Advanced Composites

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Lecture – 45
Different Boundary Conditions in Finite Rectangular Plate

Hello, welcome to Advanced Composites. Today is the 3rd day of the ongoing week, which is the 8th week of this course. Yesterday, we just started our discussion on finite plates and how we start approaching these plates in terms of their solutions. And the first step which we have mentioned is that we should try to simplify the extent of the problem, whenever we look at finite plates. Today, what we will do is we will look at these finite plates and explore their boundary conditions in more details for different types of n conditions.

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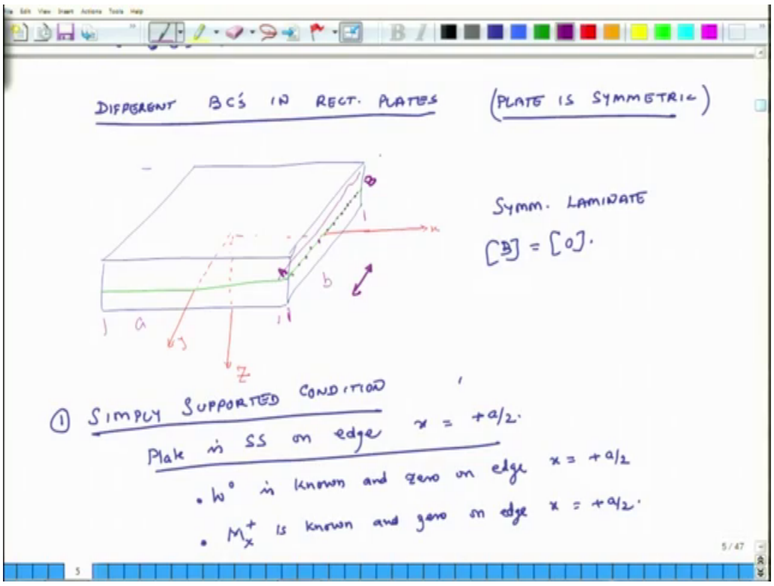

So, different BC's in symmetric in rectangular plates ok. And at least in the context of this course, we will assume that lamination sequence is symmetric. So, plate is symmetric ok, this is one assumption. So, first we will draw a picture and this is the mid plane of the plate ok. And let us say this is the x-axis, this is the y-axis and of course this is z-axis. So, the dimension of the plate in the x direction is a this dimension and this dimension is b in the y direction.

So, now we start looking at different boundary conditions and we have assumed that this is a symmetric laminate, which implies that for this plate B matrix is identically 0. So, first we will look at the first condition called simply supported condition and what it means simply supported. So, suppose we say that the plate is simply supported on edge x is equal to plus a over 2. So, what does it mean? It means that so this is the thing so, now we go back, and look at what kind of ok.

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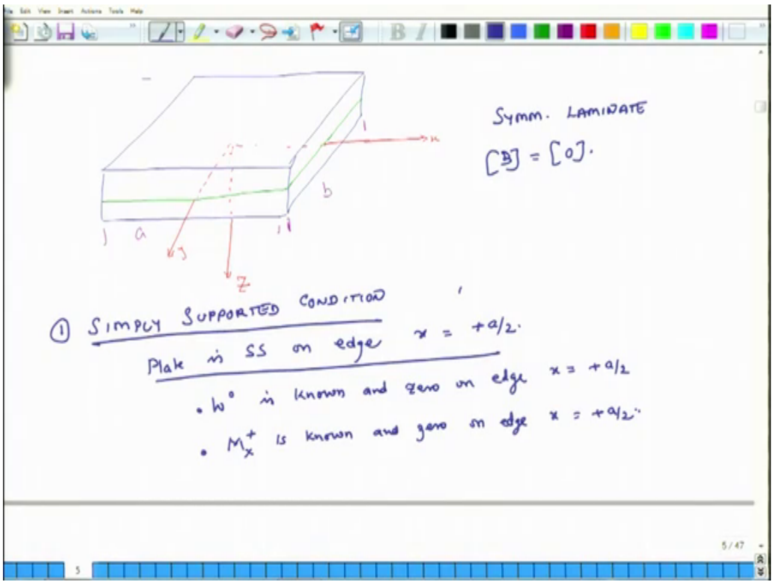

So, what it means is that w naught is known and zero on edge, x is equal to plus a over 2, so this is one thing. The second thing is the moment on this edge M x is known and zero on edge, x is equal to plus a over 2 that is what it means.

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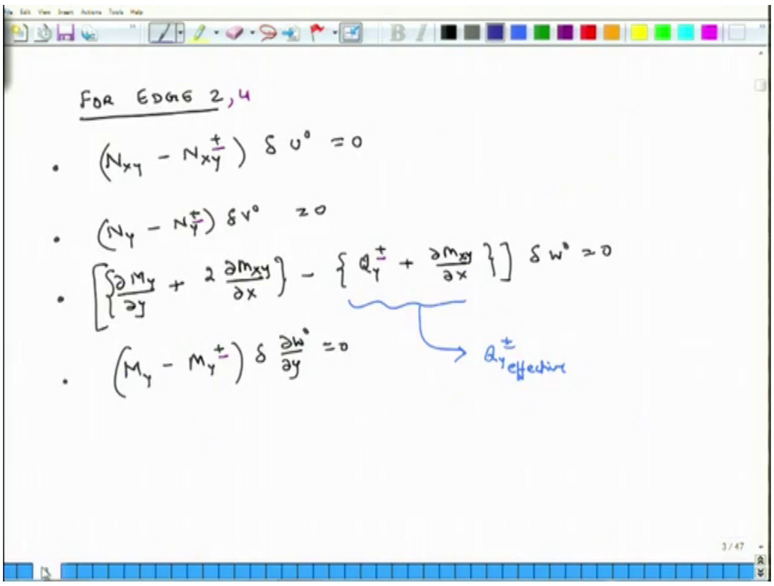

So, now let us go back and look at the boundary condition terms, we had said that each edge has four boundary conditions.

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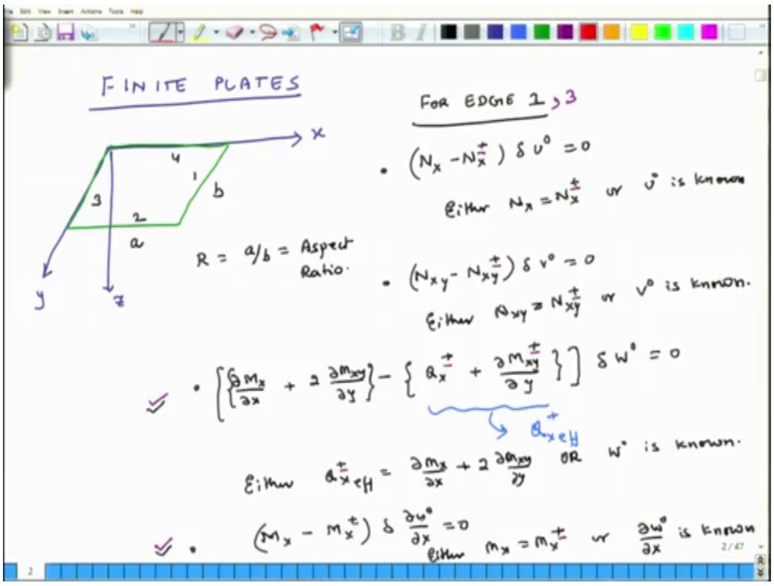

So, if you look at this, so we are talking about these three these two conditions. So, one of the from the third boundary condition equation, we say that either we know Q x effective or we should know w. So, in this case on that as w is known and because w is known its variation will be 0. The second condition is that M x is known or M x is equal to M x plus or the slope is known.

So, in this case we are saying that m x on the edges 0. Now, let us explore this further. So, when we say M x is 0, it means that M x equals 0 at all these points on the edge, because M x is a function it varies from point to point. But, on this entire mid plane at every single point M x is 0, it is not that it is integral is 0 M x is 0 at every single point.

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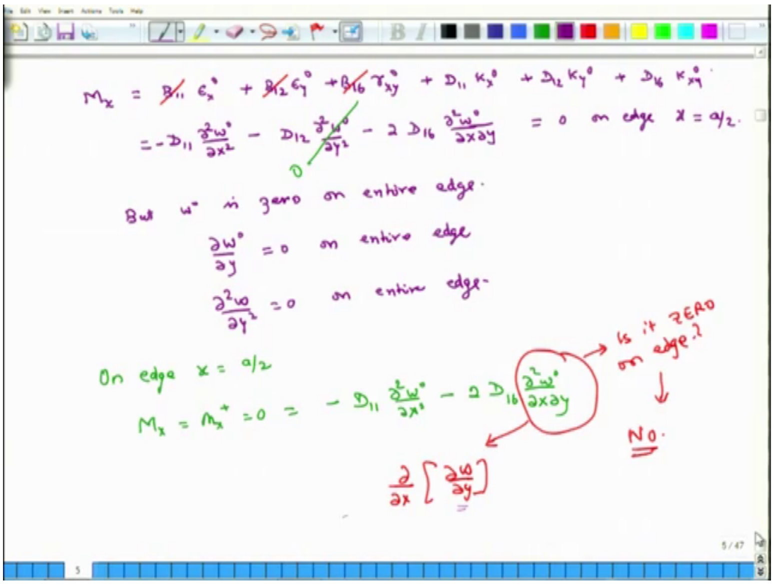

So, what is the expression for M x? So, we look at the second condition M x is equal to B 11 epsilon x naught plus B 12 epsilon y naught plus B 16 gamma x y naught plus D terms D 11 K x naught plus D 12 K y naught plus D 16 K x y naught. Now, since the plate is symmetric B 11 is 0, B 12 is 0, B 16 is 0 ok, because the plate is symmetric.

So, we expand this further, so this becomes D 11 and what is K x second derivative of w with respect to x, and it is negative. Similarly, D 12 del 2 w naught over del y square minus 2 D 16 del 2 w naught over del x del y ok. And if on the edge, so on edge this entire expression as a sum is so this is equal to 0 on edge, on edge what x is equal to which as we are talking about x is equal to plus a by 2 on edge x is equal to a over 2.

And we also know that w is 0 on this edge, so on this entire edge, so let us call this entire edge A, B on this entire edge A, B, w is 0. If w is 0, along this entire edge and what is this axis? This axis is y axis. And if w is 0 or at all the points on this edge, then it implies. So, this is there, so we will say that, but w naught is 0 on entire edge, which means that del w over del y equals 0 on entire edge right, because w is 0 on this entire edge. What is del w over del y that what is the change in w along the edge as I change y and as I am changing y, w is always 0 so, del w del y is 0.

And for the same reason del 2 w over del y square is equal to 0 on entire edge. So, if that is the case, what do we mean this term becomes 0; this D 12 related term becomes 0.

So, we say that on edge x is equal to a over 2. M x is equal to m x plus is equal to 0 and that equals minus D 11 del 2 w naught over del x square minus 2 D 12 del 2 w naught over del x x I am sorry this is should be D 16 del y. You may wonder whether this term on the edge is it zero on edge is it zero and the answer is not necessarily. Why is it not necessarily, so what is del w second derivative, so you can consider this as partial derivative with respect to x of del w over del y.

Now, del w over del y is 0 along this entire edge. But, consider another line segment, which runs parallel to this. The plate may if it is bending, here del w over del y may not be 0. And what does this mean that how is del w del y changing as I change x. So, when I change x, it may not longer be 0 ok. It may be del w, del y is 0 here on A, B, but it may not be zero on the other line parallel to it as I change x so, it does not necessarily mean that this thing is going to be 0.

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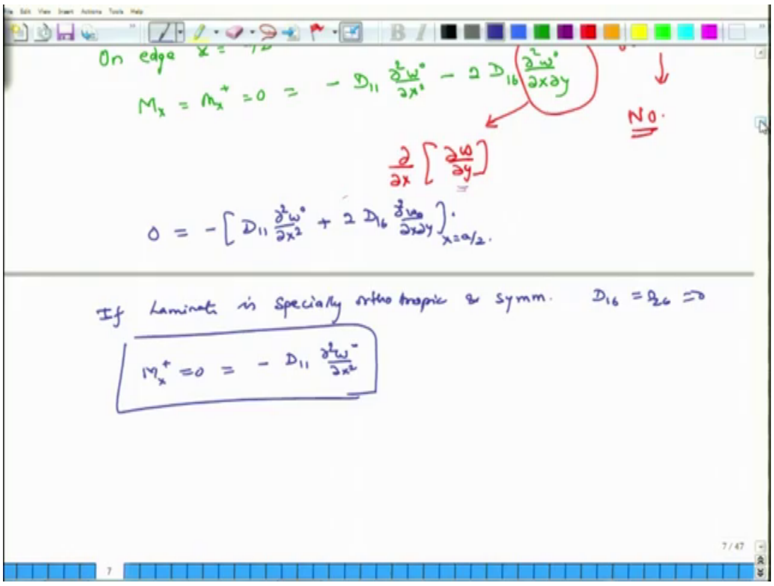

So, we cannot drop that point ok, so that is the condition. So, we will just quickly write this condition more explicitly, so which means that M x the condition for M x being equals to 0 is 0 equals minus D 1 1 del 2 w naught over del x square minus 2 D 1 6 del 2 w naught over del x del y at x is equal to a over 2. And if the laminate is especially orthotropic, so what does it mean? So, it is not only symmetric, but all the layers are either 0 degrees or 90 degrees. So, it is especially orthotropic and symmetric, then D 1 6 is equal to D 2 6 is also 0.

So, in that case M x plus equals 0, it implies that this D 6 is 0, so what I am left with is minus D 11 del 2 w over del x square is 0 ok. So, this is how we simplify the boundary condition. Similarly, if the on edge, suppose on the other edge x is 0, y is equal to b by 2 that is I am talking about this edge. If on this edge moment is 0, it means M y is 0 and we do the same analysis like that. And in that case, the important thing to be understood will be that if w is 0 and M y is 0, then del w del x will be 0 on that edge. And accordingly we make develop the relations for boundary conditions.

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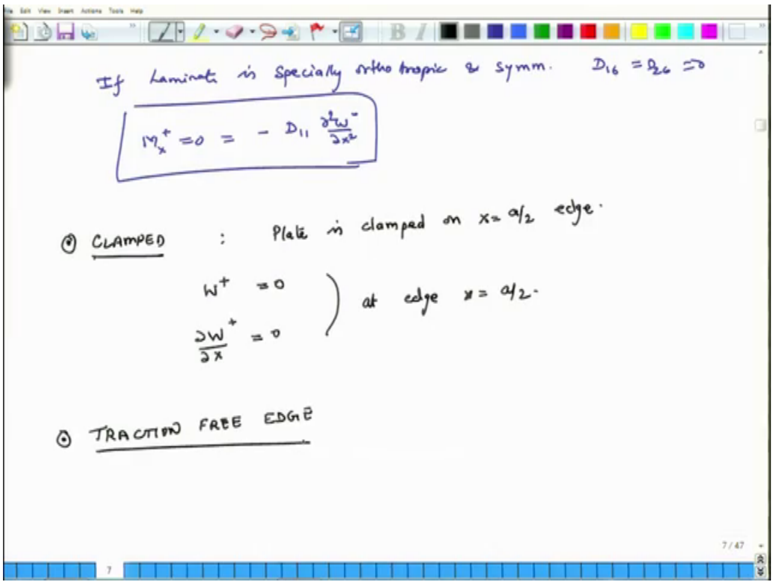

The second condition, which I would like to talk about is so we have discussed simply supported condition. The second condition is clamped condition clamped and for client condition, so let us say plate is clamped on x is equal to a over 2 edge. So, what that means is that w plus equals 0 and del w over del x is equal to 0 at edge x is equal to a over 2.

Once again, let us look at the summary of our boundary conditions. So, when a plate is clamped, we are saying that plate is clamped on this edge x is equal to plus a over 2; it means let us look at again these boundary conditions the 3rd and 4th boundary conditions, what do they say. The 3rd boundary condition says either w is known that is variation in w should be 0 or this entire thing in the bracket qa Q x effective should be known.

So, when it is clamped condition, we say that w is prescribed and it is value 0 so, variation in w is 0. And then the 4th boundary condition says that either the slope of the plate should be known at x is equal to a over 2 or I should know the moment which is being applied. So, in our case it is a clamped condition, so we do not know what is the moment, which is getting generated in the clamp, but the slope is 0 so, if the slope is 0, then that is relates to the 2nd condition, so that is there.

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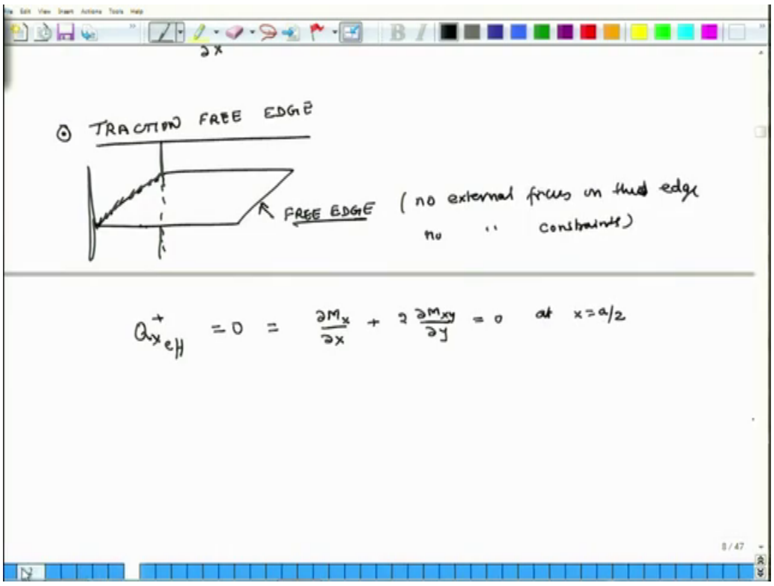

And the 3rd condition is called traction free edge so; these are the three important boundary conditions. There could be more complicated boundary conditions, but this is these are the important boundary game. What is attraction free edge? So suppose I have a plate and let us say this so the plate is let us say clamped here, but this edge has been free, this is free edge. And when I say free edge it means that, it is not seeing any no external forces on the edge and no external constraints. So, it is free to move based on the dynamics of the system so, this is the traction free condition ok.

So, let us look at again the 3rd and 4th boundary conditions. So, what does it mean, let us look at the 3rd boundary condition. It says that either we should know Q x effective or we should know the w do we know w on that edge? We do not know the value of w. So, we cannot say that variation in w is 0, but the edge is not seeing any external forces, which means that the value of Q x effective on that edge is 0. So, in a traction free edge the first condition is Q x effective is equal to 0 and that means del M x over del x plus 2 del M x y over del y should be equal to 0 at x is equal to a by 2, because that is how we have defined the q f Q x effective ok so, this is the relation.

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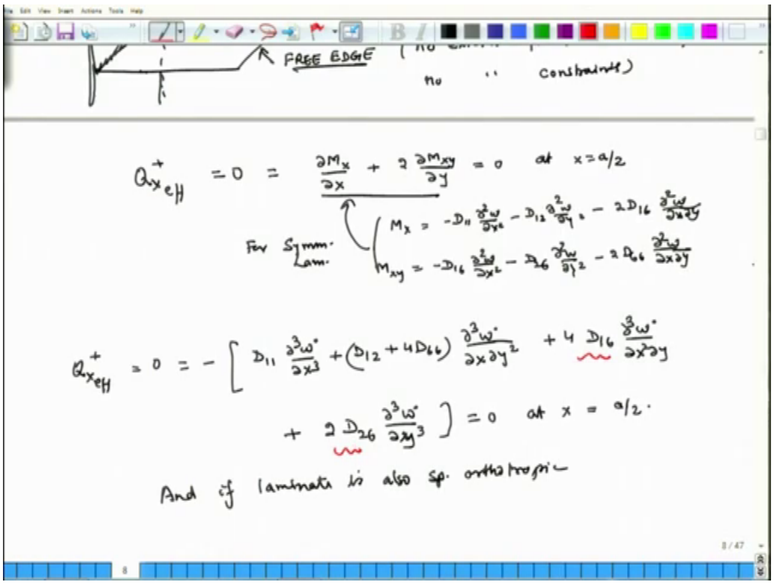

Now, if the laminate is symmetric, then M x what is M x? M x is equal to minus D 11 del 2 w over del x square minus D 1 2 del 2 w over del y square minus 2 D 1 6 del 2 w over del x del y ok. And what is M y no M x y. Because, we have to implement these things, so this is equal to minus D 1 6 del 2 w f over del x square minus D 2 6 del 2 w over del y square minus 2 D 6 6 del 2 w over del x del y so, this is again for a symmetric laminate.

So, if we put this in this equation, what we get is Q x effective is equal to 0 and that equals minus D 1 1 del 3 w over del x cube. So, these involved third derivatives, because I have to differentiate M x with respect to x plus D 12 plus 4 D 6 6 del 3 w over del x del y square minus 4 D 1 6 del 3 w over del x square del y actually this should be plus and plus 2 D 2 6 del 3 w over del x del y cube. And of course, these are all mid plane deflections so, this equals 0 at x is equal to a over 2. And if laminate is also especially orthotropic, then all these terms related to D 1 6 and D 2 6 they go away so, then this boundary condition becomes even simpler ok.

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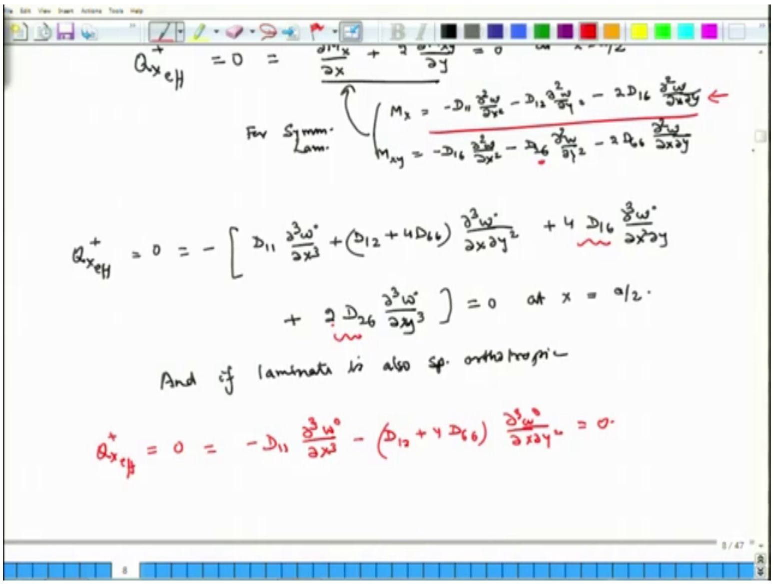

So, then what happens is that Q x effective is equal to 0 is equal to minus D 11 del 3 w naught with respect to x cube minus D 1 2 plus 4 D 6 6 del 3 w naught over del x del y square equals 0 ok. So, this is 1 boundary condition and the other boundary condition is that M x should be 0, the 4 boundary condition says either you should know the slope or you should know the moment on the edge. So, what is the moment on the edge on that on a traction free edge it has no external forces or no external moments.

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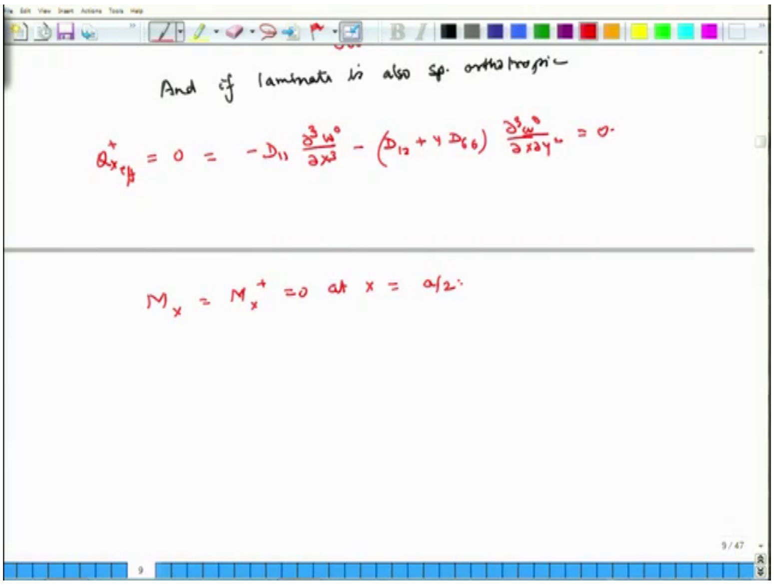

So, the other boundary condition is M x is equal to M x plus is equal to 0 at x is equal to a over 2. And once again our definition for M x is this thing, so this entire expression at x is equal to a over 2, it should be 0. So, we have discussed three types of boundary conditions, we can have more complicated boundary conditions also for semi infinite plates, where most of times we run into these types of boundary conditions either simply supported or clamped or traction free.

And in these cases, we have discussed how to handle the boundary conditions. So, tomorrow we will continue this discussion on in the info on finite plates. And we will actually start solving actual problems and see how they get handled, so that is pretty much it for today and I look forward to seeing you tomorrow.

Thank you.