 minus C 7 a over 2 plus C 8, ok. So, if you add these up if you add these up, you get C 8 equals minus 5 q a 4 divided by 384 when you add these up, and if you subtract these you get C 7 equals 0.

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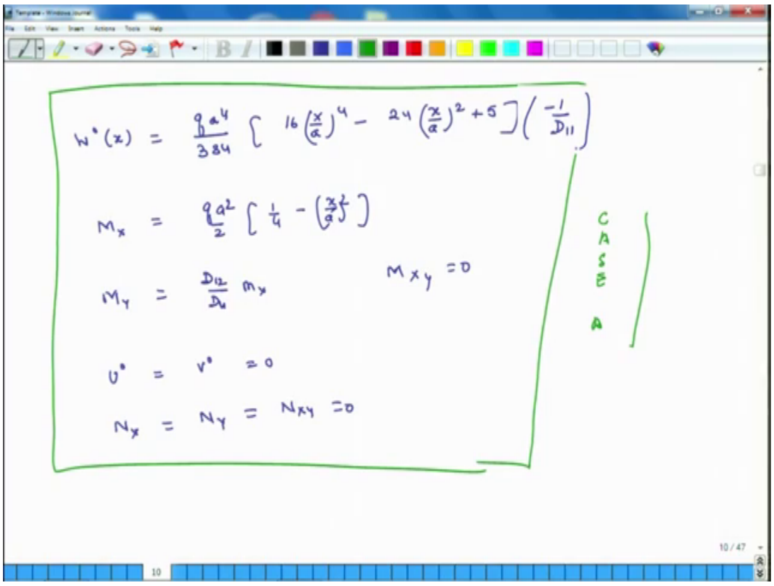

We get the relation for W as this multiplied by 1 over D 11 and the relation for M x and M y is D 12 over D 11 M x and M xy is 0 and finally, we will also write down that U naught is equal to V naught is equal to 0 and N x is equal to N x no N y is equal to N xy equals 0. So, this is the overall solution for which case? So, case A is the solution for case A. So, what we have done in this till so far? How did we arrive at the solution for case A?

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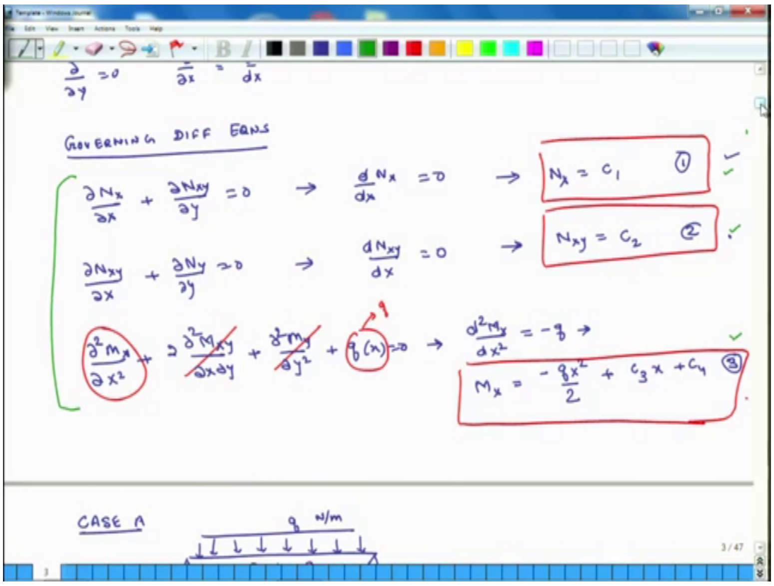

First we integrated the partial differential equations and we got expressions for N x N xy N xy M x and all these things.

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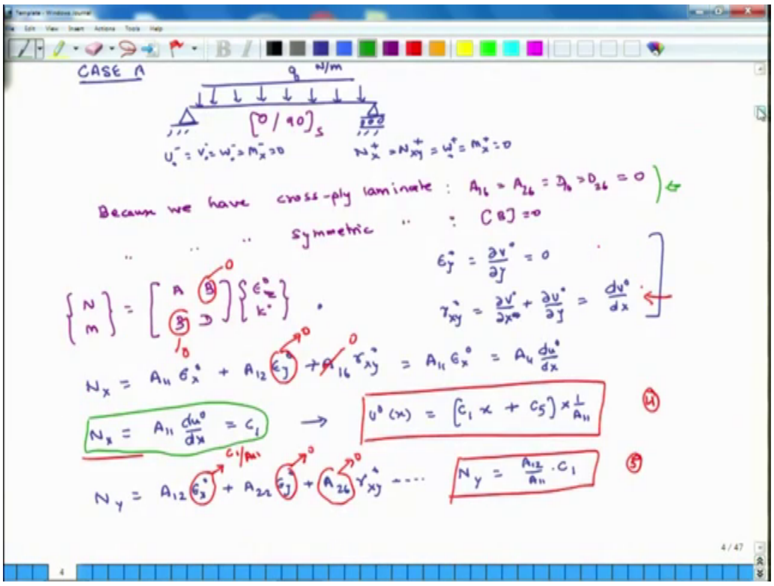

Next we implemented the condition is specific to the symmetric laminate and cross ply laminate and that give us some relations for U V and W and also it also yeah. So, we give us some relation is for U V and W and then, we started implementing the boundary conditions and the boundary condition for case A is that U V W and M x is 0 at x is equal to minus a over 2 and N x N xy W and M x, they are 0 at x is equal to plus a over 2.

So, once we implemented those 4 boundary conditions, these 8 boundary conditions, we were able to solve for 8 integration constants and once we add that, we ended up with this solution that U is equal to 0, V is equal to 0, N x N y N xy is equal to 0, w naught is given by this expression and M x is given by another non-zero expression and N y is nothing, but D 12 over D 11 times M x and M xy is 0.

So, this is the way to solve these differential equations and we will again do two more of this exercise, so that we become comfortable and familiar with it. So, tomorrow we will develop the solution for case B because we already have the solution up to this point and now, we will implement the boundary conditions for case B and once again we will see what kind of solution emerges. So, that is all I wanted to discuss for today and then, tomorrow we will continue this discussion for case B. Till then have a great day. Bye.