Advanced Composites
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Lecture – 42
Thermal Effects in Composite Laminates- (Part II)

Hello, welcome to Advanced Composites. Today is the last day of the 7th week of this course. And yesterday we just started a new topic related to Thermal Effects in Composite Laminated plates. And in that context, we have developed several equations or actually we have gone ahead and modified several sets of equations to incorporate temperature effects.

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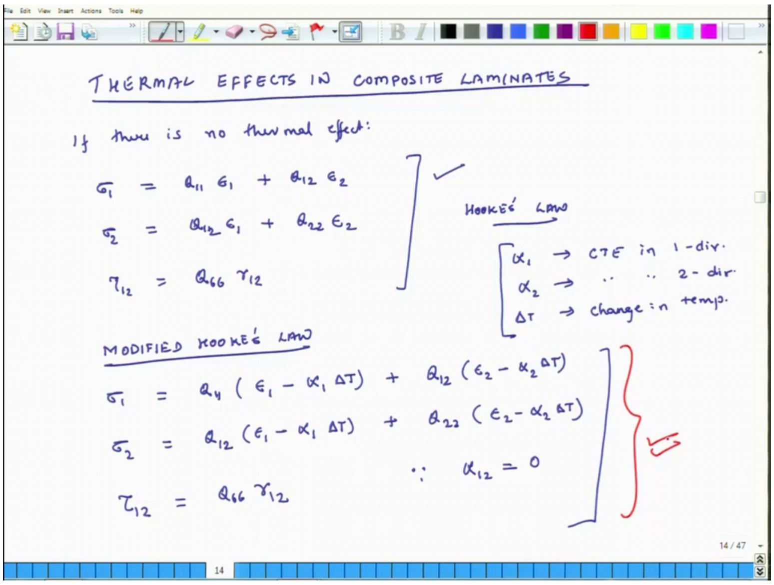

So, the first modification we did was that if there is thermal effect, then the Hooke's law relations which are these, they get modified to these side sort of relations. And here alpha 1 and alpha 2 are coefficient of thermal expansions of the material in 1 and 2 directions. And these modified relations relate stresses to strains in the system.

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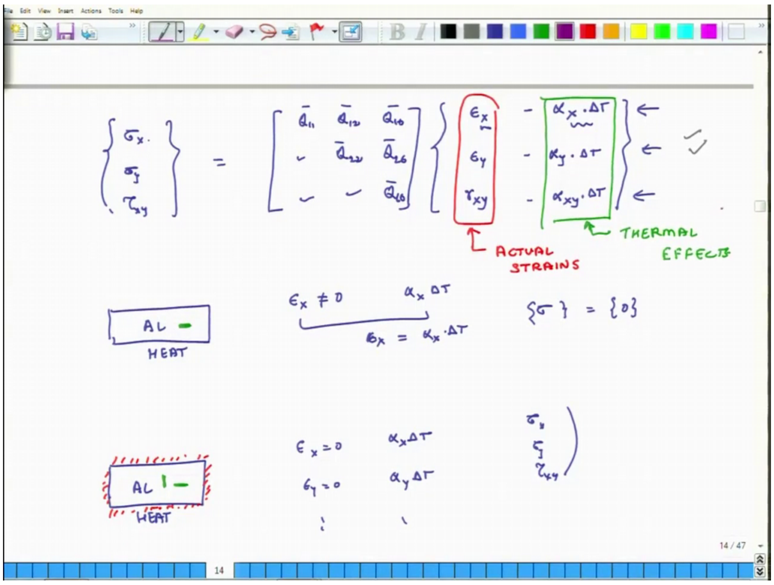

And then we made these relations more general, and we said that sigma x, sigma y, tau x y are related to the strains from based on these relations. And here the important thing to remember is that epsilon x, epsilon y, and gamma x y, they are the actual strains, which we may be able to measure, if by putting strain gauges, so their actual physical measurable strains. While alpha x, alpha y, and alpha x y times delta T, they are mathematical entities.

So, what I was saying is that the relationship between the stresses sigma x, sigma y and tau x y and the strains gets modified in the xy coordinate system through these relations. And here the strains which are encircled in the red box that is epsilon x, epsilon y, gamma x y, they are the actual measurable physical strains in the system, while alpha x delta T, alpha y delta T, and alpha x y delta T, they are thermal strains. And basically they are mathematical entities to account for thermal effects present in the composite laminates. So, this is the first set of changes, we have to accommodate to account for thermal effects. And these changes appear in the stress strain relations that is the first set of changes.

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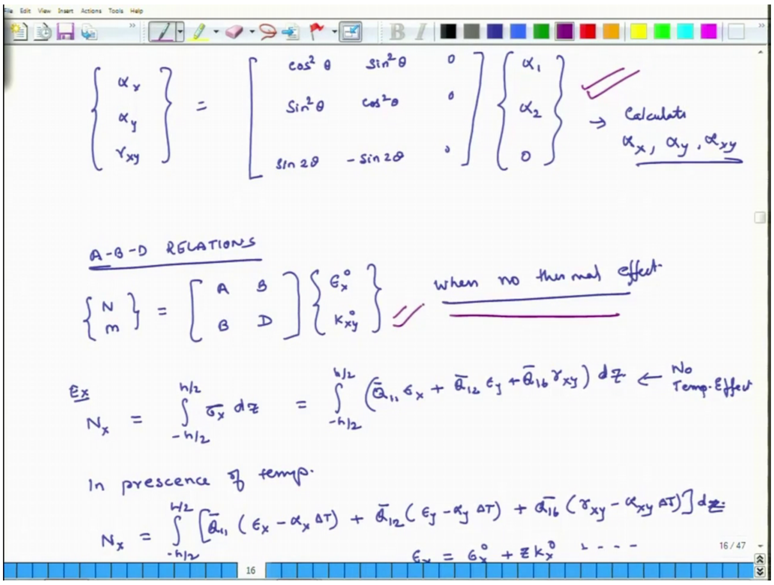

The next set of changes which we had discussed, and before we go to the next set of changes. How do we calculate alpha x and alpha y, gamma x y, it is if alpha 1 and alpha 2 which are the coefficients of thermal expansion in 1 and 2 directions of the material, they are known, then we can compute alpha x, alpha y, and gamma x y through these relations. So, we can calculate that.

So, what I mentioned earlier is that this is the first set of changes we have to account for, when we are interested in learning about thermal effects. The next set of changes appear in the relations between N and M resultants that is the moment and force resultants, and the mid plane strains and mid plane curvatures. When there are no thermal effects, then this is the governing relation.

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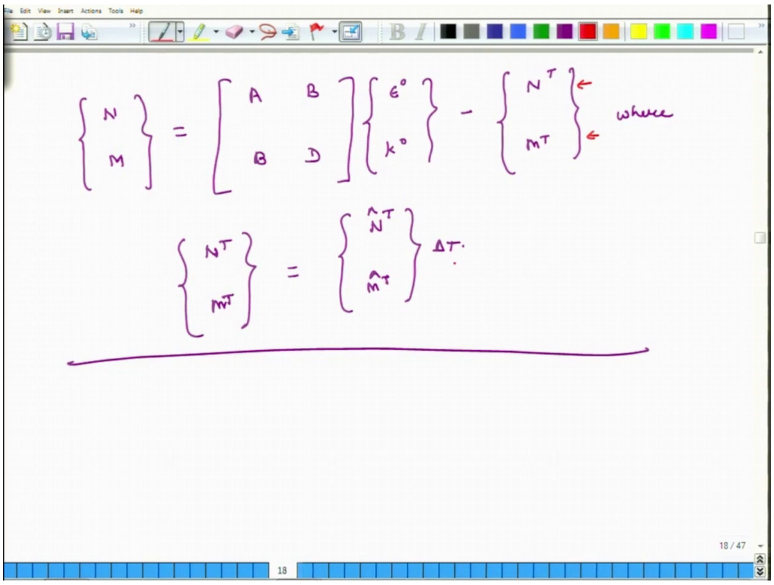

But the moment we have thermal effects the relations between N, M and mid plane strain and mid plane curvatures, they get modified. And these are the modified relations, which account for thermal effects so, this is there. And to recap, so what we have is two additional terms N T and m T. And these and terms N T and m T are account for the temperature effects, and they are nothing but multiples of N T and hat T and delta T, and M hat T and delta T.

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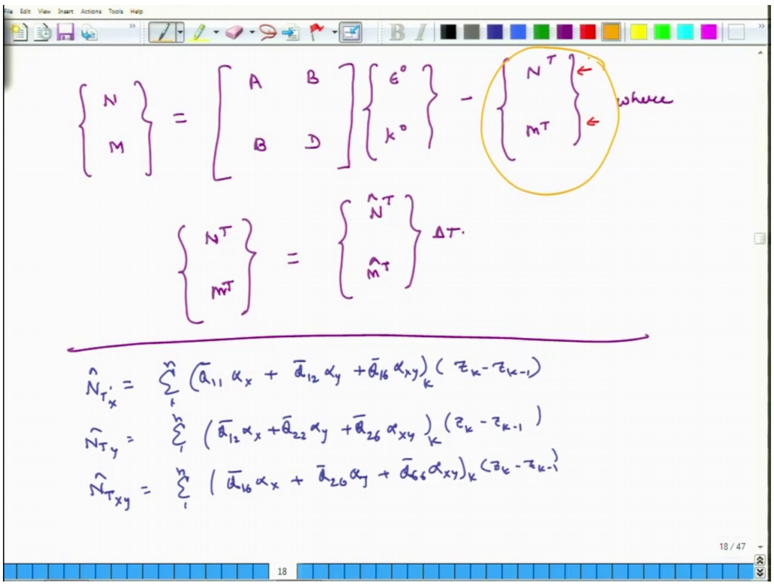

And for purposes of completeness, we will define the how we compute N T, so N hat T, and this we had developed the relation earlier, but I will just recap. So, this equals, so this is N T, and let us say this is in the x direction, then it is Q 1 1 alpha x plus Q 1 2 alpha y, and actually these are bars plus Q 1 6 alpha x y for the k th layer multiplied by the thickness of k th layer which is z k minus z k minus 1. And likewise we can also compute relations for N T y, and N and T x y. So, N T y is equal to this is again z k minus z k minus 1, and these relations are Q 1 2 alpha x plus Q 2 2 alpha y plus Q 2 6 alpha x y for k th layer, and once again these are bars.

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And in for the cross component, these are Q 1 6 alpha x plus Q 2 6 alpha y plus Q 6 6 alpha x y for k th layer times z k minus z k minus 1, and once again these are bars. And the summation is always from 1 to N layers. There are N layers, so 1 to N, 1 to N and 1 to N layers. So, this is how we define N T hats.

And I will just write one expression for M T x, and using analysis you can have expressions for M T y and M T x y also. So, M T hat in the x direction is equal to and excuse me summation from 1 to N Q 1 1 epsilon x plus Q 1 2 bar epsilon y plus Q 1 6 bar not epsilon alpha x alpha y and alpha x y. And instead of thickness, we have z k square minus z k minus 1 square divided by 2. And similarly, we can develop expressions for M T y and M T z. So, this is the second change that the relations between N, M and A B D matrices, they change, and they get modified by the presence of these terms.

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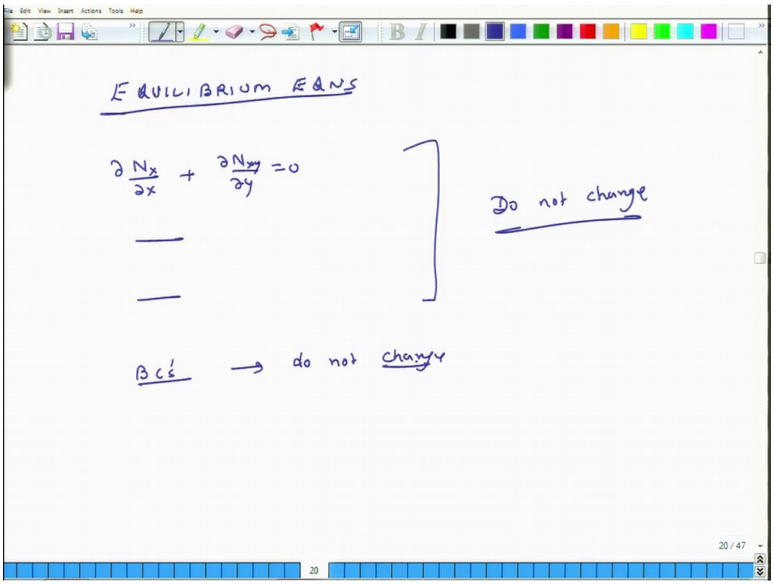

Then the next one is equilibrium equations. And what we are talking about is when we have when we are talking about the equilibrium equations, so the first equilibrium equation is del N x over del x plus del N x y over del y equals 0. And then we have second equation and third equation.

When we are talking about these equilibrium equations, these do not change, because what these equations imply is summation of forces in the x direction is 0, in the y direction is 0, in the z direction is 0, and summation of moments around y axis, x axis, and z axis are 0. So, these three equations reflect these different equilibrium conditions. So, these do not change, because these are we are talking about forces, so they do not change.

And the boundary conditions, they also do not change. So, the only thing which changes is these relations and the relation between stresses and strains. So, if we ground for these changes, then we can accommodate thermal effects relatively easily. So, now what we are going to do is we will actually solve a problem, so that we learned through an example how these type of problems are handled.

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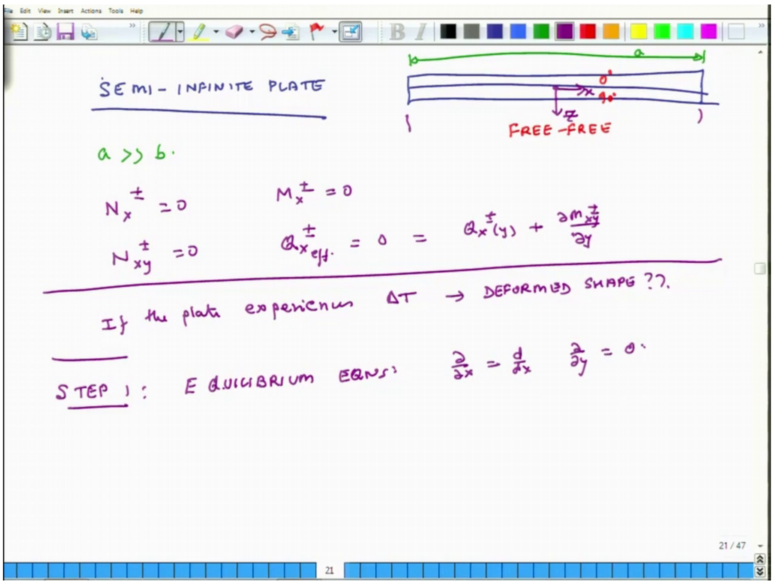

So, again we will have a semi-infinite plate, we have a semi infinite plate. So, let us say the top layer is 0 degrees, and the bottom layer is 90 degrees and it is a free-free plate. So, it is a free-free plate, meaning that there are no restrictions on any of the edges of the plate. And the third thing is that this length is a, and the width of this plate is very small compared to a, which is the length of the plate. The coordinate system as we have always done is located at the center, and the z axis is pointing downwards.

So, given that this is a free-free plate, what are the boundary conditions? So, if you go back and look at the boundary conditions, each edge has to have four boundary conditions. And because this is semi-infinite plate, we really care about this edge, and this edge. So, what are the boundary conditions? The first one is that N x at both the edges is 0, because it is free-free, so there are no external forces acting upon the system. Then N x y for both the edges is again 0, then there are no moments external moments at the edges. So, M x plus at both the edges is again 0.

And the fourth one is that Q x effective. And the definition of Q x effective is what, so at plus edge and minus edge is also 0. And we had defined what is Q x effective, it was Q x on the edge plus partial of m x y with respect to y, and Q x is a function of y. So, all these entities on positive edge and all negative edge they are 0 anyway. So, these are the things. So, these are the four boundary conditions.

So, now the question is if the plate experiences a temperature change of delta T, we have to find out its deformed shape. How do we find its deformed shape? So, we have to essentially find u v and w for the plate, once it has been subjected to a temperature of delta T. So, the 1st step is we do equilibrium equations. And when we are doing these equilibrium equations, we realize that del over del x is equal to d over d x and del over del y equals 0, because the plate is very long compared to its width.

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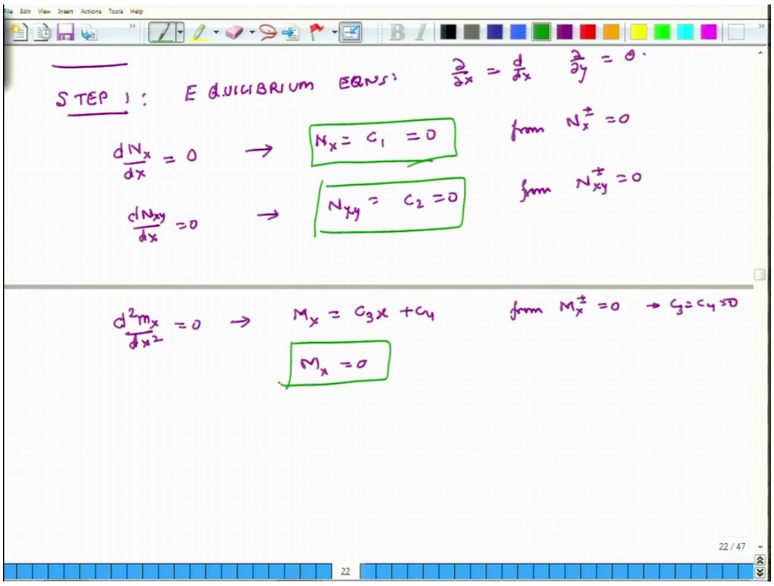

So, we say the first equation is d N x over d x is equal to 0, this is from the first equilibrium equation. And this gives us N x is equal to C 1 right, but the plate is not being subjected to any external stresses. So, at x is equal to plus a by 2 or minus a by 2, this value should be 0. So, this is equal to 0 from (Refer Time: 14:18). The second equation is d N x y over d x is equal to 0. So, this gives us N y no N x y is equal to C 2. And again from the boundary conditions, we get C 2 is also equal to 0.

The third equation is d 2 m x over d x square equals 0. So, this gives us M x is equal to C 3 plus C 4, C 3 x plus C 4, but once again from M x plus and minus is equal to 0, we get C 3 is equal to C 4 is equal to 0. So, M x is equal to 0. And so what it means is that when a plate of this type un symmetric laminate, if it un symmetric laminate. If it is subjected to a temperature change, there will be no N x or N xy or M x inside the plate, because this is what it is telling us, and it makes sense because the plate is free. So, it does not experience any internal stresses, because it is free to move. So, it will assume whatever deformed shape it has, and that is what these equations tell us.

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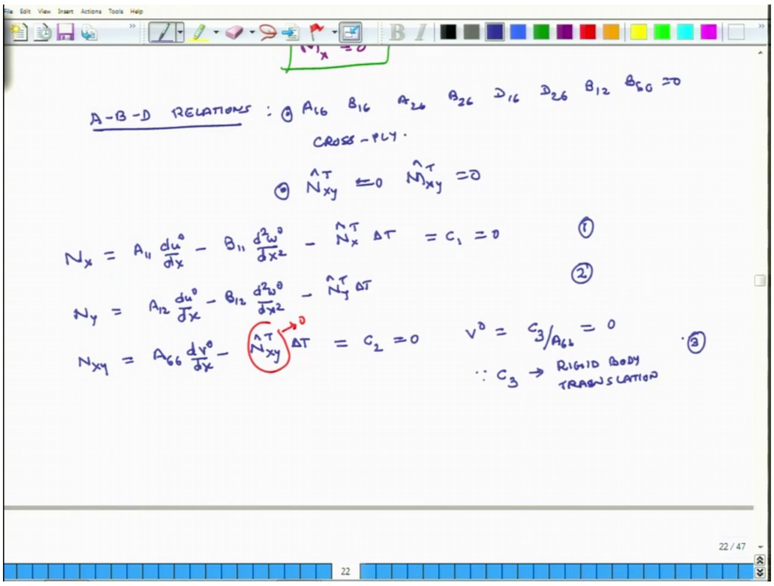

So, now our aim is to compute the displacements in the plate. And what we will do is we will use these relations and express this these relations in terms of u, v, and w, and then compute the and find displacement functions. So, because so but before we do that, we realize that so we use the A-B-D relations for N x, N y, and M x. See these are the three things, we computed N x, N x y, and M x. So, we expand N x, N x y, and M x in terms of u, v, w and delta T.

But, before we do that we note that in A-B-D relations, A 16, B 16, A 26, B 26, D 16, D 26 B 12 and B 66 are 0, because it is a cross ply, cross plane wait a minute. Also, so this is one thing, also we note that N x y T hat is equal to 0 and M x y T hat is equal to 0. And why is it 0, because we look at the definition. What is N x y? N x y is Q 16 bar plus Q 26 bar alpha y plus Q 66 bar alpha x y.

Now, for a cross ply laminate, all the plies are either 0 degrees or 90 degrees. So, Q 16 bar is 0, Q 26 bar is 0 for a cross ply laminate. So, first two terms go away. And because the ply laminate is cross ply alpha x y also for each layer is 0, because the layer is either 0 degrees or 90 degrees. If you remember, in the very beginning we said that if we are material axis is aligned, alpha 12 for a you know for this material axis system is 0, so because of these reasons both N x y hat and M x y hat the temperature components they are 0.

So, now we write an expression for N x, so N x is equal to A 11 d u naught over d x minus B 11 d 2 w naught over d x square. So, these are the terms, we have already developed for in case e and c and d right. But, now we will have an additional term related to temperature, so that is minus N x times delta T T in this N x hat. And so this is equal to C 1, and this is equal to 0 or actually let us call it 1. Then N y is equal to A 12 d u naught over d x minus B 12 d 2 w naught over d x square minus N y T delta T this is hat is equal to yes, so this is there. So, this is equation 2.

And N x y is equal to A 66 d v naught over d x minus N x y T times delta T and this is hat, but we said that this thing is 0. So, this is and this entire thing equals C 2, and this is equal to 0. So, essentially what it means is that v naught is equal to C 3 v naught is equal to C 3, because when I integrate this equation v naught becomes v naught is equal to C 3 or C 3 by A 66 C 3 by A 66. Now, what does this mean? What it means is that the plate has the same it could have the same displacement at all points in it, because it is a constant thing.

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So, what that means is that if I am just moving my plate, suppose the plate is just moving at some uniform velocity uniform velocity, then it has some same displacement you know. So, not uniform velocity, it is just a question of the rigid body motion rigid body motion. So, I can as well assume C 3 represents rigid body motion. So, suppose the plate is initially here and it just moves like this. So, it is a rigid body motion, but if I it just moves as a rigid body nothing changes in the plate right; nothing changes at plate.

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So, the state of the plate in this case or this case is the same, so I can as well assume that this is equal 0, because C 3 represents rigid body translation.

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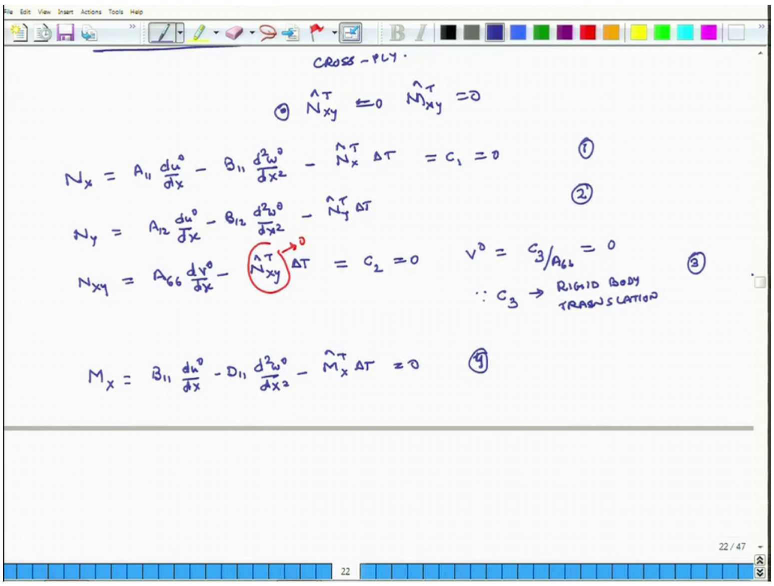

So, we can just consider that rigid body translation is 0. So, these are the three equations related to N's. And then let us look at one more equation M. So, M x is equal to B 11 d u naught over d x minus D 11 d 2 w naught over d x square minus M x T times delta T equals 0. So, this is equation 4. So, these are the four equations, and then we can also develop expressions for M y and M x y.

And what we will do next is we will use these equations, because these are equations in u, w and v, we will use these equations to compute u and w, v. We have already computed, and we have found that v is 0. So, we will likewise similarly compute expressions for u naught and w naught. And then we will like to look at the behavior, and how these relations affect the shape of the plate, so that is what we plan to do going forward. Today's time is over, so we will continue this discussion in the next class. So, till then have a great weekend bye.