Advanced Composites
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Lecture – 43
Thermal Effects in Composite Laminates (Part-3)

Hello, welcome to Advanced Composites. Today is the start of the 8th week of this course. And over this week we will continue our discussion in the context of solutions for these partial differential equations, which were generated to represent equilibrium of present laminated composite plates.

While last week we have focused exclusively on semi infinite plates and their solutions. And at the close of the last week we started touching up on the issue of thermal stresses and thermal strains, because these types of stresses and strains are very important specially in context of laminated composite plates because, they are associated with the manufacturing of these plates.

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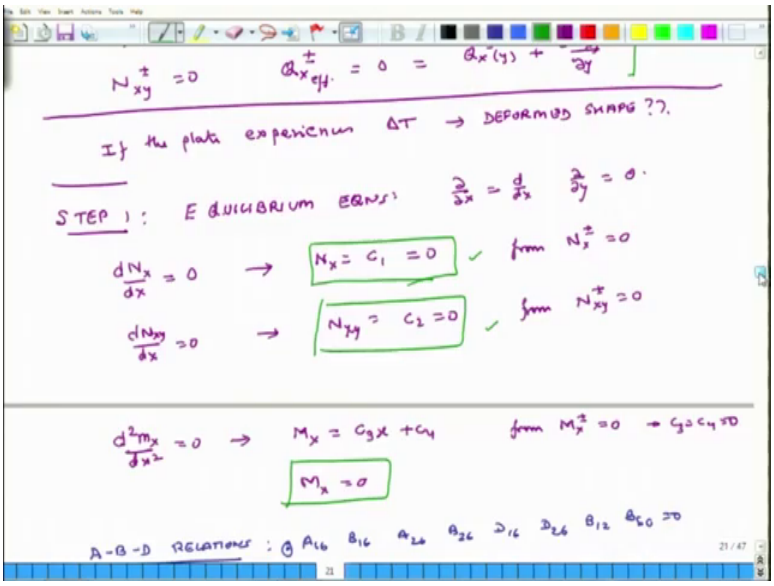

So, in that context we had developed equations of equilibrium and also modified ABD relations for these types of plates which have inbuilt thermal stresses and strains; and then we started to solve a particular problem related to semi infinite plates so that it helps us understand the issues at hand better in context of thermal stresses. So, the problem which we were discussing was this one. And here we have an infinite semi infinitely long beam. The dimension in the length direction is a, the dimension in the other direction that is it is with b.

And we have assumed that a is extremely large compared to be and the plate is free to move so it is not constrained on either of the edges. So, because it is free to move the boundary conditions associated with such a plate are given here that on both the edges x is equal to plus minus a over to the external value of N x that is N x plus or N x minus is 0 also N xy N x and N x plus is 0 and also bits because it is not seen any external moments or shear forces. So, M x is 0 and also Q x effect is 0.

So, for these boundary conditions there will be proceeded to solve the governing differential equations. From the first differential equation which represents equilibrium of forces in the x direction; we got N x is equal to C 1 equals 0; from the second governing equation we got N xy equals C 2 equals 0. So, these are the two governing integrations of the first two other governing equations.

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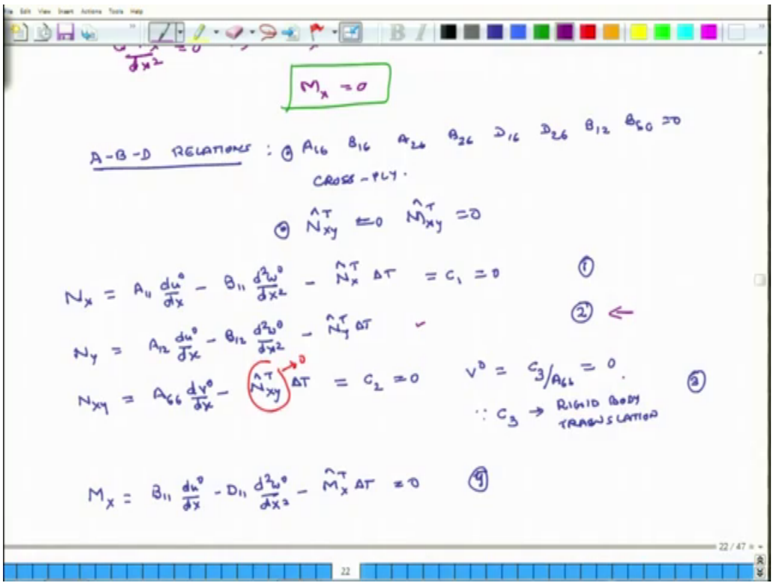

And from the third governing differential equation which represents equilibrium of forces in z direction as well as equilibrium of moments along x and y axis. We got d 2 M x over dx square by equals 0, and then we integrate that we got M x equals C 3 x plus C 4 and because M x is equal to 0 on both the edges of the plate at x is equal to plus minus 2, plus minus a over 2. The values of integration constants C 3 and C 4, they turn out to be 0. So, M x is equal to 0. So, in overall sense N x equals 0, N xy equals 0 and M x equals 0; and that makes sense also because the plate which we are discussing is not seeing any external forces and it is free to move. So, we should not expect any internal forces and moments in the plate.

So, once these equilibrium equations were integrated, then we proceeded to linking these values of N x, N xy, M x, M xy and so on and so forth with mid plane strains and curvatures. And through mid planes strains and mid planes curvatures to mid plane displacements u, v and w naught. So, what we had done in the last class was that we integrated the we stated that N x equals A 1 1 d u naught over dx minus B 1 1 bw over dx square minus N x T hat times delta T equals C 1. Similarly, we also developed an expression for an N y and N xy.

And from the second equation this one is said that I am sorry from third equations for N xy, we said we came up with an expression that V naught equals C 3 over A 6 6 and that equals to be 0. And then we were working on the forth equation for M x, so that states M x equals B 1 1 d u naught over dx minus D 1 1 d w second derivative of w with respect to x minus M T in the x direction hat times delta T. And now what will do is we will write down the other two relations for M xy and M y.

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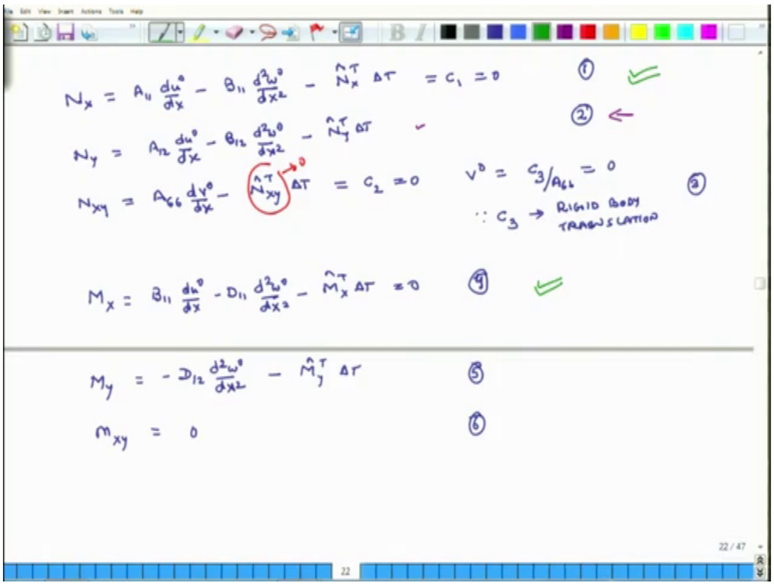

So, M y equals minus D 1 2 d 2 w naught over dx square because why is it that because all the b s associate terms for the relation of M y, they involve what D 1 2 which is 0, D 1 6 which is 0, and B 6 6 which is 0. So, B terms do not come into the picture and for M y if the D terms associated are D 1 2 so that is non 0 and so is d 2 w over dx square. And then the next term would be D 2 2 times curvature in y direction which is 0 because the plate is semi infinitely long in x direction. And the third term will be D 1 6 times k x y, so D 1 6 is 0. So, this is the only term which is left. And the and of course, there is a temperature related term, so that is M y hat T times delta T. And the sixth equation is M xy equals and when we expanded we find that either in all the terms equals 0, so this is equal to 0. So, these are the 6 equations.

Now, our aim is to find expressions for u naught, v naught and w naught. V naught we have already showed that out that values works out to be exactly 0 throughout the length of the plate. So, for u naught and v naught we consider equations 1. So, when we observe equation 1, it has u naught and w naught and everything else in the equation is known, A 1 1 is known, B 1 1 is known, M x T we can compute because we know the lamination sequence and alpha 1 and alpha 2 of the material and delta T is also known.

So, the first equation involves only u naught and w naught. And the fourth equation is same is true for fourth equation also. So, essentially what we will do is we will integrate these equations with respect to x 1. So, we will get an expression in u and w dw over dx and then solve for u naught and dw dx.

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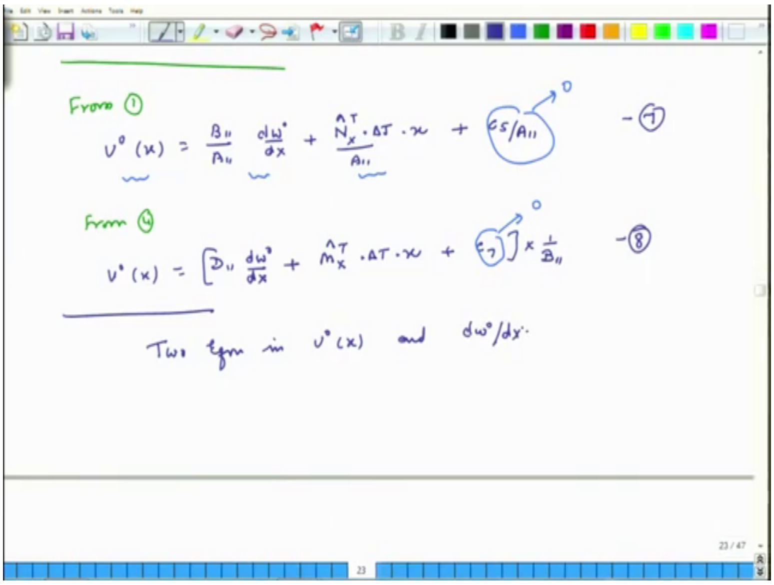

So, we get from 1 we get u naught x is equal to B 1 1 over A 1 1 dw naught over dx plus N x T hat times delta T times x because that I have integrated it once. And then I get an integrated integration constant, so that is C 5 over A 1 1. The other thing is so from likewise from 4, we get another expression for u naught and that gives us D 1 1 d w naught over dx oh so they should be here also A 1 1. And from the fourth equation, what we get is D 1 1 times dw over dx plus this is the equation we are considering. So, M x T delta T M x T times hat delta T times x plus another integration constant C 7 and this entire thing gets multiplied by 1 over B 1 1. So, this is equation 7 and this is equation 8.

Now, consider the case that if delta T is 0, if delta T is 0 then the plate should not exhibit any u naught. If delta T, if temperature is 0, the only reason plates configuration is going to change it is free, it has not any external forces, so the only reason plate is going to change it is configuration and exhibit u v and w is because of temperature difference. So, in case delta T is 0 then u naught should be 0 also the plate should not exhibit any curvature, so dw not over dx should be 0.

So, essentially what am trying to say is that this thing will be 0, this term will be 0 and because delta T is 0 this should be 0. So, the own an which implies that C 5 again represents rigid body motion and we can set it as 0. So, we said this as 0. Similarly, we look at equation four and we see that and we again do the same thought experiment that when delta T is 0, u naught should be 0, dw over dx should be 0 so and the only nonzero term left is c seven. So, for this equation to be satisfied this also has to be set to 0 because C 7 and c C 7 and C 5 represent rigid body translation of the plate.

So, once we said these to be 0 what we see is that these modified equations after they are said to be 0 or equations in u naught and dw over dx. So, we have two equations in u naught x and dw naught over dx. These are simultaneously equation simultaneous equations. So, we can solve for u naught and dw over dx.

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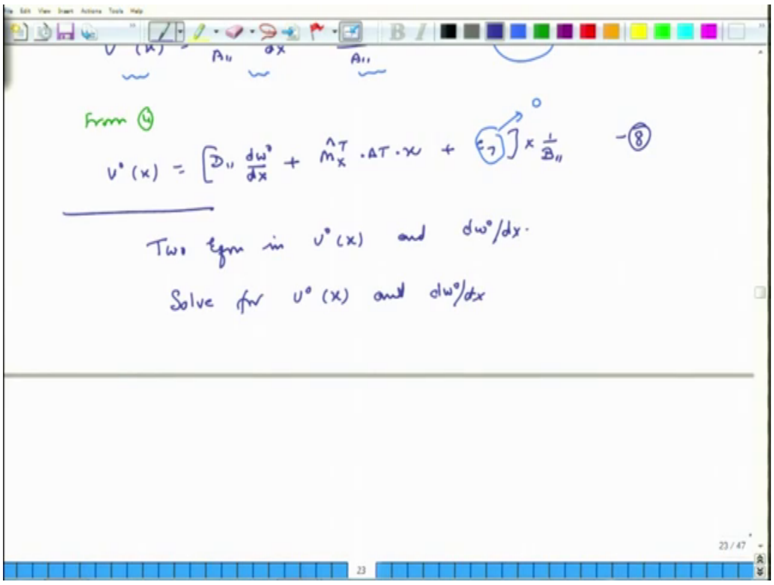

So, we solve for u naught x and dw naught over dx. So, when we solve for this is what we get.

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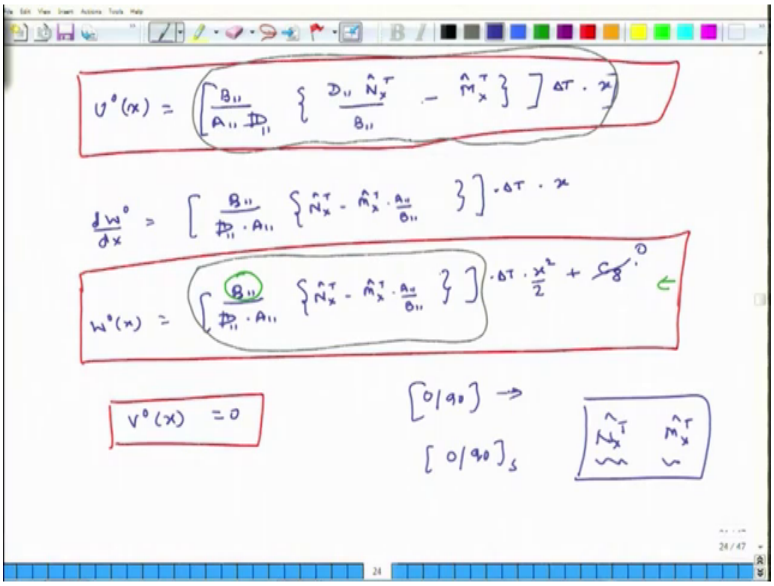

U naught x equals B 1 1 over A 1 1 script D 1 1 times D 1 1 N x hat t over A 1 1 minus M x t hat this entire thing multiplied by delta T x. And dw over d x works out to be 1 over D 1 1 times so that is times delta T times x, so that is the slope of the plate at any given point. So, I integrate so this is so this is one important equation. And this is the equation the next equation is for slope. And if I integrate this equation for slope that is dw d x, I get w naught as a function of x is equal to this entire expression times delta T times x square over 2 plus another integration constant C 8.

So, once again what is how do we find out the value of C 8, C 8, we again think of a scenario that when delta T is 0, then the plane should not bend at all and that can happen if C 8 is 0. So, this is 0. And C 8 represents rigid body motion of the plate; it does not represent any deformation of the body. So, this is the expression for u naught, then we also have an expression for v naught and then so v naught x is equal to 0. So, these are the relations.

Now, we will discuss these relations for a couple of minutes, and then move onto the next topic. So, consider these and think about a scenario that when delta T is 0, when delta T is 0 what does the equation tell us when delta T is 0 then all these terms they may not be 0. But if delta T is 0 then all these terms are not 0, but they get multiplied by delta T, so u naught is 0. So, when there is no temperature change then there is no u naught and which makes sense. Similarly the same is true for the second case also, because the entire solution is a multiple of delta T, so that is one thing to understand.

The second thing I would like to state is that suppose the plate was symmetric suppose the plate was in this case we said that the plate is un symmetric. So, it is 0, 90. And because of this type of plate because of this type of plate 0 90 the plate will tend to bend when I heat it. It is like a bimetallic plate the side glue and piece of aluminum steel together and when I heat it, it will try to bend it is bimetallic because different materials have different coefficients of thermal expansion in different directions. So, this kind of plate will bend.

Suppose the plate was symmetric let us say it is 0 90 symmetric, and then we heat it, then we would not expect the plate to bend, but delta T will be non-zero, but then what will make the plate not bend. The reason in that case will be that when we compute the values of N x T, and M x T these values they will work out to be 0, when we compute these values. Because what are the definitions of these things, so if we compute these things we will find that these terms will work out to be 0. And once that happens so what was I was saying this that this delta T as it becomes 0 the plates do not exhibit any deformation.

The other thing is what happens when the plate is symmetric and if delta T is non-zero, then what would happen. So, in that case we should look at these relations for u naught and w naught, so when the plate is symmetric in both the expressions for u and w we have in the new replicator both the relation for u, the entire displacement function is multiplied by this term B 1 1 and when the plate is symmetric then B 1 1 will be 0, so u will be 0. Similarly, when the plate is symmetric B 1 1 is 0. So, when B 1 1 here is also an expression for w. So, again w naught will also be 0.

So, this again needs a physical requirement. And in this way very methodically and logically we can develop expressions for mid plane stresses and mid plane strains and thermal stresses and thermal strains in context of plates which are un symmetric and if they are treated to temperatures. We can also use the same approach to calculate internal stresses for even symmetric systems when they are subjected to temperatures, so that is what I wanted to discuss today. And tomorrow we will move onto our different topic and we will start looking at plates which are not necessarily semi infinite in dimensions. So, we will start approaching plates of those types and then we will see how they can be solved.

Thank you very much and have a great day.