Advanced Composites
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Lecture - 40
Semi-infinite Plate Loaded in the x-direction (Part-II)

Hello welcome to Advanced Composites. Today is the 4th day of the ongoing week which is the 7th week of this course. And yesterday we started a new problem; that is case e, in which we had a semi-infinite plate in the x direction, and this plate was is unsymmetric in nature. One of it is ends is pinned and fixed and the other end is pinned, but on roller supports. And such a plate is being subjected to an in plane force.

And the force resultant in the x direction is uniform externally applied force resultant is N bar. So, with that problem formulation what we have done till so far is, we have come up with an expression for w naught x which is this entire relation.

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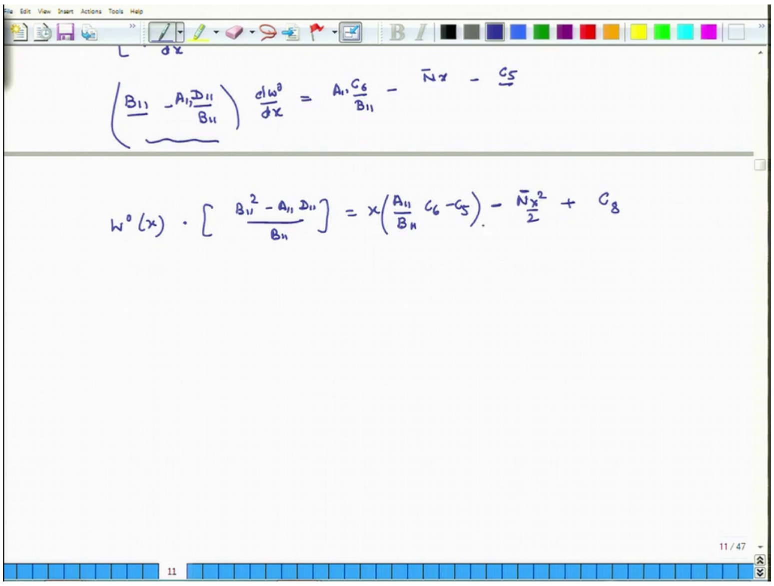

And now what we plan to do is, we want to evaluate the value of these integration constants C 6, C 5 and C 8 from w naught x.

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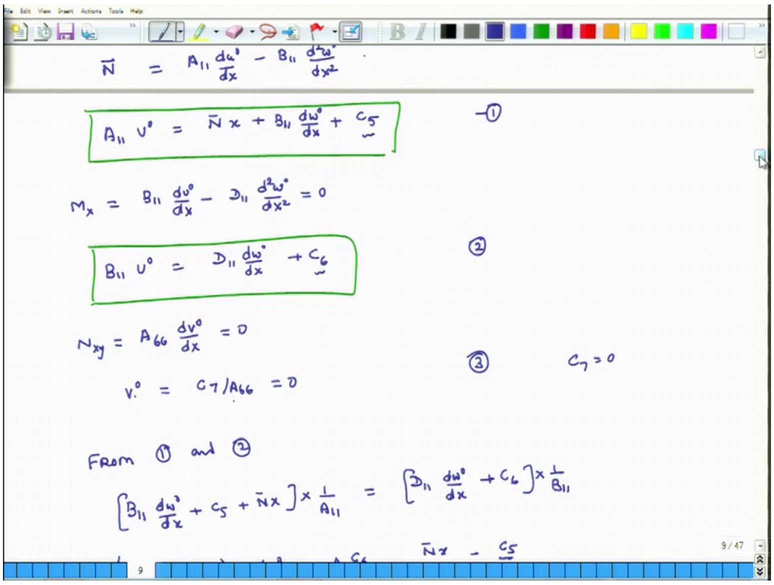

And then later we will also use the same thing, and compute the relation for U naught.

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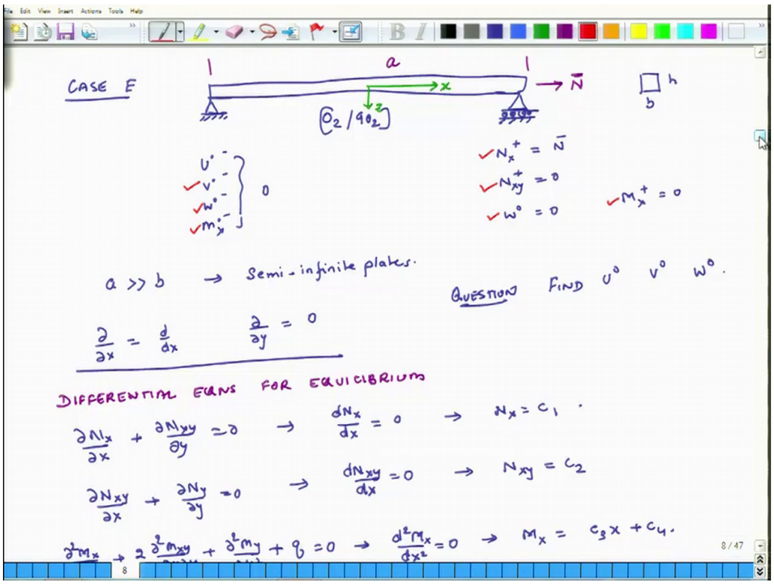

So, the boundary conditions for this plate, the ones which are checked in red we have already applied. We will now apply 2 boundary conditions. And these 2 boundary conditions are on W naught, and let us see that once we apply these 2 boundary conditions, what does the relation for w naught come out as?

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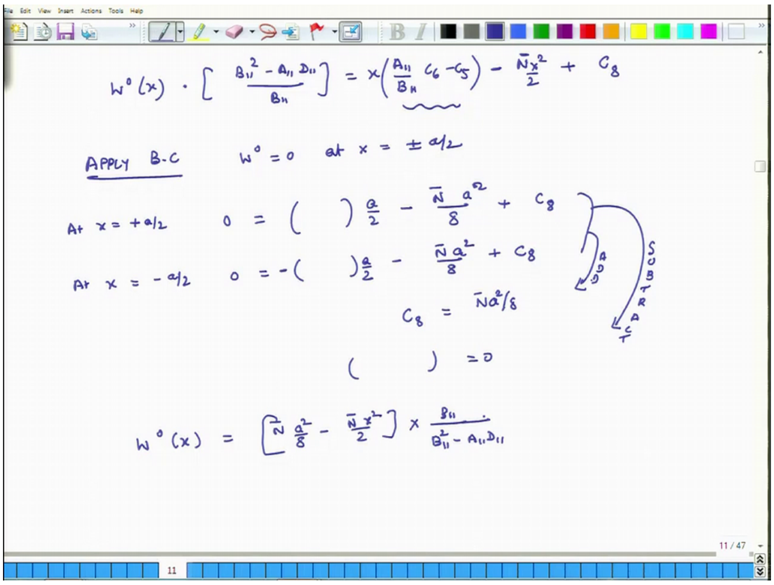

So, apply B C and the boundary condition is that W naught is equal to 0 at x is equal to plus minus a over 2. So, what that means is that so, if I apply at x is equal to a over 2, what do I get? I get 0 equals this entire thing times a over 2 minus N bar a square over 8 plus C 8. And if we apply the other boundary condition x is equal to minus a over 2 I get this same parameter, but here I have a negative times a over 2 minus N bar a square over 8 plus C 8, ok.

So, we have 2 equations, one for a over 2 one for negative a over 2. If I add them or if I add them, then I get C 8 is equal to N a N bar a square over 8. So if I do, if I add these boundary conditions and if I subtract them, I get this entire parameter in the expression this. A 1 1 over B 1 1 times C 6 minus C 5, this is equal to 0 right. So, once I get this, I can say that W naught over x is equal to N bar a square over 8 minus N bar x square over 2 into B 1 1 by B 1 1 minus A 1 1, D 1 1 this B 1 1 square. This is my expression ok. And I can just reorganize this again.

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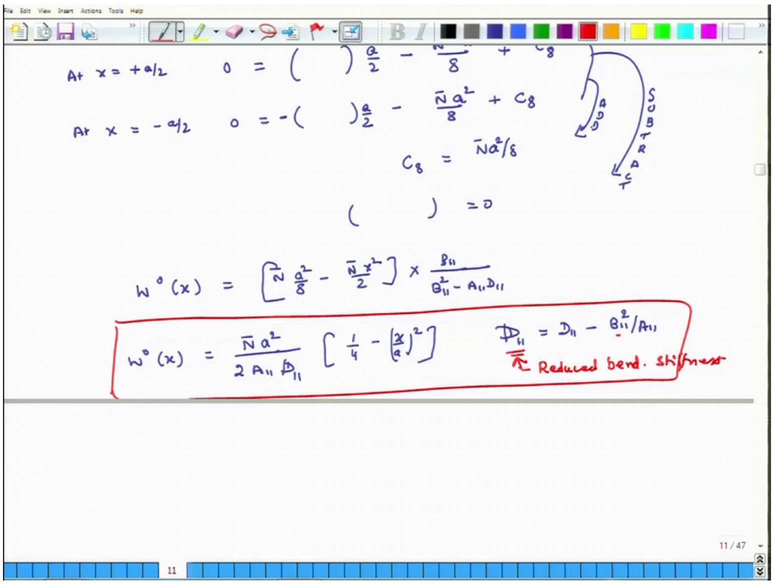

So, if I have to reorganize it, what I am going to do is I am going to take N a square over 2 outside the bracket. 2 and I will call A 1 1, but also I will take it out, and this is the script D 1 1. And in the parenthesis, I will be left with is 1 over 4 minus x over a square. And here D 1 1 is equal to D 1 1 minus B 1 1 square over A 1 1; this D 1 1 script so, the same definition which we have used earlier. So, we once again see that the in the denominator, we do not get D 1 1 for a unsymmetric laminate, rather we get this D 1 1 script. And we had said earlier that this is known as reduced bending stiffness. And essentially, what it means is that the stiffness of this plate it gets reduced because of the presence of B 1 1. If B 1 1 was not there, then the plate would be more stiff, and it diminishes because of the presence of B 1 1 ok.

So, this is the expression for D 1 1. So, there is one small actually the sign here was incorrect. So, I will fix that.

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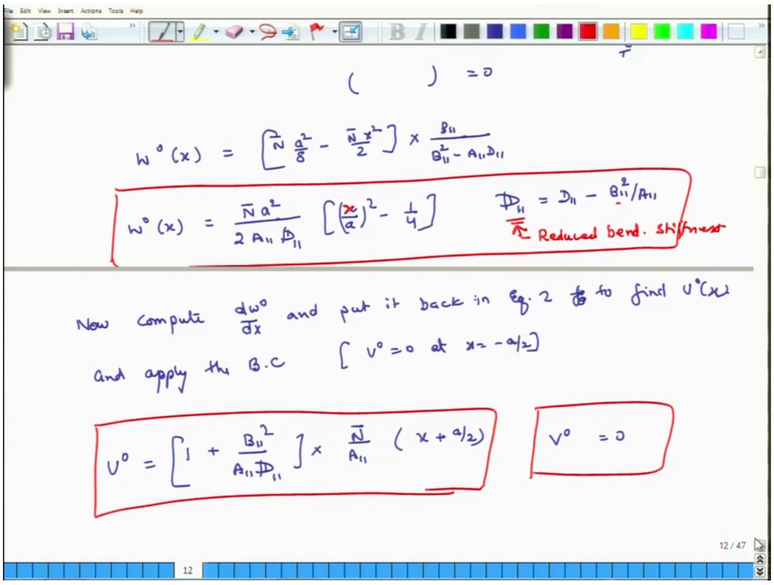

So, this is x over a whole square minus 1 over 4 this is there. So, this is the expression for w naught x, and if you now go back so now, you can compute dw dx from here, and you plug the value of dw over dx in the expression for u naught.

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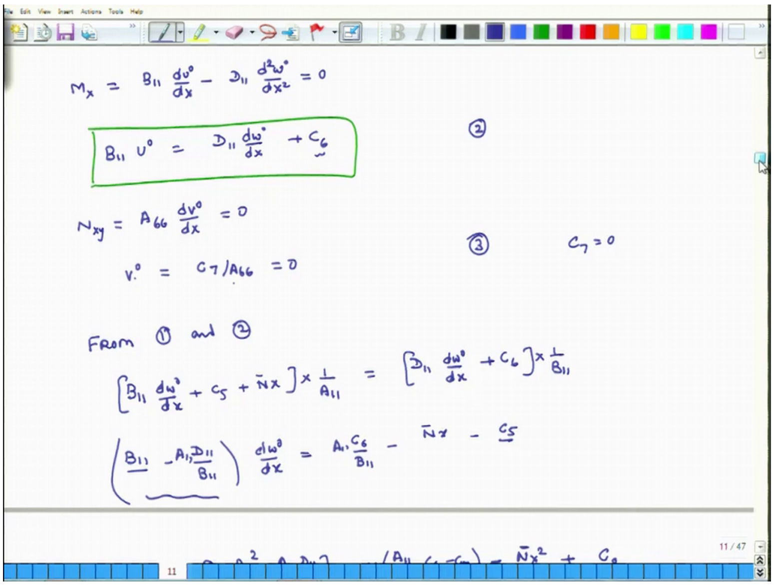

You can plug it in one of these equations, you can plug it in here ok. And once you plug it in so, what I am saying is; so now, compute dw naught over dx, and put it back in equation 2 to find U naught x that is a simple mathematical process and what and then apply the boundary condition.

So, which boundary condition we use? We use u naught is equal to 0 at x is equal to minus a by 2. And then what you end up getting is, U naught is equal to 1 plus B 1 1 square over A 1 1 D 1 1 script, into n over A 1 1 and bar times x plus a over 2. So, this is the expression for U naught. And the expression for V naught is 0. So now, we have solved this problem ok. So, we have an expression for u naught V naught and W naught. Now we will make some observations here. So, one thing I omitted accidentally in the expression for w naught is, that there is a B 1 1 here, but I omitted it here. So, this B 1 1 should have been here. That says error which you guys should note.

So, we will make some observations.

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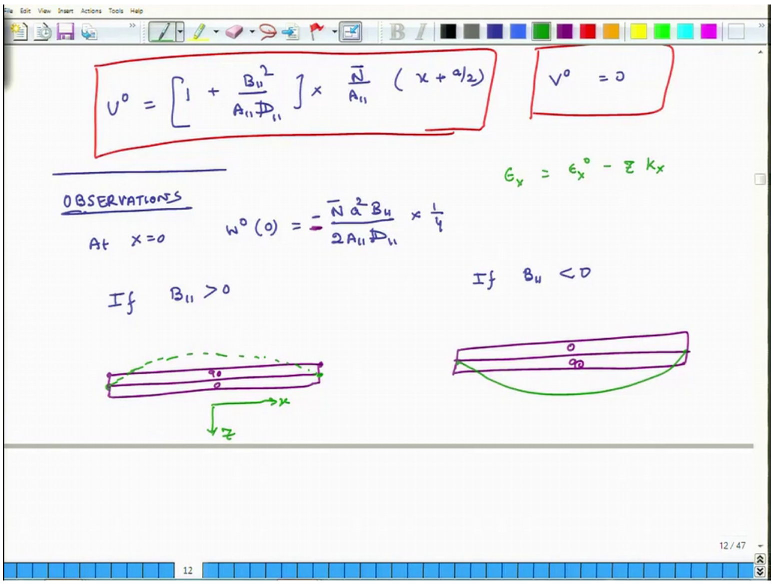

The first thing is that at x is equal to 0; which is at the mid of the plate let us compute the value of w naught. So, when x is equal to 0, w naught at 0 equals N bar a square be 1 1 divided by 2 A 1 1 D 1 1 into and there is a negative sign times 1 by 4, ok. So, in this case once again we see that depending on the sign of B 1 1. W naught will be either positive or negative. So, what that means is, that if B 1 1 is greater than 0 and our original plate is like this. So, when it is greater than 0 it means what? That the 90 degrees flyer, this is 90 and this is 0, then B 1 1 is greater than 0, we had computed the value of B 1 1 huh.

So, if that is the case and if B 1 1 is greater than 0. And of course, N bar is positive, N bar is positive a square is positive A 1 1 is positive D 1 1 is positive everything is positive. Then if B 1 1 is greater than 0, then the plate will bend in which direction? The plate will exhibit a negative deflection; which means it will bend so, it will bend something like this. Why is it going to bend? In the so, we are saying it is going to bend in the negative direction, because in our entire coordinate system this is x and this is z. Z is always downwards, if you remember throughout our development coordinator system when we calculated abd matrices, and we developed the relation that epsilon x is equal to epsilon x naught minus z k x, in this z axis was always going down from the mid plane ok.

So, if it goes down then it would be considered a positive displacement. But it has to be negative, we are saying because B 1 1 is positive, N bar is positive, a square is positive, but there is a negative sign here. And because there is a negative sign, it means that the mid plane this flection of the plate is going to be negative; which means it is going to bend upwards, ok. It is going to bend upwards. And if B 1 1 is less than 0 so, if it is less than 0, then the plate configuration will be something like this, 0 on top and 90 on the bottom. And in this case, the plate will bend something like this. So, the first interesting thing is that, what is the problem?.

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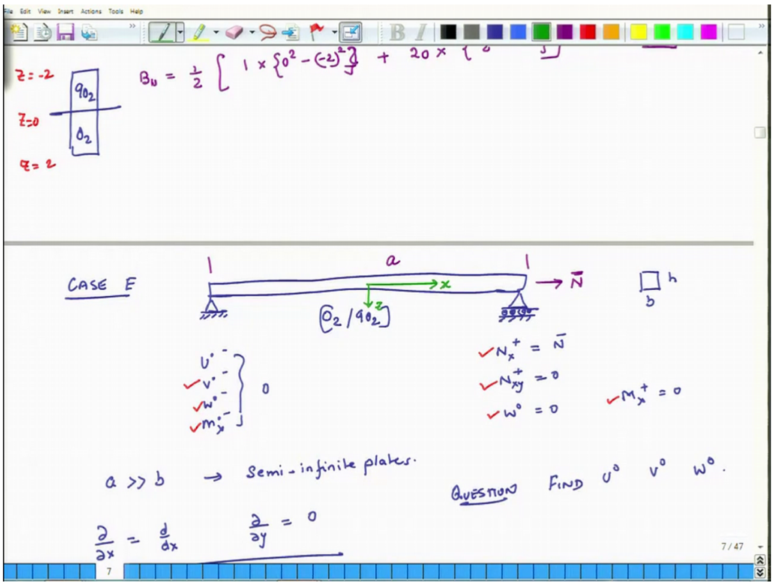

The problem is that we are subjecting a flat plate to a tensile load. And normally you would not expect the plate to bend, it would only get stretched. But because of the presence of B matrix the plate bends.

And that there so, that is the first thing because of the presence of B matrix, and the direction of the plate bending is determined by B 1 1 in this case. So, this is mathematically how it works out. The physical reason why plate bends in this case upwards and in the second case when B 1 1 is less than 0 is, because so, let us so, the physical reason.

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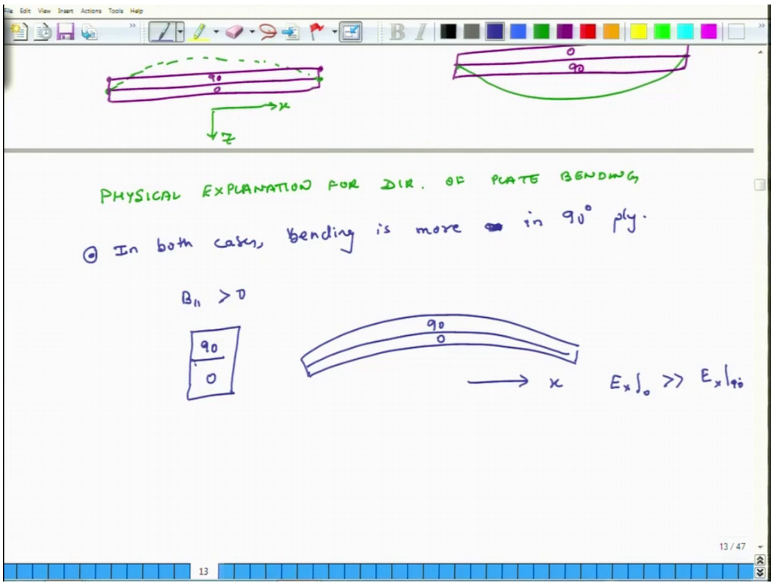

Physical explanation for direction of plate bending; in both the cases we note bending is more on our in 90-degree ply, right? It is more in 90-degree ply; why why do I say that? Consider this case. So, consider case B 1 1 is more than 0. B 1 1 is more than 0 the plate configuration is like this. You have 90 degree on top and 0 on bottom, and when it bends it is bending like this.

So, 90 is on top and 0 is on the bottom; which means there is more stretching happening on the 19 in the 90-degree play, right. Because it hasn't and there is less head stretching happening in the 0-degree play. And that makes sense because 0-degree play is stiffer in the x direction. It is E L in the E X; E X for 0-degree ply is very large compared to E X for 90-degree ply, ok. So, it stretches in the so it tries to stretch less and 90-degree ply tries to stretch more because it is more compliant.

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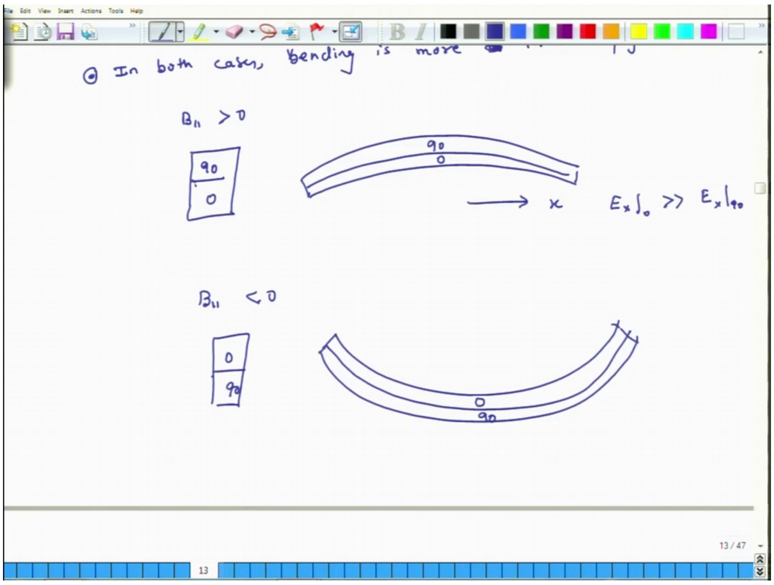

And the same thing happens when B 1 1 is less than 0, what happens? The order of the lamination sequence it reverses. So, in this case, the plate bends like this.

So, once again 0-degree ply is on the inside and 90-degree ply is on the outside. Once again here also the 90-degree ply stretches more than 0-degree ply stretches less. So, this is the physical reason why B 1 1 influences the direction, because it provides some kind of preference to less stiff plys, and less preference to more stiff plys. So, that is the role of B 1 1. So, this is what I wanted to discuss today. Tomorrow onwards which is the next 2 lectures, we will venture into a new area of this course. And we will start discussing thermal stresses and thermal strains in context of laminated composite plates.

So, we learn how to handle thermal stresses and thermal strains. And whatever we are going to discuss tomorrow and maybe a day after tomorrow onwards, that is not only applicable for composite plates. But it is also going to be applicable for plates with isotropic materials, if they are made up of several layers and each layer is having a different material. So, that is what we plan to start discussing tomorrow onwards.

Thank you and I look forward to seeing you tomorrow. Bye.