Advanced Composites
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Lecture - 34
Particular Solution for Semi-Infinite Plate
(Case B)

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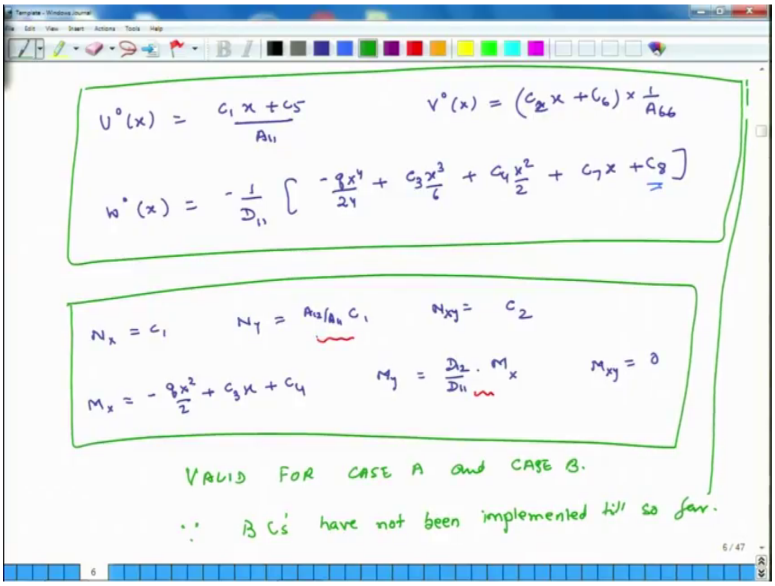

Hello. Welcome to Advanced Composites. Today is the 4th day of the ongoing week, which is the 6th week of this course. And what we will do is we will develop a solution for the second situation for a semi infinite plate that is for case B. Once again, in this case, the lamination sequence is a still symmetric; so, these equations, which we have developed. They are still valid for U, V, W, N x, N y, N x y, M x, M y. and M x y, they are still valid.

But, what will happen is that maybe the values of U 1 U 2 no these integration constants, they may change they may or may not change, but it all depends on the nature of boundary conditions. So, what we will do is, we will once again implement the boundary conditions and look at the solution of the system, and see what it tells us. So, these are the, this is the starting point, and let us look at the solution.

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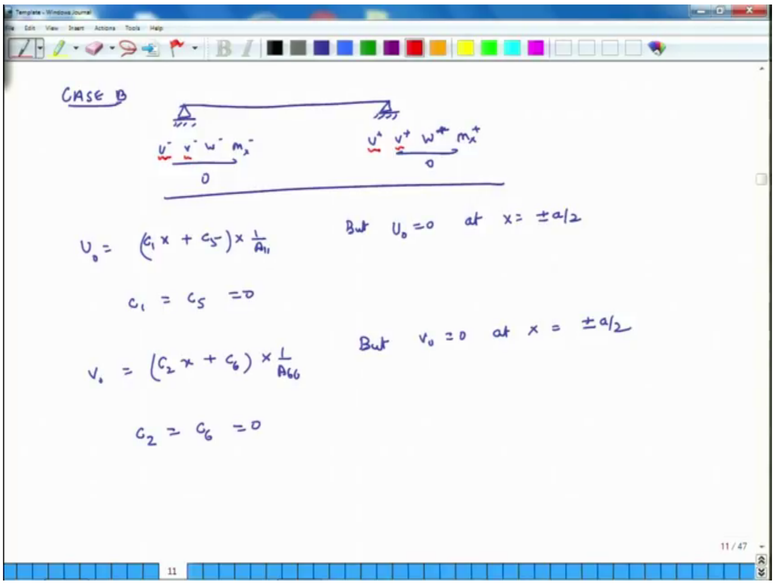

So, once again we are going to do this for case B, and the boundary conditions are such that U minus V minus W minus M x minus and U plus V plus W plus and M x plus, they are all 0. And the lamination sequence again we said that it is not changing, it is still 0 90 symmetric. So, the equations shown in this slide, they are still valid. So, now let us look at so, we will start now implementing the boundary conditions. So, we will start right away from U; so, U naught is equal to C 1 x plus C 5 into 1 over A 1 1.

And we know that that U is equal to 0 at x is equal to plus minus a by 2 small error of notation this should be U subscript, so this is equal to 0. So, if I implement these boundary conditions, I get that C 1 is equal to C 5 is equal to 0. If I apply x is equal to a by 2 U is 0, I get one equation, and then another equation is C 5 minus C 1 a over 2 is 0. So, from this, I get this thing as 0.

The second equation is V x is equal to C 2 x plus C 6 divided by A 6 6. So, V naught is equal to C 2 x plus C 6 into 1 over A 6 6. And then we know that V naught is equal to 0 at x is equal to plus minus a by 2. So, from this, we get C 2 is equal to C 6 is equal to 0, we get this once again. The third condition is so, what have we done till so far, we have implemented four boundary conditions; we have implemented these BC, all the four BCs and U and V. Now, we are still left with the boundary conditions related to w and m x. So, let us look at the other condition. So, first we will try to implement the condition for m x ok. So, m x is equal to minus q x square over 2 plus C 3 x plus C 4.

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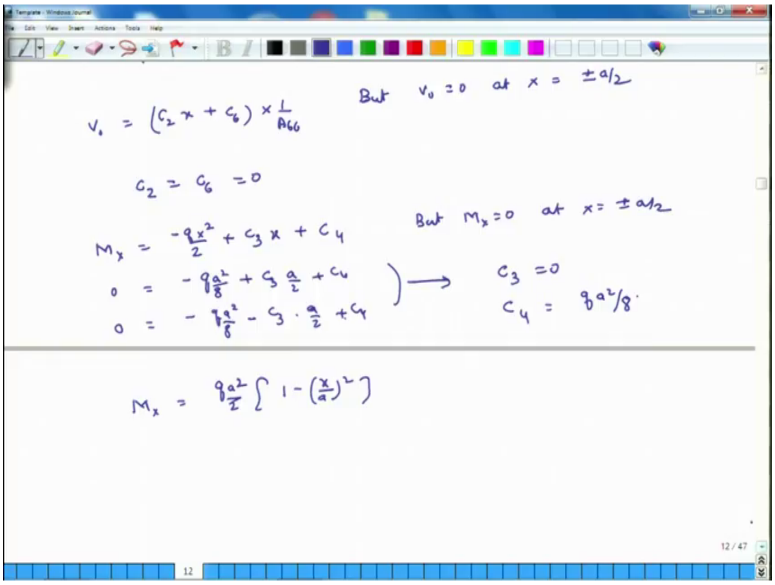

So, we will implement it M x is equal to minus q x square over 2 plus C 3 x plus C 4. And we know that M x equals 0 at x is equal to plus minus a over 2. So, we get two equations 0 equals minus q a square over 8 plus C 3 a over 2 plus C 4, and the other one is 0 equals minus q a square over 8 minus C 3 times a over 2 plus C 4. So, from these, we once again get C 3 equals 0, and C 4 equals q a square over 8. So, I end up with M x as q a square over 2 1 minus x over a whole square. So, now we have implemented six boundary conditions. So, we have also implemented the BCs on M x, and the two boundary conditions, which are left are related to W out of plane displacement. The relation for W is this long thing.

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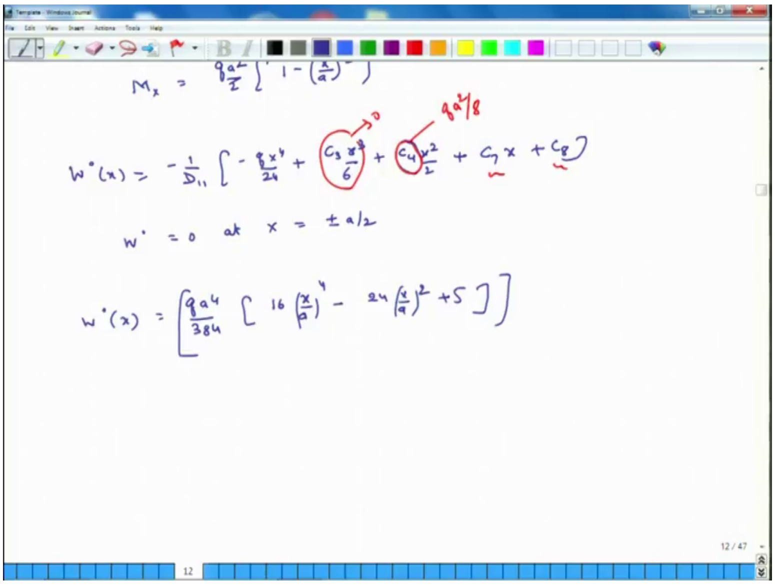

And if I write this, what I find is so, let us write it down W naught, so, this is equal to minus 1 over D 1 1 minus q x 4 over 24 plus C 3 x cube over 6 plus C 4 x square over 2 plus C 7 x plus C 8. And we have also already said that C 3 is 0 here from here. So, so this term, it goes to 0. And now, we implement the boundary conditions that W naught equals 0 at x is equal to plus minus a by 2 ok. And we also know that C 4 is q a square over 8. So, this thing is q a square over 8 ok.

So, now we have two conditions and two unknowns, C 7 and C 8, and we can calculate that. And I and if we do all the math correct, this is what we get. So, I will just write down the final result. So, W naught x equals q a 4 over 384 16 x over a 4 minus 24 x over a square plus 5; so, this is the solution, we get from this. So, we will come.

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Yes. So, there should be no there is no D 1 1, this is what it is. So, let us compile all the results in this case.

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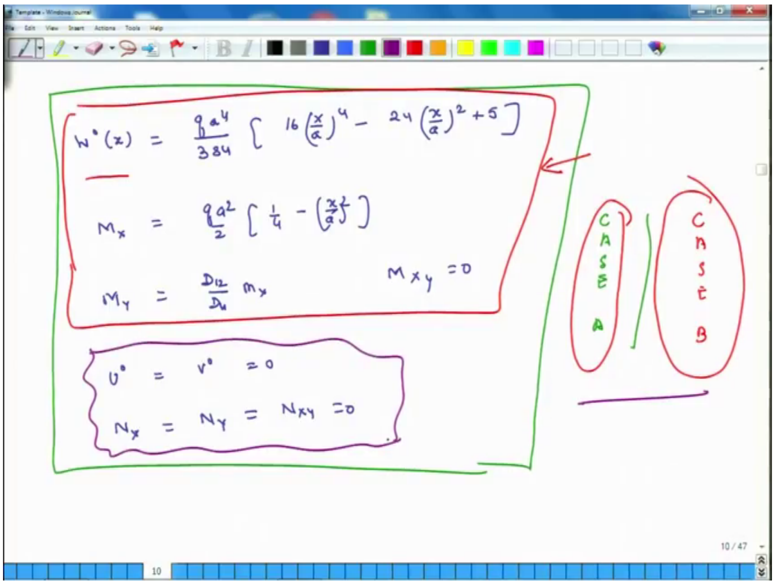

And if you compile all the results, you will find that the solution is identical as this one. So, this was the solution for case A; and this is also the solution, when we solve and implement all the boundary conditions for case B. So, what we find in this case is that the solution for case A and case B is identical. And now, you will be wondering that why is it the case, because the boundary conditions in both the cases are different.

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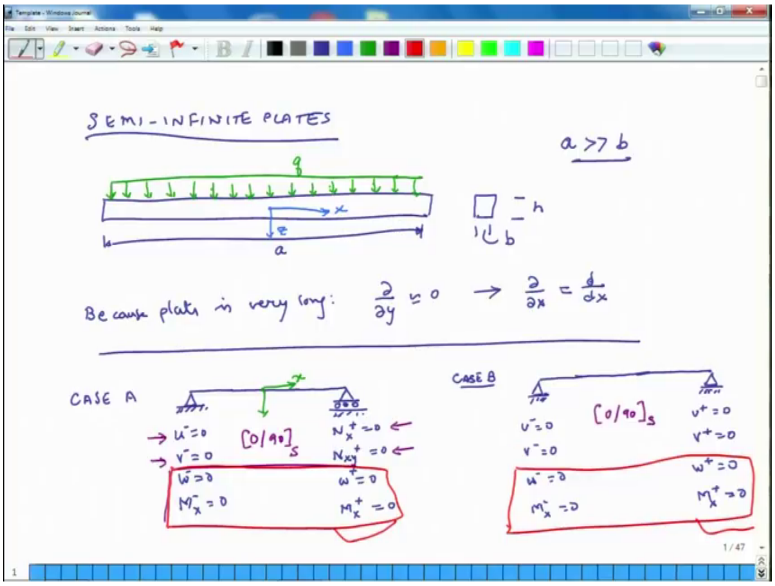

The boundary conditions if you remember in case A, are different than those in case B is specifically at x is equal to plus a over 2 end ok. So, you may be wondering that what is the problem, what is the reason why, despite the fact that these two plates have different boundary conditions, the solution comes out as same that is the question. So, what we will do is that we will spend next few minutes trying to explain, why is it that the solution is different. Let us go back to our original differential equations.

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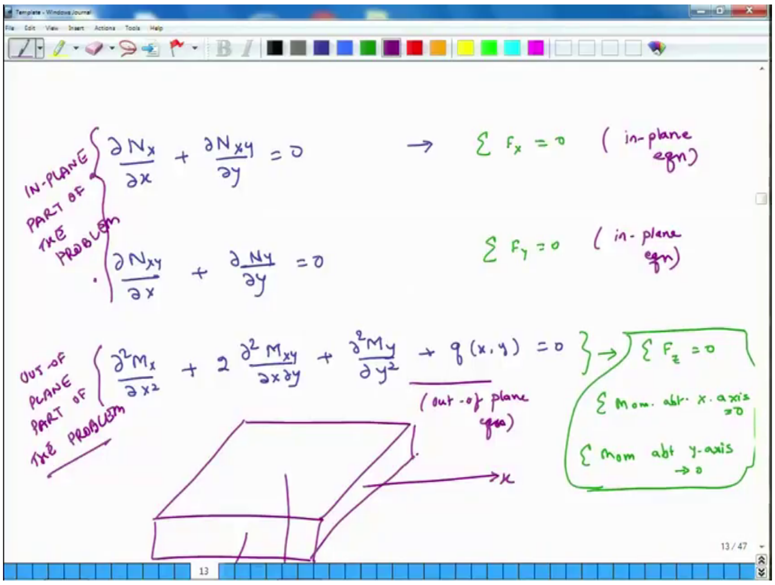

So, what are the governing differential equations, del N x over del x plus del N x y over del y equals 0, del N x y over del x plus del N y over del y equals 0, and del 2 M x over del x square plus 2 of del 2 M x y over del x del y plus 2nd derivative of M y with respect to y plus q x y is equal to 0. These are the three differential equations. And the way we had developed these three differential equations was that this first equation was related to the force equilibrium condition that this is sum of all the forces in the x direction is 0 right.

The second differential equation came out, when we equated all the forces in the y direction to be 0, if you remember during the derivation. And this differential equation is essentially a combination of three differential equations, which we have combined together. What were those three differential equations related to sum of all the forces in z direction is equal to 0. The second one was sum of moments about x-axis that equals 0.

And the third equation was sum of moments about y-axis, there sum was 0 ok. So, this third differential equation we had shown is a combination of three other differential equations. They have they were all first order differential equations, we combined them, and we came up with the third differential equation. Now, when you look at this, so this is F x is equal to 0, F y is equal to 0; so, if I look at my plate, this is my x-axis, this is the y-axis and this is the z-axis. So, when I add up all the forces in the x direction to be 0, then I get this first differential equation. What does it mean that the first differential equation relates to forces which are in the plane of the plate, because they are acting in the x direction. So, these forces are in plane of the plate [FL]. So, this is an in-plane equation.

Second one is sum of forces in the y direction that equals 0. And the consequential differential equation is the second one, which is del N x y with respect to x plus del N y with respect to y 0. So, here all the forces are in the y direction, again they are in the plane of the plate. So, again this is again in-plane equilibrium in plane equation. But, all the remaining three equations, which are sum of F z is equal to 0. And movements, see movements can never be, see these movements were about x-axis. So, if there is something about x-axis, the moment will be out of the plane. And similarly, movements about y-axis, will also be out of the plane; and F z is also out of the plane.

So, this third equation this third equation is out of plane equation this is the out of plane. Actually, I should not call it so, so the first equation and the second equation, they tell us about in-plane equilibrium. And the third equation tells us about out of plane equilibrium. So, these two, they tell us about in-plane part of the problem. And the third equation tells us about out of plane problem. So, if you solve, so that tells, so that is about out of the plane problem the third equation; and the first two equations are about in plane part of the problem. So, this is one thing to understand first two equations in part in plane part of the problem; the third equation is out of plane or part of the problem.

Next, we know now in cases A and B, we said that the solution for case A and B is the same I mean we have found it is not that we are just saying it we have calculated and it comes out to be the same. And what are cases A and B in both the cases A and B the lamination sequence is symmetric lamination sequence is symmetric which means that B matrix for case A, and B matrix for case B 0 ok. Now, in plane part of the problem involves N x and N xy, an N xy and N y in plane part of the problem means these first two differential equations which involve only N x, N x y, and N y; out of the plane problem involves M x, M x y and M y and q ok.

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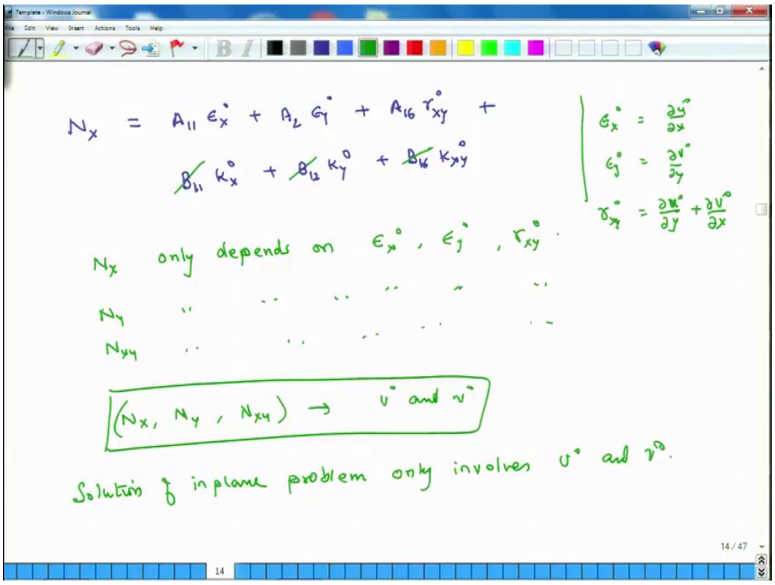

Now, let us look at N x y N x. From our A B D equations what is N x, N x is equal to A 1 1 epsilon x naught plus A 1 2 epsilon y naught plus A 1 6 gamma x y naught plus B 1 1 k x naught plus B 1 2 k y naught plus B 1 6 k x y naught. And because the matrix is symmetric these terms go away the B terms go away. So, N x so what; that means, is that N x only depends on epsilon x naught, epsilon y naught, and gamma xy naught, because B matrix is 0. Similarly, N x, N y and N xy, we can say the same thing about all the answers ok.

Now, we know that epsilon x naught is del U over del x, epsilon y naught is del v naught over del y, and gamma x y naught is del U naught over del y plus del v naught over del x. So, we can say that N x, N y and N xy, they only depend on u and v, mid plane u and mid plane v ok. This is because so which means that if we look at the in plane problem relates to the first two equations these involve only N x, N N y, N x y and M xy. The solutions for N x, N y, N x y they only depend on the nature of u and nature of v ok, when v matrix is 0 ok.

So, I can make a further statement that solution of in plane problem only involves u naught and v naught, it does not involve w naught. If B was not 0, then it would have also involved w naught; but in this case B 0, so it involves only u naught and v naught. Similarly, we look at the third equation the third equation is involves only M x, M x y, M y and q ok.

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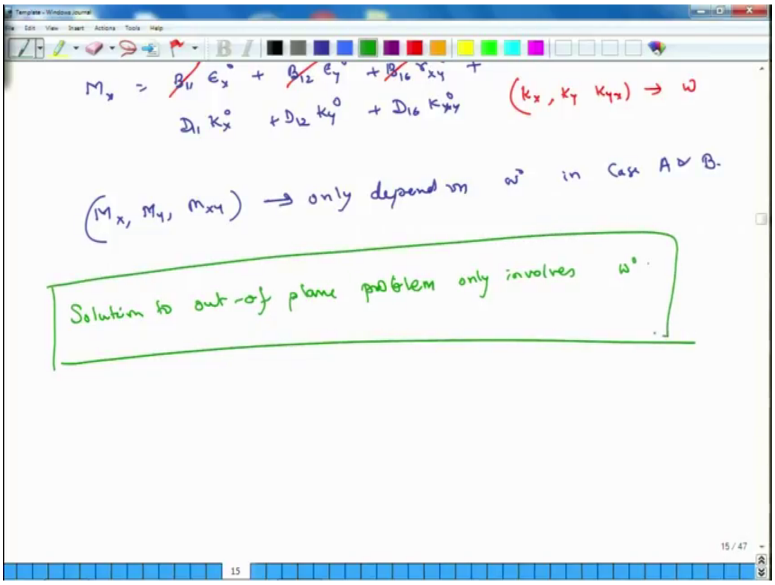

And so we will see for instance M x is equal to what B 1 1 epsilon x naught plus B 1 2 epsilon y naught plus B 1 6 gamma x y naught plus D 1 1 K x naught plus D 1 2 K y naught plus D 1 6 K xy naught. Now, again because the lamination is symmetric, these things go away; and K x we know they only depend on W, they only depend on W. So, we know that M x and for similar logic M y and M xy, they only depend on W, when the when. So, in case A and B, in case A and B, they only depend on W.

And if you look at the third equation which is out of plane equation, it only involves M x, M y, M x y and q. So, we can also write that the solution to out of plane problem only involves it only involves w naught ok. So, this is one statement solution to out of plane problem involves only w naught. And the other statement we made this solution to in plane problem only involves U naught and V naught. So, the solution for U naught and V naught W does not influence W, and the solution for W does not influence U and V this is what me it means.

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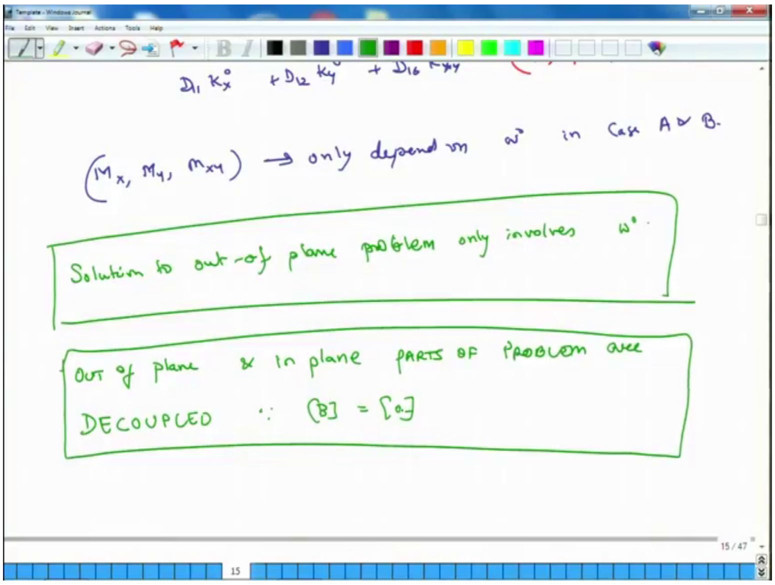

So, in this case because the lamination sequence is symmetric the out of plane and in plane parts of problem are decoupled; out of plane part of the problem and in plane part of the problem are decoupled. The solution 4 W does not influence the solution for U and V, and solution for U and V does not influence the solution for W. This is happening because the lamination sequences because this B matrix is 0 ok, B matrix is 0, so that is one case. So, solution of U and V does not influence W and vice versa.

Now, our original question is why is the solution same for case A and case B. So, to see that let us look at this picture again. What this means is that. So, the solution for case A, and solution for case B will have an in plane solution the in plane solution will involve U and V. And it will have an out of plane solution and that will involve w. And the in plane solution and out of plane solution are decoupled because in both the cases the say lamination sequence is symmetric.

Now, let us look at these boundary conditions and see what are the in plane boundary conditions. So, u is in plane boundary condition, v is in plane boundary condition, N x is in plane boundary condition and N y is N xy is in plane boundary condition. What are out of plane boundary conditions, these two are out of plane boundary conditions. So, in case B and case A, out of plane boundary condition, I have put it in red box ok.

Now, when you look at the out of the plane problem for case A and case B, you see that out of the plane boundary conditions are identical W is 0 at both ends and M x is M x is also 0 at both the ends. So, out of plane boundary condition is identical, also the lamination sequence is identical the length is identical. So, if the geometry, so the out of plane problem the boundary conditions are same, and also the lamination sequence is same, length of the plate is same and because so the solution will also be the same. Because this solution will not get influenced by in plane boundary conditions, it will not get influenced by in plane boundary conditions because the situation is decoupled ok.

So, for that reason you see that the expression for w is same expression for. So, this is the out of plane boundary out of plane solution and it is same out of plane solution why is it same because the out of plane boundary conditions are same, geometry is same, and lamination sequence is same ok. So, the solution will also be same ok. The solution will also be the same that is the thing.

The other thing is that we also see that in plane solution is also same. So, this is the in plane solution, now in plane solution is same because effectively in the in plane I am not putting any external forces. If I am not putting any external forces, then everything will be 0. So, for this reason the out of the we see that for case A and B, out of plane as well as in plane solutions are same even though the boundary conditions are different. This will not happen when we start dealing with non symmetric lamination sequences. And we will actually see this. So, this is what I wanted to discuss in today's lecture. And we will meet once again tomorrow and continue this discussion further.

Thank you.