Advanced Composites
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Lecture - 35
Particular Solution for Semi-Infinite Plate
(Case C)

Hello welcome to Advanced Composites. Today is the 5th day of the on going week which is the 6th week of this course. And, what we will do today is we will start our discussion in the context of case C and case D of the Semi-Infinite Plate Problem. Now, in both these cases unlike cases A and B, the lamination sequence is not symmetric and we will see that how this influences the overall solution.

One thing based on the discussion which we had just yesterday, one thing we will very clearly see is that the in plane part of the problem and the out of plane part of the problem their solutions will be coupled. So, that is one thing we will see and then we will also see some other things which emerge out when we actually solve this problem.

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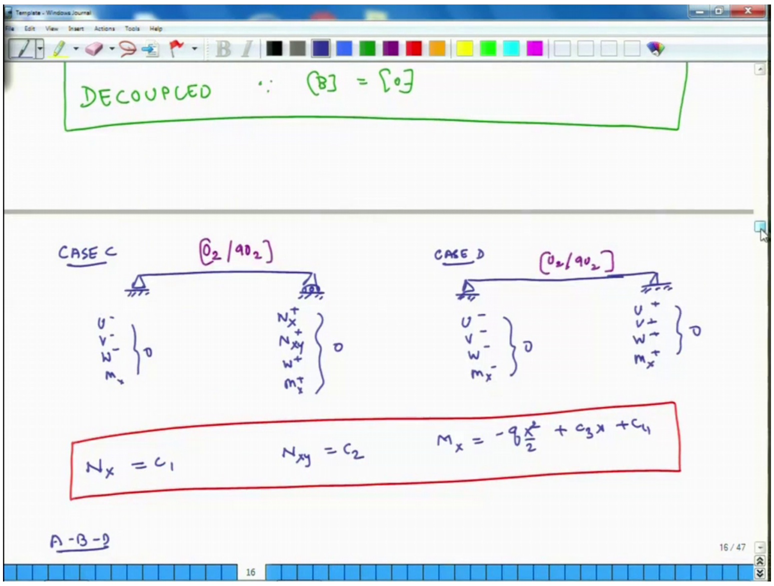

So, let us recap case C; case C is one end is pinned and fixed and the other end is pinned, but on rollers. So, here U V W and M x they are all 0 and on the other end you have N x N xy W plus and M x plus, they are all 0 and on case D both the ends are pinned as well as fixed. So, U V W M x they are 0 at both the ends and the lamination sequence is 0 2

90 2 symmetric in both the cases. So,

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I am sorry this is not symmetric it is 0 2 90 2 so, 0 2 and 90 2. So, number of layers is still the same 4 layers 2 0 layers and 2 90 layers like the earlier 2 cases A and B, but the lamination sequence is not symmetric. So, first we will very quickly solve the differential equation.

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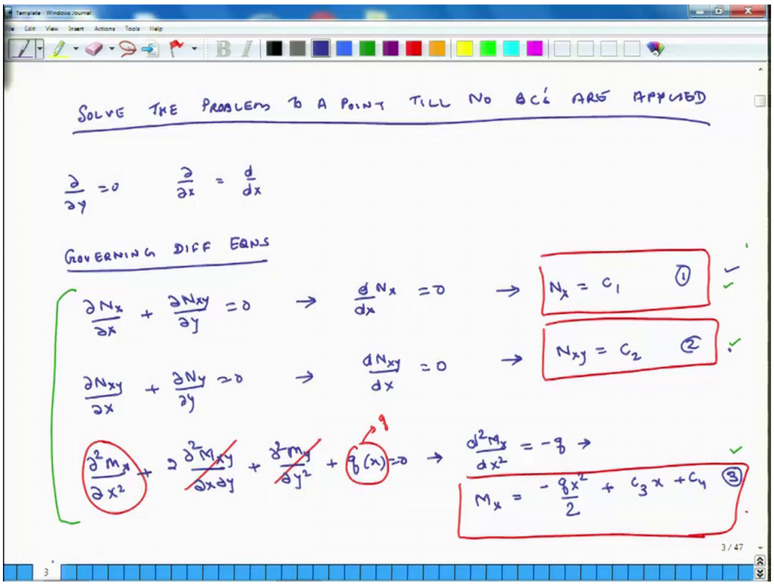

The solutions for differential equation as we discussed earlier, they are going to be the same. Because, when we are doing these solutions, we do not consider the effect of lamination sequence and neither did we consider the effect of boundary condition. So, if that is the case then these 3 equations still hold. So, we will directly write these equations later. So, our 3 equations which we get from the differential equation is N x is equal to C 1 N xy is equal to C 2 and M x is equal to minus q x square by 2 plus C 3 x plus C 4.

So, these equations are still valid. Next we have to be now more careful when we are doing this problem. So, let us look at A B D relations, A B D matrices. Now, in this system all the layers are either 0 degrees or 90 degrees. So, it is a cross ply laminate ok, it is a cross ply laminate.

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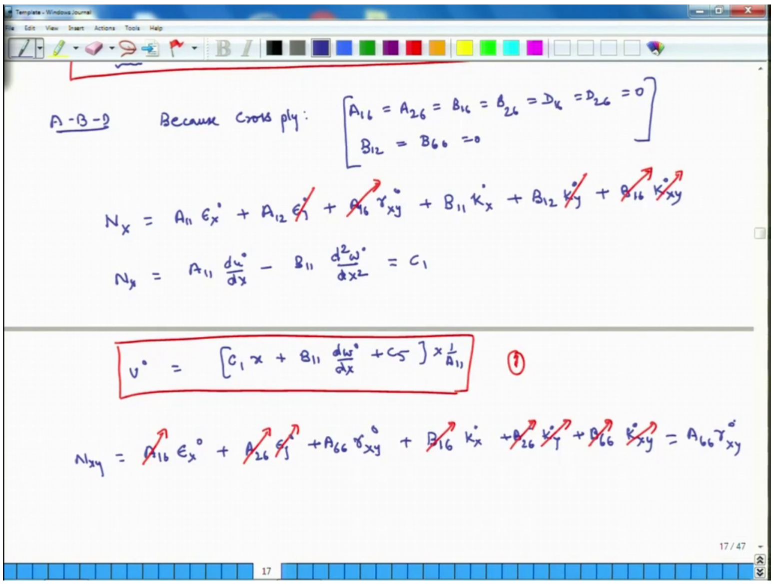

So, when if it is a cross ply laminate because, it is a cross ply laminate we can say that it is A 1 6 is equal to A 2 6 is equal to B 1 6 is equal to B 2 6 is equal to D 1 6 is equal to D 2 6 is equal to 0 ok. And, also because this is 2 layers of 0 and 2 layers of 90 we can also show that B 1 2 is equal to B 6 6, this is also equal to 0 ok. So, if that is the case then what do we; so now, we start developing relations for N x N y N xy. So, what is it N x is equal to A 1 1 epsilon x dot plus A 1 2 epsilon y dot plus A 1 6 gamma xy at the mid plane. A 1 6 is 0 oh I am sorry, but they will also B terms plus B 1 1 K x plus B 1 2 K y plus B 1 6 K xy. So, now, things start becoming messy.

So, now let us see which terms can be dropped off, A 1 6 is, 0 B 1 6 is also 0 epsilon y is 0 because del over del y is 0. So, this is 0 K y is 0 because, again it is related to partial second derivative of w in the y direction and K xy is also 0 because, K xy is del 2 w over del x del y. So, what this gives us is equal to A 1 1 d naught over d x minus B 1 1 d 2 w naught over dx square N x. So, now you see the coupling very explicitly N x is a function of u and w. In the symmetric laminate this B term would have been 0. So, N x would only depend on u, but here N x depends on u as well as on w. So, it is coupled problem and we know that N x is equal to C 1 from this red box N x is equal to C 1.

So, we integrate this equation once and we get U naught is equal to C 1 x plus B 1 1 dw naught over d x plus C 5 into 1 over A 1 1 ok. So, we will call this equation 1. Now, let us look at the relation for N xy. So, N xy is equal to A 1 6 epsilon x naught plus A 2 6 epsilon y naught plus A 6 6 gamma xy naught plus B 1 6 K x plus B 2 6 K y plus B 6 6 K xy and let us see which terms get dropout in this equation. So, A 1 6 is 0 A 2 6 is 0 because, it is a cross ply laminate; B 1 6 is 0 B 2 6 is 0 and B 6 6 is also 0. We have mentioned this and actually also K xy is 0 epsilon y is 0 so, all these things are 0. So, essentially this boils down to A 6 6 K excuse me gamma xy everything else goes away.

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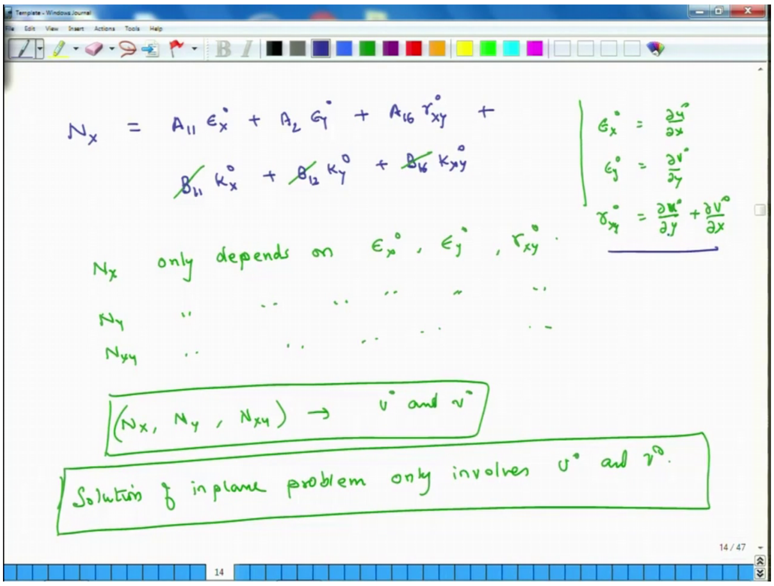

And gamma xy if you remember somewhere back we had said gamma xy is del u over del y plus del v over del x, but del u over del y is 0. So, it is del v over del x only.

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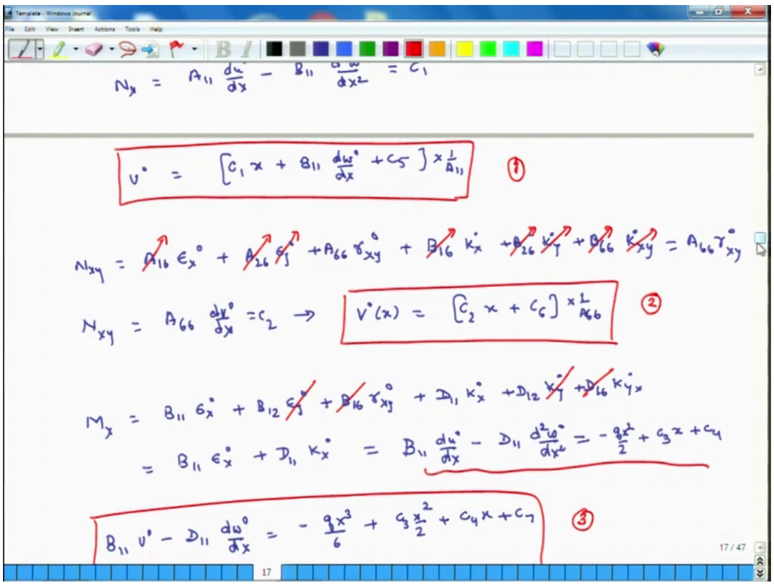

So, we can say that N xy is equal to A 6 6 dv naught over dx. So, this gives us V 2 I am sorry V nought which is the function of x is equal to and what is N xy; N xy is C 2 we have already said that earlier right from this equation N xy C 2. So, this is equal to C 2 x plus C 6 into 1 over A 6 6. So, this is the expression for V, this is the expression for V. Next look at M x; so, M x equals B 1 1 epsilon x naught plus B 1 2 epsilon y naught plus B 1 6 gamma xy naught plus D 1 1 K x plus D 1 2 K y plus D 1 6 K xy ok.

Again we see which terms can be dropped out B 1 6 is 0 D 1 6 is 0 and K y is 0. So, what I am left with is B 1 1 epsilon x naught plus oh actually epsilon y is also 0. So, B 1 1 epsilon x plus D 1 2 K x that is all I am left and this is equal to I am sorry this is should be D 1 1. So, this is equal to B 1 1 d u naught over dx minus D 1 1 d 2 w over dx square ok. Now, this is equal to M x, but we also have another expression for M x which is this one. M x is equal to minus q x square plus C 3 x plus C 4 which is valid for all the 4 cases so, I putting in there; so, minus q x square over 2 plus C 3 x plus C 4.

So, if I integrate this whole thing ones and I rearrange my things what I get is B 1 1 U naught minus D 1 1 dw naught over d x is equal to minus q x 3 over 6 plus C 3 x square over 2 plus C 4 x plus C 7. So, this is my third equation, this is the third equation. So, let us quickly see now, we have expressions for U V and W. So, what we see is that equation 1 2 and 3 U involves dw over dx so, it is a coupled problem. V does not involve any other thing and the third equation is B 1 1 times U naught minus D 1 1 times dw over d x. So, again there is a coupling between U and W in the third equation ok.

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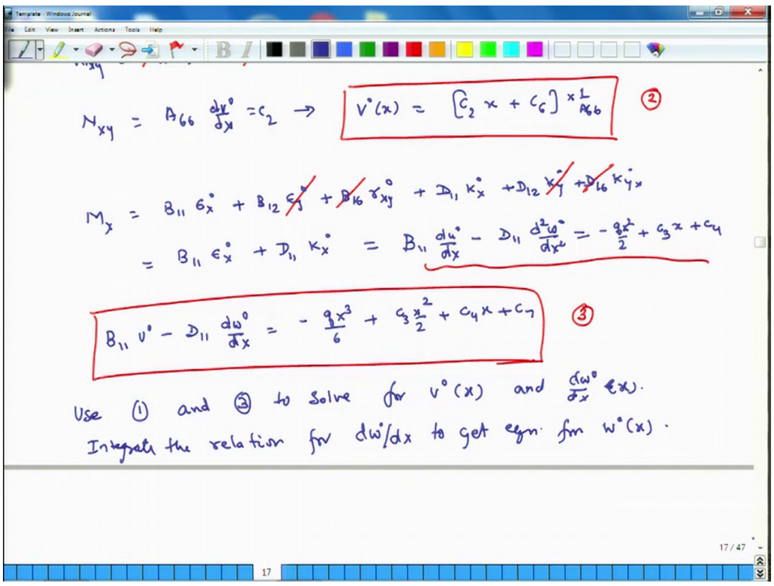

So, what we can do is we can solve. So, what we can do is we can use 1 and 3 to solve for U naught as a function of x and dw naught over d x as a function of x. These are simultaneous equations so, I can solve for U naught and dw over dx ok. So, I will get one expression for U naught and another expression for d w over d x and then integrate the relation for dw over d x to get equation for W naught as a function of x ok. So, if I do all the math; so, you can do all that math that fairly standard 12th standard mathematics of linear simultaneous equation. So, if we do what we end up with is a relation which looks something like this, I will write it.

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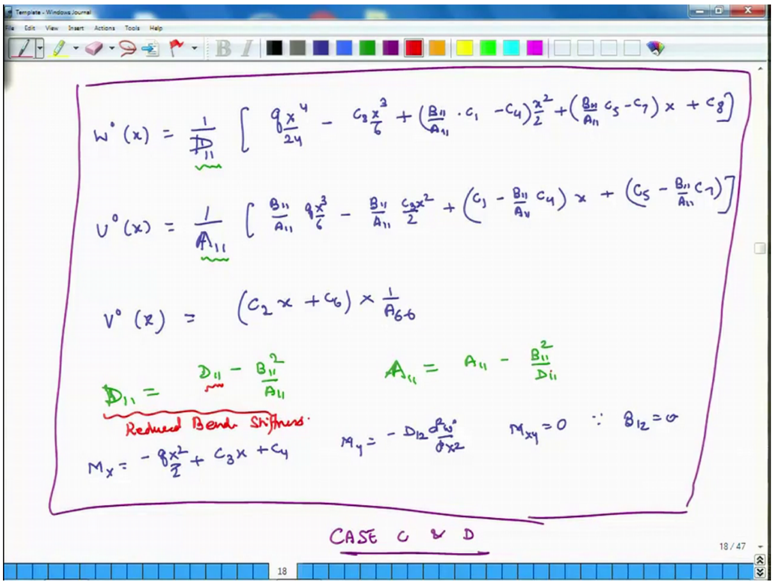

So, W naught x is equal to 1 over D 1 1. Now, this is not regular D this is a difference symbol D. So, it is it is not regular. So, I will define what this big D 1 1 is ok. It is not like our regular D times q x 4 over 24 minus C 3 x cube over 6 plus B 1 1 over A 1 1 times C 1 C 4 x square over 2 plus B 11 over A 1 C 5 minus C 7 times x plus C 8 ok. And the relation for U naught x is equal to 1 over A 1 1, again this is a different A it is not like our regular A. And, this times B 1 1 over A 1 1 q x cube over 6 minus B 1 1 over A 1 1 C 3 x square over 2 plus C 1 minus B 1 1 over A 1 1 C 4 x plus B 1 1 over A 1 1 C 7 actually ll better. So, it will be C 5 minus B 1 1 over A 1 1 C 7 and then of course V.

What is V naught? It is equal to C 2 x plus C 6 into 1 over A 6 6 and I have defined these you know script D 1 1 and script A 1 1. So, what are these? This D 1 1 equals regular D 1 1 minus B 1 1 square over A 1 1 and this is script 1 1 equals A 1 1 minus B 1 1 over D 1 1 square and so, these are U V and W’s. And then what is M x, is still minus q x square over 2 plus C 3 x plus C 4. M y is still minus D 1 2 D 2 w naught over d x square, M xy is still 0 because, B 1 2 is 0. So, this is the solution for both cases. So, this is the solution for case C and case D because, we have not implemented the boundary conditions still so far. So, this is the solution which is common 2 cases C and D.

Once again this solution has a lot of unknowns C 1 C 2 C 3 C 4 C 5 C 6 C 7 and C 8 and those 8 unknowns will be computed once we start implementing the 8 different boundary conditions. 2 more points, this term this D 1 1 script D 1 1 this is called Reduced Bending Stiffness ok; it is called reduced bending stiffness. Why is it reduced bending stiffness? Because, D 1 1 by itself is positive and B 1 1 is so, the effective bending stiffness in the expression for W, in the expression for W we have 1 over this script D 1 1. If B 1 1 was not there then this is script D 1 1 would be same as D 1 1, but because this B 1 1 is there its effective value becomes reduced.

So, the denominator instead of D 1 1 becomes script D 1 1 and that is a smaller number; when it is a smaller number the value of W will increase. So, what this shows? This relation even though we have yet not implemented the boundary conditions; what it shows is that the presence of D matrix tends to increase the deflection of the plate. In the symmetric laminate this B was 0. So, the deflection would be less because, it will be just D 1 1. In unsymmetric laminate because B 1 1 is non-zero, it reduces the stiffness. When it reduces the stiffness happens this plate bends more ok.

Same thing also happens to script A 1 1, its value is A 1 1 minus B 1 1 square by D 1 1. So, here also the value of U also tends to become larger because of presence of B matrix. So, this is what I wanted to discuss today. Tomorrow we will start implementing the boundary conditions and see what the actual solutions look like.

Thank you.