Advanced Composites
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Lecture - 39
Semi-infinite Plate Loaded in the X-direction (Part-I)

Hello welcome to Advanced Composites. Today is the 3rd day of the ongoing, that is the 7th week of this course. And yesterday we just finished a detailed discussion on case C and D and how the value of n plane displacement that is u, it is sin changes based on the sign of b 11. Today we will do another problem related to Semi-Infinite Plates and in this case, the case the plate is not loaded in the transverse direction, but rather in the n plane direction.

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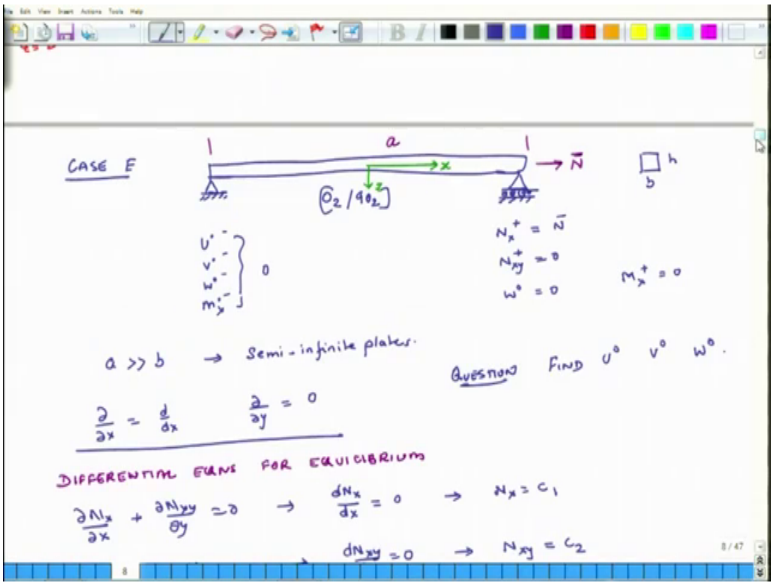

So, let us look at the definition of the problem so, let us call it case E and here we have a plate which is fixed at one end, I mean pinned at one end, but the pin connection is not rolling. So, it is fixed and the other end is on rollers so, this is the plate. The lamination sequence of the plate is 0 to 92 so, 0 layer is on the top and then, we have 90 degree less on the bottom and then, this plate is being subjected to N x at this edge.

So, I am pulling this plate and let us say that the value of this tensile N force resultant is N bar. So, the question is so before we do we write the problem definition, the question is what the boundary conditions? The boundary condition at this edge is U naught V naught W naught and M x naught, all of these are 0 so, at the negative edge. And at the other end N x plus is equal to N bar which is known N x y plus is equal to 0.

Because I am not applying any external shear, W naught is equal to 0 at x is equal to a over 2. So, I am still saying that this is positive x, this is positive z and M x, I am not applying any external moment, the plate is free to rotate at the pendant. So, it is having external moment at the positive edge is also 0. The other thing like we did in the case of earlier overall the length, the plate is a and if I look at the cross-section of the plate, so the height is h and the width is b. So, I say that a is very large compared to b so, that is our approximation for semi-infinite plates, yes the approximation for semi infinite plates.

So, because a is very large compared to b, we can say that dell over dell x is equal to d over dx and dell anything it is differential in the y direction is 0. So, this is exactly the same thing which we had assumed in cases a, b, c and d because this is a semi-infinite plate in the x direction.

If it was in the y direction, then dell over dell y would be d over d y and dell over dell x will be 0. So, now we want to, so the question is find U naught V naught and W naught? So, we start with the differential equations of equilibrium for equilibrium.

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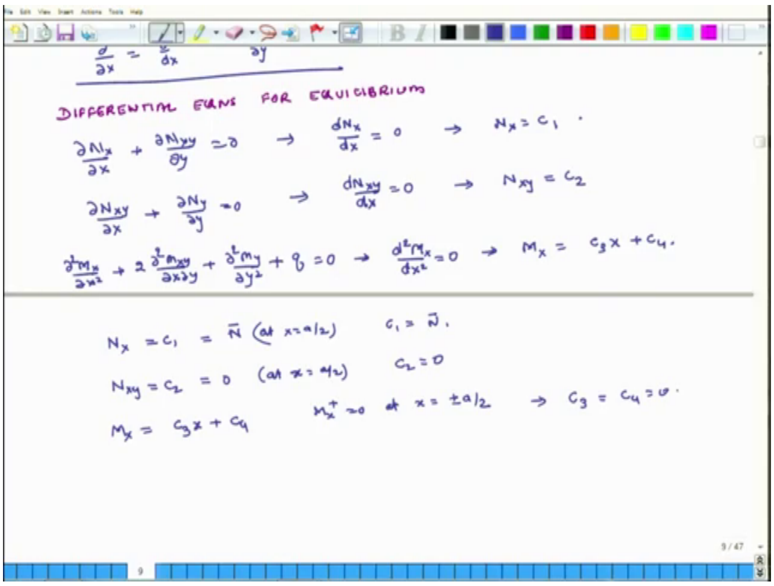

So, the first differential equation is dell N x over dell x plus dell N xy over dell y equals 0, but dell over dell y 0. So, this simplifies to dN x over dx is equal to 0 or N x is equal to

C1. 2nd equation is dell N xy over dell x plus dell N y over dell y equal 0, this simplifies to dn xy over dx is equal to 0. So, N y N xy equals C2 and the 3rd governing differential equation is dell 2 M x over del x square plus 2 dell 2 M xy over dell x dell y plus del 2 M y over dell y square plus q is equal to 0.

So, in this case q is 0, because the plate is not having any external transfers load and also, these partial derivatives with respect to y are 0. So, this equation simplifies to d 2 M x over d x square is equal to 0. So, what that tells me is that M x is equal to C3 x plus C4, ok so, this is what we get from the differential equations.

Now, we apply some boundary conditions, we will not apply all the boundary conditions. So, we have C1 C2 C3 C4 and we will select some specific boundary conditions, so that we can calculate the values of these constants. So, first we select N x plus is equal to N bar so, we look at this equation. So, it says N x is equal to C1 and that is equal to N bar at x is equal to a over 2 so, C1 is equal to N bar.

The next one is N x y equals C2, but at x is equal to plus a over 2. Its value is 0 at x is equal to a over 2so, C2 is equal to 0. The 3rd thing is M x is equal to C3 x plus C4, but we know that M x plus is equal to 0 at x is equal to plus minus a over 2. So, this gives me C3 is equal to C4 is equal to 0, because I am applying two boundary conditions at x is equal to plus a over 2 and minus a over 2 M x plus is 0 or let us do this explicitly.

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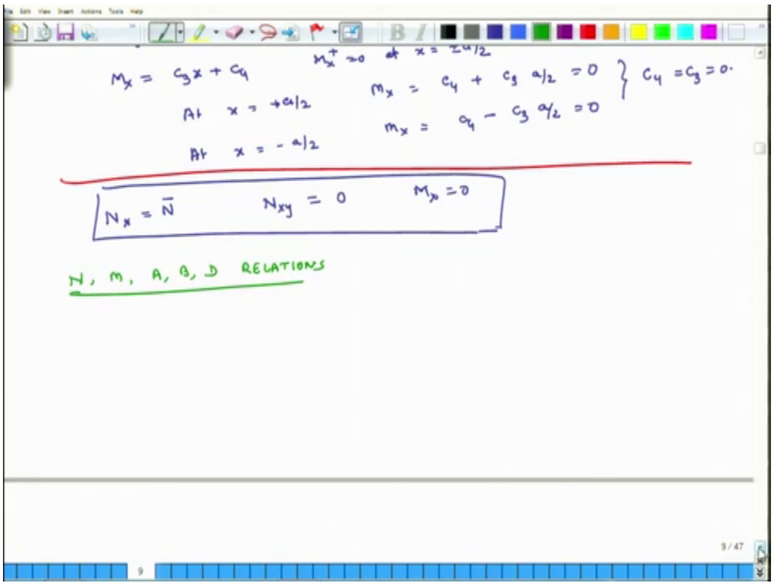

So, we first at x is equal to plus a over 2, what is it? M x is equal to C4 plus C3 a over 2 and that is value 0 and at x is equal to minus a over 2 M x is equal to C4 minus C3 times a over 2 is equal to 0. So, these give us C4 is equal to C3 is equal to 0. So, once we have applied, so which boundary conditions we have applied? We have applied the boundary condition on N x, on N x by on M x plus and on M x minus we have applied 4 boundary conditions.

4 boundary conditions are still remaining to be applied so, after we had done with these 4 boundary conditions, what we are left with is N x is equal to N bar N xy is equal to 0 and M x is equal to 0. Now, we develop expressions for N x N x y and M x in terms of A B D matrices. So, we will develop N M A B D relations.

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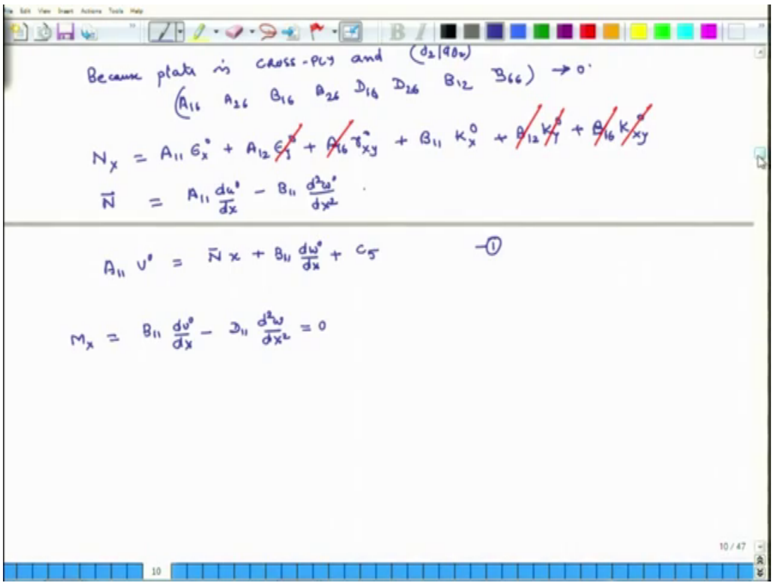

And before we do that we note that because plate is cross ply and 0 to 92 a lot of terms in A B D mattresses are 0. What are these terms? A16 A26 B16 B26 D16, D26 B12 and B66, all these terms are 0, because it is a cross ply laminate ok. This is important B12 and B66 is also 0 for cross ply.

So, with this understanding now let us write down what is the relation for N x. N x is equal to, so I will write it as A11 epsilon x naught plus A12 epsilon y naught plus A16 gamma x y naught plus B11 K x naught plus B12 K y naught plus B16 K xy naught. This is the expression for N x and what we see is epsilon y naught is 0, because it is a partial derivative of v with respect to y, A16 is 0, because the laminate is cross ply B12 is 0, because we just said that it is a cross ply laminate and also K y is 0 and also B16 is 0 and also K x y is 0.

So, this simplifies to an epsilon x naught is partial derivative of u with respect to x, but because the plate is semi-infinite, it is du over dx and the other term which gets involved is B11 and K x is what minus d 2 w over d x square. And the value of N x is N x bar so, this gives us, so from this we get if I integrate this equation once and rearrange, what I get is A11 U naught equals N bar x plus B11 dw naught over dx plus C5 ok so, let us call this equation 1.

Similarly, if I expand M x, so M x is equal to B11 du naught over dx minus D11 d 2 w over dx square because, all other terms B16, all other terms are involving epsilon y gamma x y K y and K x y, they will be 0. So, this is the only term left and this is equal to 0, why is it 0? It is because we have seen that M x comes out to be 0 in the plate.

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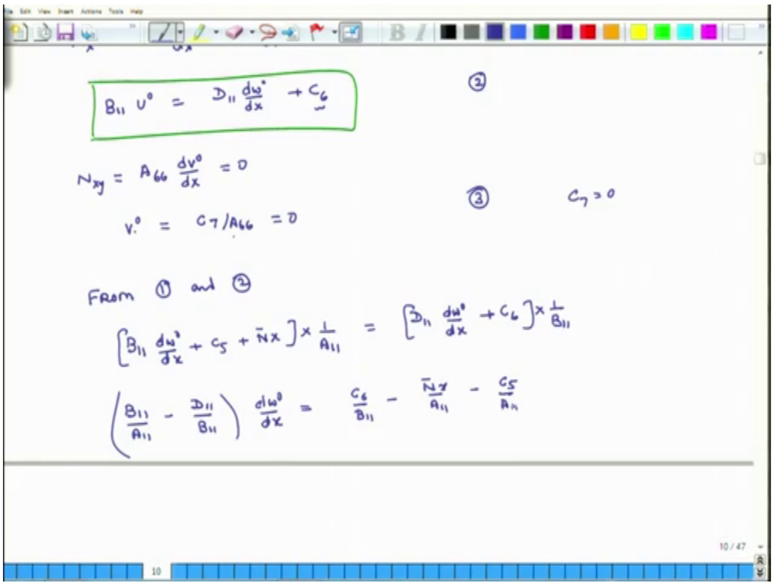

So, we apply this condition and when we integrate it once what we get is B11 U naught is equal to D11 dw naught over d x plus C 6, this is equation 2. So, we have expanded on N x, we have expanded on M x and next now look at N xy. So, N xy is equal to the only thing which is left is A66 d v naught over dx, all other terms drop out. So, this is coming from A66 times gamma xy and gamma xy is d partial derivative with respect to x plus partial derivative of y u with respect to y.

So, the other component is 0 so, this is all we have left with and this is equal to 0. So, from here we get v naught equals C 7 by A66 ok so, this is equation 3. So, now how many integration constants we have? C5 C 6 and C 7 and we will get one more integration constant when we integrate w because, right now we only have derivatives of w, but before we go there what is the value of C 7?

So, we look at this boundary condition that at x is equal to minus a over 2, v naught is 0. So, this means that C 7 is 0 so C7 also it works out to be 0. So, now what we are left with are two important equations which are coupled equations in U and dw naught over dx ok. So, if I solve for dw naught over dx from equations 1 and 2, so basically, if I equate if I put the value of U naught in from equation 1 into the other equation, so what I get is from 1 and 2 what I get is B11 dw naught over dx plus C 5 plus N bar x into 1 over A11 is equal to D11 dw naught over dx plus C 6 into 1 over D11 ok. So, this is I have eliminated U naught from here and this is an equation only in dw d x.

So, if I come rearrange this equation, what I get is B11 minus B11 over A11 minus D11 over B11 dw naught over dx is equal to C 6 over B11 minus N bar x over A11 minus C 5 over A11. And if I multiply this entire equation by A11 essentially what I get is, so I am multiplying the entire equation by A11. So, it goes away from here denominator and it appears in A11 and it goes away from these two denominators also. So, what I do is now I integrate to get w, I have to integrate this to get w, I have to integrate it.

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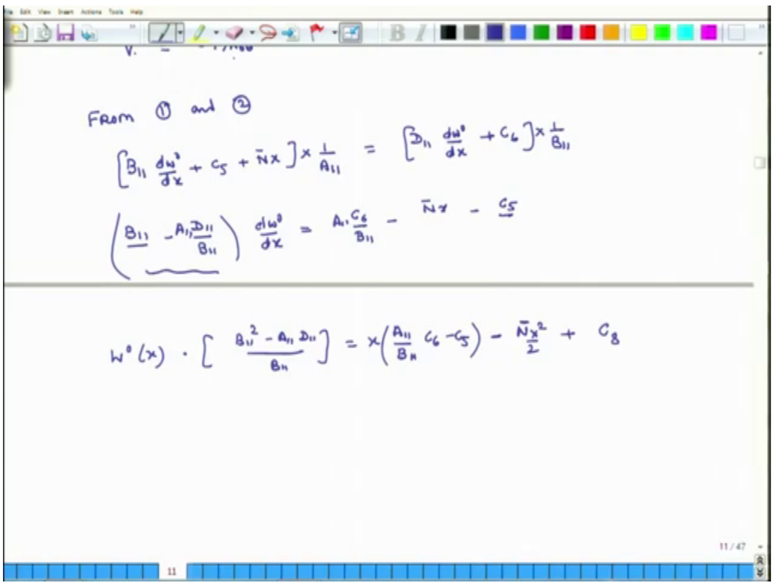

So, what does it imply? It means that w naught x and this I can rewrite it as B11 square minus A11 D11 divided by B11 is equal to A11 by B11 C 6 minus C 5 minus it is N x square over 2. There is an x here plus another integration constant C 8.

So, this is the expression for w naught and this is actually N bar. So, what we will do is, we will stop here and we will continue this discussion on this particular problem case 5 and we will complete the solution for this case 5 or case E in tomorrow's lecture. So, thank you and I look forward to seeing you tomorrow.

Thanks.