

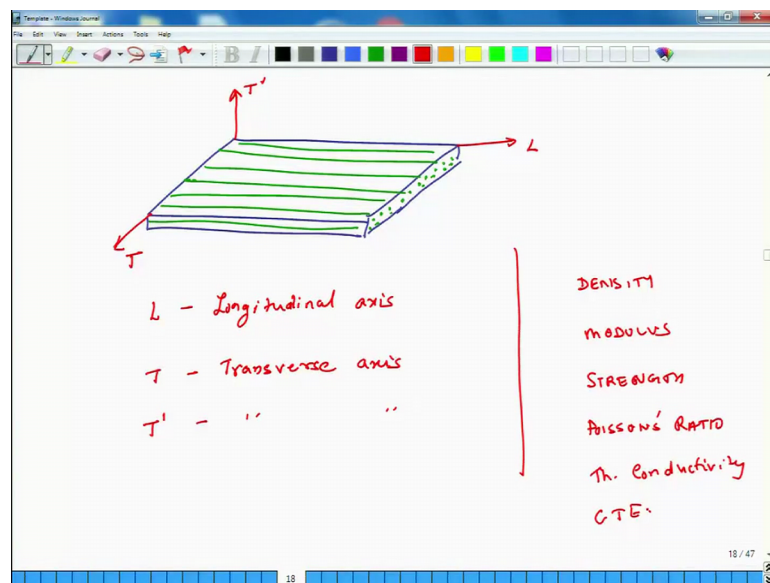
Advanced Composites
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Lecture - 04
Properties of Single Layer Continuous Fibre Composites Part-II

Hello, welcome to Advanced Composites. In this course, we have just started discussing material properties associated with anisotropic and generally orthotropic materials. And what will we do, starting today is start discussing about different expressions related to Single Layers of Continuous Fibre Composites.

So, we will start discussing about different properties of single layer composites.

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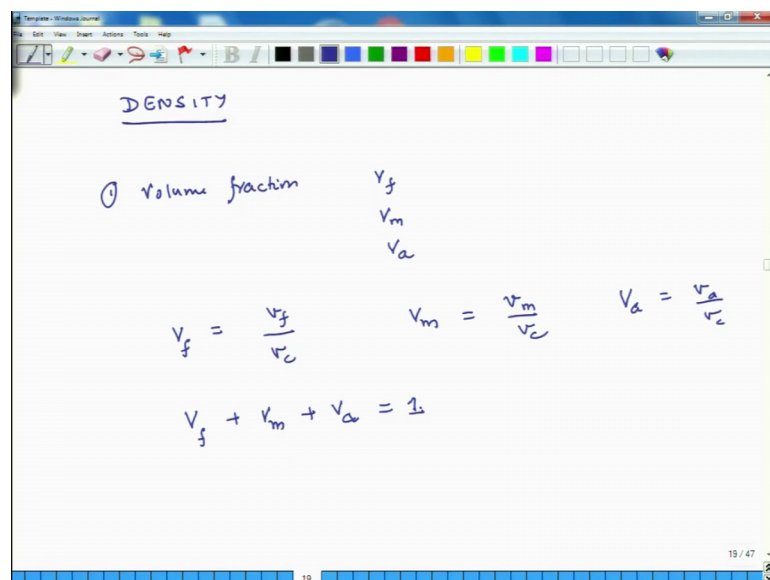
So what type of composite materials we will discuss? So, we will consider that the material is a thin layer ok. So, thin layer and it has continuous fibres. So, this is so these fibres their cross sections appear on this plane and on this plane again we have a fibre, which is visible and our axes system is this is my L axis, this is my T prime axis and this is my transverse axis.

So, L axis is known as longitudinal axis, T is transverse axis and T prime is also transverse axis. And the picture which I have drawn is for a single layer continuous fibre composite and in this case this is like a; and if my load on this composite is aligned to

these axis then that will be the case of special orthotropy. So, what we will start discussing today starting from today is; we will figure out how to calculate density, modulus, strength, Poisson's ratio and thermal conductivity and finally, coefficient of thermal expansion.

So, we will learn how to compute these in some cases we will actually derive the expressions, in other cases we may not necessarily derive the expressions because some of these expressions have also already been derived in the introductory course, but this is the overall stuff which we plan to capture.

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So, we will start with density. And before we density we will introduce 2 terms; the first one is volume fraction.

So, what is volume fraction? So, you can in a composite you can have a volume fraction of fibre V_f , you can have volume fraction of matrix material V_m and you can have volume fraction of air. So, what is volume fraction of fibre? Capital V_f it is the volume actual volume in a piece of composite material, actual volume in a piece of composite material divided by the total volume of the composite material. Likewise, volume fraction of matrix is V_m over small V_c . So, it is the ratio of volumes of matrix and the composite volume fraction of air is V_a over V_c . So, if you add all these 3 volume fractions V_f plus V_m plus V_c it all $V V a$. So, these all these volume fraction of fibre matrix and air they should always equal 1.

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② Mass fraction.

$$M_f = \frac{m_f}{m_c} \quad M_m = \frac{m_m}{m_c}$$
$$M_f + M_m = 1$$

DENSITY

$$m_c = m_f + m_m$$
$$\rho_c V_c = \rho_f V_f + \rho_m V_m$$

The second term is mass fraction, this is not that often used but it is still a useful entity to entity and the definitions are similar, So, mass fraction of fibre is mass of fibre divided by mass of composite, mass fraction of matrix equals mass of matrix divided by mass of composite and then of course, the mass of air is almost 0 so we ignore it.

So, in this case $M_f + M_m = 1$ because of mass of air is virtually 0. So, with these we will develop an expression for density of a composite material. So, we know that if I have a piece of composite; let us say its total mass is m_c then that equals mass of fibre plus mass of matrix. And what is mass of composite? Mass of composite is nothing but density of composite overall density of the composite materials times the volume of composite material.

So, that equals mass of fibre and mass of fibre is what; overall density of fibre times. So, these volumes should be in smaller letters because small letter indicates volume large case letter indicates volume fraction. So, anyway so $\rho_f V_f + \rho_m V_m$.

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DENSITY

$$m_c = m_f + m_m$$

$$\rho_c V_c = \rho_f V_f + \rho_m V_m$$

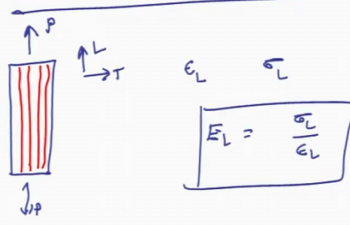
$$\rho_c = \rho_f \left(\frac{V_f}{V_c} \right) + \rho_m \left(\frac{V_m}{V_c} \right)$$

$$\rho_c = \rho_f V_f + \rho_m V_m$$

And I divide this entire expression by V_c . So, I get density of the composite material equals $\rho_f V_f$ over V_c plus $\rho_m V_m$ over V_c , but V_f over V_c is volume fraction of fibre and this is volume fraction of matrix. So, my expression for composite is ρ_c equals $\rho_f V_f$ plus $\rho_m V_m$. So, that is my expression for composite material density.

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LONGITUDINAL MODULUS E_L



$$E_L = \frac{\sigma_L}{\epsilon_L}$$

$E_f = E_m = E_c$ — in L direction

$$P = P_f + P_m$$

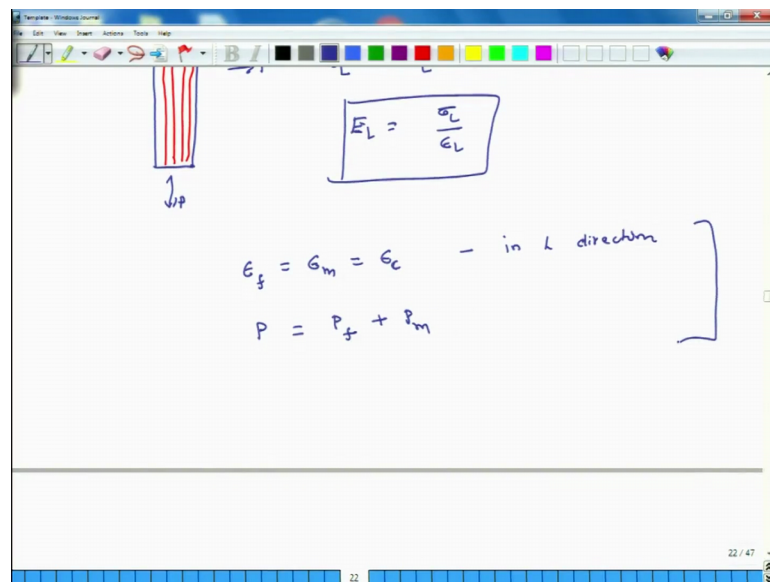
The next parameter we will discuss will be longitudinal modulus. And we designate longitudinal modulus as E_L . So, physically what does it mean? What it means is; that if I have a composite material and this composite material is a single layer, reinforced with

long continuous fibres and let say my fibres are in this direction running like this, and the load is also applied in the L direction.

So, here I am applying load P and because of P it causes a strain epsilon L, so this is my L direction this is the L direction and this is T direction. So, because of this external load P I generate a strain in the length longitudinal direction epsilon L, and the stress in applied in the longitudinal direction is sigma L then E L is defined as; sigma L by epsilon L. This is assuming the behaviour of matrix and fibre is perfectly elastic, otherwise this will be a differential expression.

So, how do we compute epsilon L and E L? Now to compute E L we may we see this situation and we come to 2 conclusions. One is that whatever is the strain in fibre the same strain in the longitudinal direction will also exist in matrix and same strain will also exist in overall composite. So, we say that epsilon fibre equals epsilon matrix, equals strain in composite in l directions. So, this is one and the other one is that the total load is P and this load is partly shared by fibres. So, let us say the amount of load shared by fibre is P f, and plus the load shared by matrix and that is P m.

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So, this is these are the 2 assumptions based on the physics of the situation.

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From ② $P = \sigma_c A_c = \sigma_f A_f + \sigma_m A_m$

$$\sigma_c = \sigma_f V_f + \sigma_m V_m$$

Apply ① $E_L \epsilon = E_f \epsilon V_f + E_m \epsilon V_m$

$$E_L = E_f V_f + E_m V_m$$

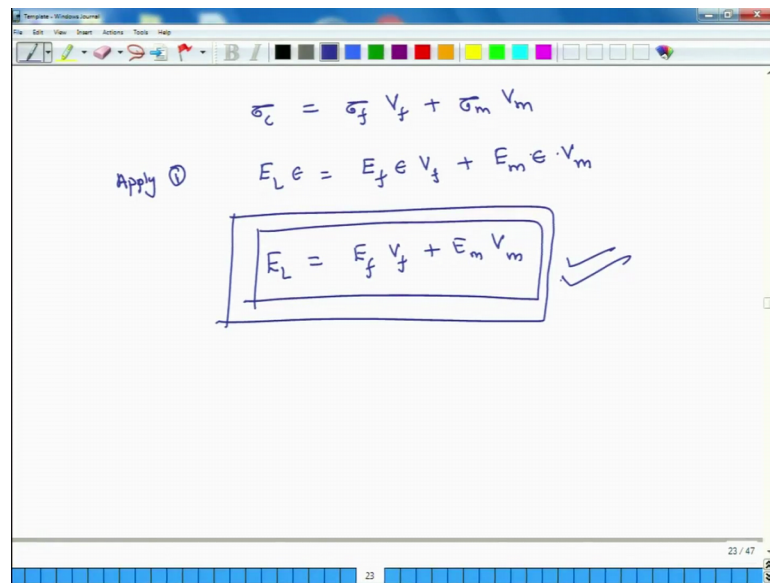
Now, let us look at equation 1 and this is equation 2. So, from 2 what do we say this P is equal to stress in the composite which is σ_c , times area of the composite A_c . So, this is the right side of the left side of the equation and that equals stress in load shared by fibre. So, load shared by fibre is what? Stress in fibre σ_f times area cross sectional area of fibre. And likewise is stress in matrix times area of matrix. And if we divide this entire expression by A_c we get σ_c is equal to σ_f volume fraction of fibre plus σ_m times volume fraction of matrix.

So, this is what we get from 2 and then now we apply 1 in this equation ok. How do we apply one in this equation? We know that the stress in composite is nothing but Young's I mean the modulus of composite in the L direction. So, actually I will call it E_L times the strain in composite and a strain in composite fibre and matrix everything is same. So, this is ϵ and that equals σ_f , σ_f is a stress and stress in fibre is what? It is Young's modulus E_f times strain in fibre, ϵ times V_f plus stress in matrix which is E_m times strain times V_m . And ϵ is common on in all the terms so it goes away.

So, ultimately we are left with the expression for E_L and that is equal to $E_f V_f$ plus $E_m V_m$, $E_f V_f$ plus $E_m V_m$. So, this is based on these 2 assumptions; that the strain and fibre matrix and composite it is same when the composite of this type is loaded in the longitudinal direction.

And the other assumption we made is that the total load experienced by composite it is nothing but the sum of loads shared by fibre and the load shared by matrix material.

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The image shows a screenshot of a presentation slide with a whiteboard background. The whiteboard contains the following handwritten text:

$$\sigma_c = \sigma_f V_f + \sigma_m V_m$$

Apply ①

$$E_L \epsilon = E_f \epsilon V_f + E_m \epsilon V_m$$

$$E_L = E_f V_f + E_m V_m$$

The equation $E_L = E_f V_f + E_m V_m$ is enclosed in a hand-drawn rectangular box with a double arrow pointing to the right. The slide also features a toolbar at the top with various drawing tools and a status bar at the bottom showing '23 / 47'.

And using these 2 assumptions and principles of linear elasticity, we were able to come to this expression for E_L , which is Young's modulus which is the modulus of the composite in the L direction.

Now, how do we know whether this is this relation and our assumptions are actually true. Well the way to check that out is by doing a lot of experiments and over last several decades people have done a very large number of experiments. And the predicted value of E_L for as computed from this relation it comes to be very close than to experimentally observed values.

So based on this agreement between a very large number of experimental data and predictions from this relation we can be fairly certain and confident that if our system is linearly elastic, then we safely predict the modulus of a unidirectional composite in the longitudinal direction from this relation, which is $E_L = E_f V_f + E_m V_m$. So, that concludes our discussion for today. Tomorrow which is our next lecture we will extend this discussion and continue this forward.

Thank you.