Advanced Composites

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Lecture - 37
Solution for Governing Equation Related to Semi-infinite Composite Plate

Hello good morning today is the start of the 7th week of this course. This is the course on Advanced Composites and last week we did it last week we did a couple of examples related to Solutions for Governing differential Equations Related to laminated Composite Plates. And specifically we had solved for 2 cases or actually 3 cases case A, case B, case C for a semi infinite plate.

In case A and case B the plate was having a symmetric lamination sequence and in case C and case d; the plate is having a lamination sequence which is not necessarily symmetric. And we had also developed the solution for case C and today we will continue that discussion and we will complete the discussion on all the 4 cases.

So, we will also as a part of that exercise develop the solution for case D.

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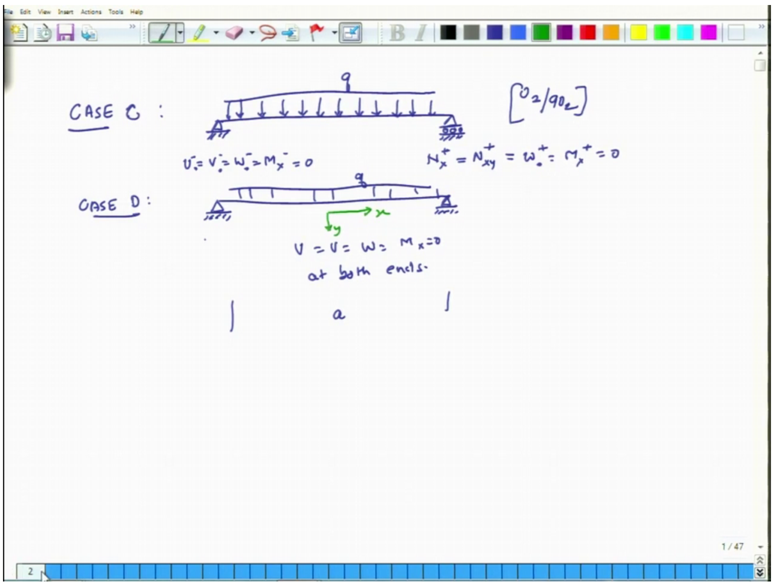

So, just very quickly recap, case C; was case C in this we had a beam pinned and fixed at one end and pinned and sitting on roller bearings at the other end. And then of course, this a uniformly distributed load on the plate and this plate is semi infinite in nature and the constant load intensity was q, which is a constant and lamination sequence we said is 0 2 90 2.

So, and then in case D for which the solution still remains to be developed; the boundary conditions the only thing different about this is that the boundary conditions are different. So, it is like this; so, the boundary conditions at this edge are U minus is equal to V minus is equal to W minus is equal to M x minus equals 0 and here because the plate is free to move N x plus is equal to N x y plus and then of course, W is 0.

So, this is W plus is 0 is equal to M x plus is equal to 0 and in the other case D it is still loaded by q. So, but the boundary conditions are that U is equal to V is equal to W is equal to M x is equal to 0 at both ends that is the difference in boundary conditions.

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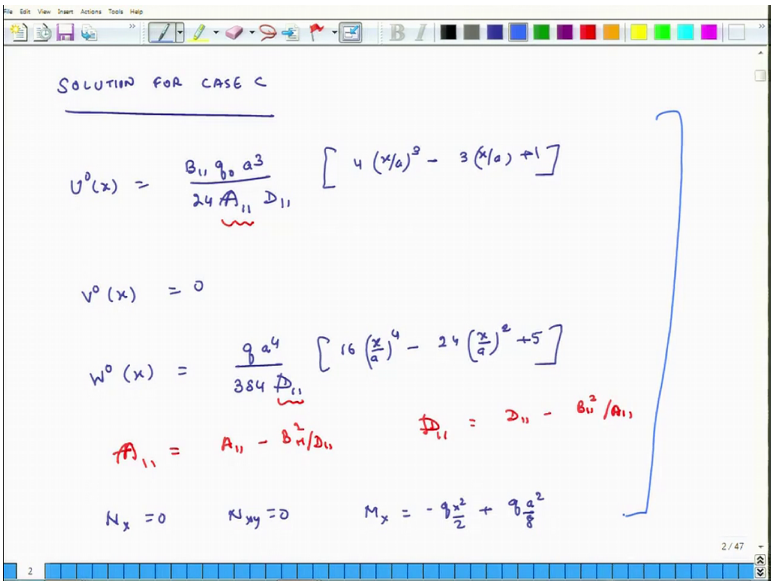

So, the solution for case C; we will just recap the solution for case C by solving the differential equations; it was U naught of x equals B 11, q naught a cube and a is the length of the plate; this is the length of the plate is a and just to recap the coordinate system is located at the center.

So, x is 0 at the midpoint of the length of the plate. So, with that understanding U naught x; we had calculated was B 11 q naught a cube by 24 script A 11 times D 11 times 4 x by a; cube minus 3 x over a plus 1 ok. And V naught x we calculated that at 0 and W naught x it comes out to be q a 4 divided by 384; script D 11 16 x over a to the 4 minus 24 x over a to the power of 2 plus 5.

And these script A 11 and D 11 what are these? So, this A 11 is defined as A 11 square minus I am sorry it is not square A 11 minus B 11 square over D 11 and the script D 11 is defined as D 11 minus B 11 square over A 11. So, this is the solution for case C and then the other things, we had also calculated was that for case C we found that N x is equal to 0, N x y is equal to 0 and M x is equal to minus qx square over 2 plus qa square over 8; so, that is the moment resultant.

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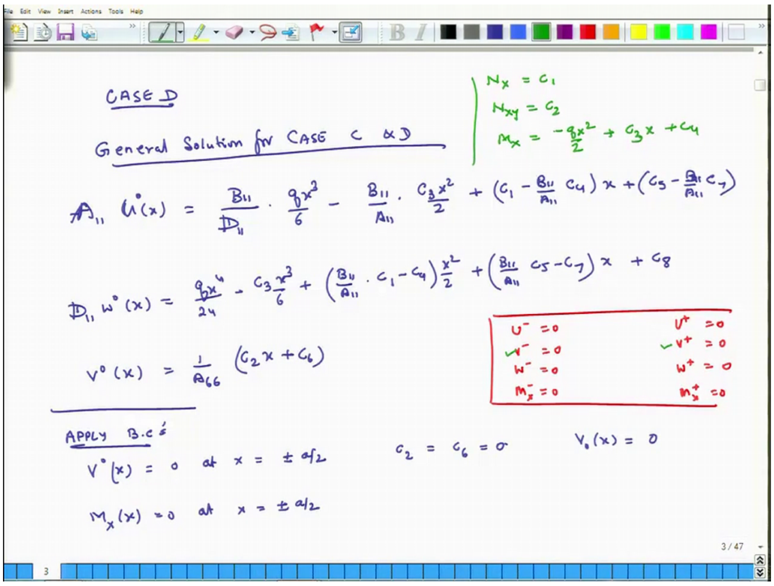

So, now we proceed to case D and we had developed some general solutions in the last class; so, the general solution for case C and D we had developed, it was A 11 script times U of x equals B 11 over D 11 times q x cubed over 6 minus B 11 over 2 I am sorry over A 11 times C 3 x square over 2 plus C 1 minus B 11 over A 11 times C 4 times x plus C 5 minus B 11 over A 11 times C 7 ok.

And D 11 times W naught x equals qx 4 over 24 minus C 3 x cube over 6 plus B 11 over A 11 times C 1 minus C 4 times x square over 2 plus B 11 over A 11 C 5 minus C 7 times x plus C 8; so, that is the expression for W, this is the general solution.

So, this solution is good for both case C and case D the difference will be the values of these constants integration constant C 1 through C 8. And V naught x is equal to 1 over A

66, C 2 x plus C 6 ok. So, now, we apply the boundary conditions for case D; so, we go back and look at what the boundary conditions are?

For case D U mid plane displacements U V and W and m x are 0 at both the ends ok; they are 0 at both the ends. So, we apply these; so, these are 8 boundary conditions, 4 boundary conditions and 1 and 4 boundary conditions on the other end. So, we use these 8 boundary conditions to find out these 8 integration constants. So, these are the 3 equations; so, now, we find these integration constants C 1 to C 8 by applying these 8 boundary conditions for case D.

So, the first thing we do is we know that; so, we apply BC s. So, we know that V naught x is equal to 0 at x is equal to plus minus a over 2. So, if we use this we find that C 2 equals C 6 equals 0 and that gives us I am sorry; so, this should not be subscript [FL] this mid plane.

[FL].

[FL] Anyway so, this V naught x is equal to 0; so, this is the first thing. So, what we have done is let us list down these 8 boundary conditions here; U minus V minus W minus M x minus so, all of them are 0. And then on the positive edge; U plus V plus W plus and M x plus all of them are 0. So, what we have implemented here 2 boundary conditions these 2 have been implemented.

So, now we have to apply 6 other boundary conditions. So, the other thing is that M x which is a function of x is equal to 0 at x is equal to plus minus a over 2. So, which means this expression; so, M x we know that we had calculated as so, we will write 3 more expressions; so, sorry for this less organization we had also done it in the general solution N x is equal to C 1; N x y is equal to C 2 and M x we had calculated is equal to minus q x square over 2 plus C 3 x plus C 4.

So, in this we will apply the second boundary condition at this boundary condition M x is equal to 0 at positive edge and negative edge.

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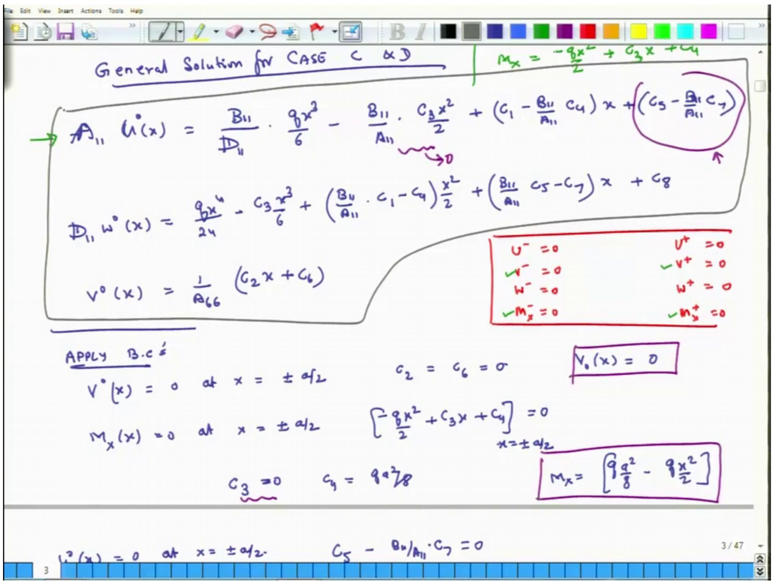

So, which means that minus q x square over 2 plus C 3 x plus C 4 is equal to 0 at what? x is equal to plus minus a over 2 ok. So, if we implemented these boundary conditions what we find is that C 3 is equal to 0 and C 4 is equal to q a square over 8. So, anyway; so, this is one solution for V and for the relation for M works out to be M x is equal to q a square over 8 minus q x square over 2; so, this is the relation for M x ok.

So, now with this we have implemented 4 boundary conditions 2 on V and 2 on M x; we have implemented 4 boundary conditions. The next thing we do is we implement the conditions on U; so, we know that U is equal to 0 at x is equal to plus minus a over 2; so, this should be superscript. So, if that is the case the expression for U is this long thing and if x is equal to 0; at plus minus a by 2 what we will find is that this term will be 0 and also we have found that C 3 is 0.

So, this term also goes away C 3 is 0 because we found that by implementing the condition on M x. And when we implement the condition on U is equal to 0 at x is equal to plus minus 2 this term C 1 minus B 11 over A 11 times C 4; this becomes 0 because what that means, is that this term. So, not this one this term will be 0; so, if we implement these conditions and do the math essentially what I will do is I will directly write the relations.

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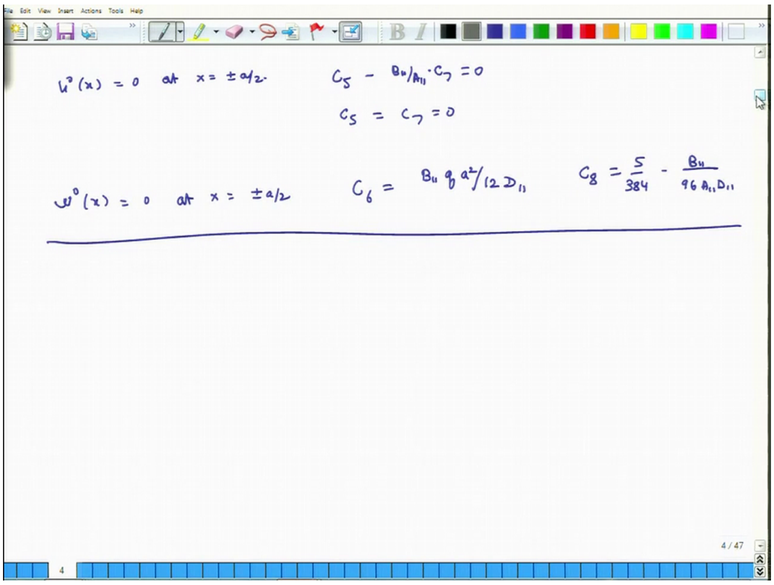

So, what it means is that C 5 minus B 11 over A 11 times C 7 is equal to 0 ok. And that can and the other thing which comes out is that C 5 is equal to C 7 is equal to 0. So, this is what you get and then we when we implement the conditions on W which is W naught is equal to 0 at x is equal to plus minus a over 2; we can calculate the value of C 8 and C 6.

So, C 6 comes out to be B 11 q a square by 12 D 11 and C 8; it works out to be 5 over 384 minus B 11 over 96; A 11, B 11 ok. So, with these once we put these constants back into the equation; the general solution for case C and D, which is this is the general solution for case C and D.

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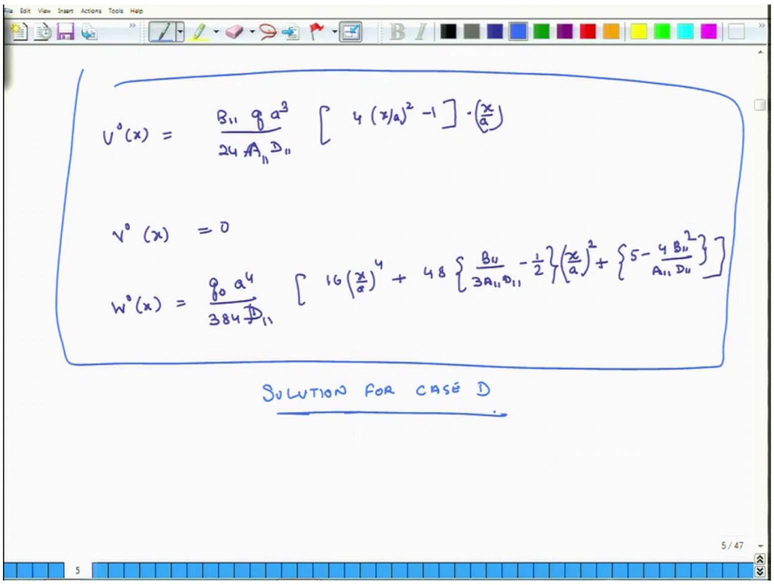

So, we put all these constants back into the thing what we end up getting I have the following relations U naught x is equal to B 11 cube a cube; 24 by subscript A 11 D 11 into 4 over x over a square minus 1 times x over a. The relation for V naught x is 0 and the expression for W naught x is q naught a 4 over 384 script; D 11 times 16 x over a 4 plus 48 times B 11 by 3 A 11 D 11 minus half times x over a whole thing square plus 5 minus 4 B 11 square divided by A 11 D 11 ok.

So, these are the solutions for case D. Now when we look at this solution and we compare it with the solution for case C. So, this is the solution for case C; we find that the expressions for U naught x and W naught x are totally different. This is remarkably different than when we compared the solutions for case A and B and when we were comparing the solutions for case A and B; even though the boundary conditions were different, the solution came out to be the same exactly same.

And that we ascribe to the fact that in case A and B the out of plane problem and the in plane problem were decoupled. But in this case the solution for U depends on W solution for W depends on U and so on and so forth and for this reason case C and case D have significantly different solutions. So, this is one important thing which we can take away from this thing; that in situations where the problem is decoupled out of plane and in plane problem are decoupled, if the out of plane problems boundary conditions are same then the solution will be same.

But, in this case the out of plane problem and the in plane problem they are coupled and because of this coupling the solutions for U V and W for case C are significantly different than that for case D; so, this is what I wanted to talk about today.

We will continue this discussion tomorrow also and we look at the nature of some of these deflections and see how they vary along the length of the; plate and on what parameters do those deflections depend. So, that is all what I wanted to cover for today and we will meet once again tomorrow.

Thank you very much bye.