Advanced Composites
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 46
Example Based On a Finite Rectangular Plate (Part-I)

Hello, welcome to Advanced Composites. Today is the fourth day of the ongoing week. And today we will actually solve a particular problem related to finite sized plates, the finite size composite plates. So, let us have the problem definition first.

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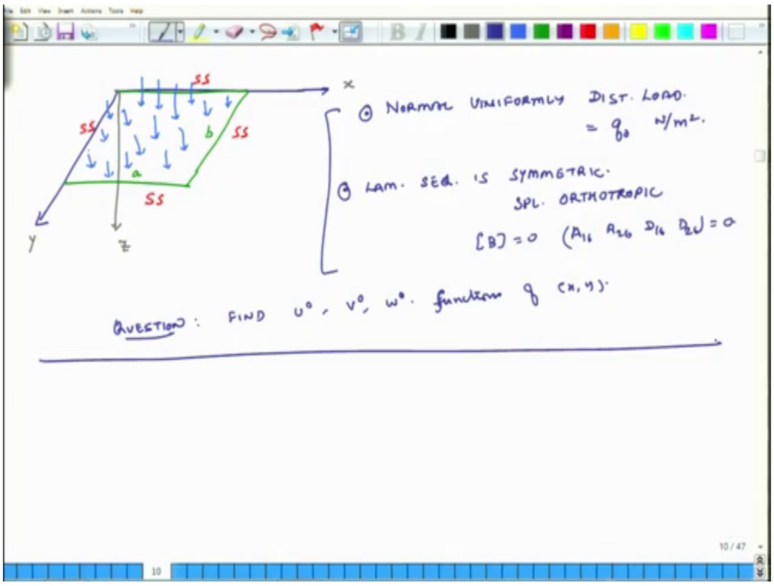

So, the problem is that we have a rectangular plate and we will choose our coordinate system slightly differently, so that is the x-axis, z-axis and y-axis. And the plate is located at the corner. So, the origin is not at the center of the plate, but it is at the corner of the plate, because it makes the mathematics slightly easy that is all. So, this dimension of the plate is a; and in the y direction it is b long ok. And the boundary conditions are such that all the edges are simply supported, all the edges of the plate are simply supported. And then finally, we have another condition that the plate is seen a normal distributed load. So, it seemed normal distributed load and so normal uniformly distributed load and its value is q naught Newton's per meter square ok. So, the total load will be q times a times b which is a and b is the area of the plate. So, this is there.

The other thing which we know about the plate is that lamination sequence is symmetric and it is also especially orthotropic. So, if it is specially orthotropic, what does that mean essentially what that means is that the B matrix is 0 because of symmetry and also A 1 6, A 2 6, D 1 6, D 2 6, all of these guys are also 0.

So, simply supported plate on all the four edges, it is simply supported acted upon by normal transversely distributed load. The lamination sequence is symmetric and also the layers are the lamination sequences especially orthotropic, so that is there. So, then the question is find u naught, v naught, w naught as functions of x and y we have to calculate that. So, this is the problem definition.

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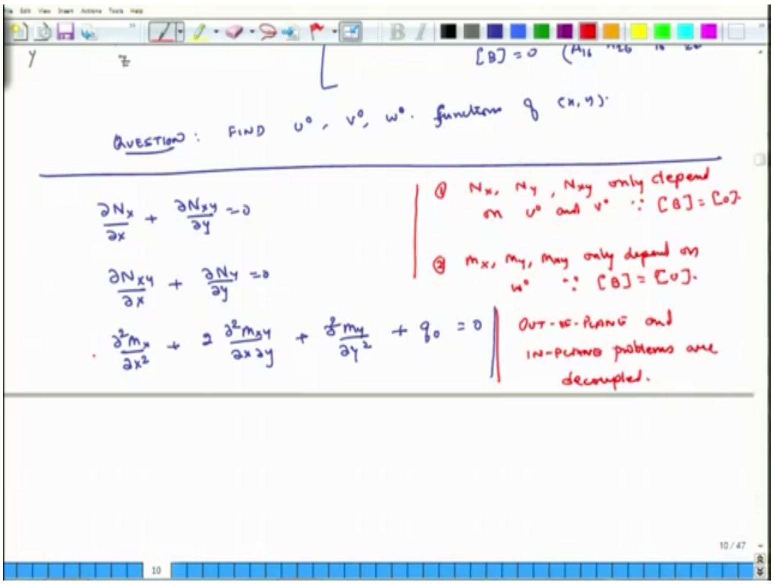

So, we will look at the governing differential equations del N x over del x plus del N x y over del y equals 0; del N x y over del x plus del N y over del y equals 0. And the last one is del 2 M x over del x square plus 2 del 2 M x y over del x del y plus del 2 M y over del y square plus q naught equals 0. Now, when we look at these equations we make following observations, one; N x N y N x y only depend on they only depend on u naught and v naught because B matrix is 0 ok. Second M x, M y, M x y only depend on w naught because B matrix is 0. So, the solution of the third equation does not influence the solution of first two equations and the solution of the first two equations does not influence the solution of the third equations, because B matrix is 0.

So, in because of this simplification we can say that this is out of plane problem out of plane third equation is the out of plane problem. And first two equations are IN plane problems. So, out of plane and IN plane problem are decoupled. So, the solution for w naught is not related to the solution for u naught and v naught and vice versa; u naught and v naught are interlinked, but the solution for w naught does not influence the solution for u naught and v naught and the solution for u naught and v naught does not influence the solution with w, because the B matrix is 0. So, with this understanding so first thing what we will do is we will try to solve the out of plane problem.

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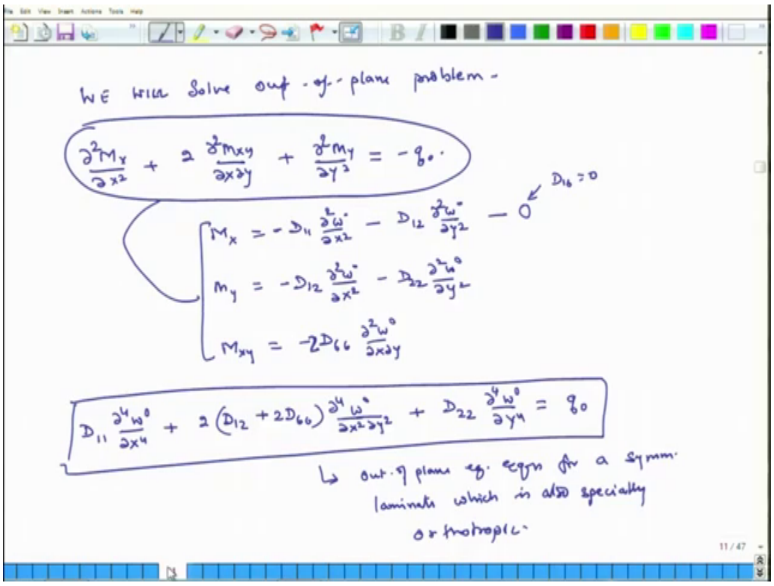

So, we will aim will solve the out of plane problem. So, once again this is del 2 M x plus del x square plus 2 del 2 M x y over del x del y plus del 2 M y over del y square is equal to minus q naught. And we know that M x, what is M x? M x is equal to D 1 1 del 2 w naught over del x square minus D 1 2. So, there is a minus also here, del 2 w naught over del y square. And the third term is 0 because D 1 6 is 0, because D 1 6 is 0 because the plate is especially orthotropic.

Similarly, M y equals minus D 1 2 del 2 w naught over del x square minus D 2 2 del 2 w naught over del y square. And the D 2 6 term is again 0. And M x y is minus D 6 6 del 2 w naught over del x del y. And there is a 2 here, because again D 1 6 and D 2 6 are 0. So, these are the simplified terms. So, we put these into this equation and so in this way we will develop an expression for w. So, what does this become? It becomes D 1 1 del 4 w naught over del x 4 plus 2 D 1 2 plus 2 D 6 6 del 4 w naught over del x square del y square plus D 2 2 del 4 w naught over del y 4 equals q naught. The negative sign from both sides it goes away. So, this is the out of plane equilibrium equation. So, this is what out of plane equilibrium equation for a symmetric laminate which is also especially orthotropic.

So, if there is any plate whatever shape it is it will be governed by this particular equation. If the plate is symmetric in lamination sequence and it is also especially orthotropic; any plate, it can be rectangle or non rectangular. And q could be a variable or constant; in this case q is q naught which is constant uniformly distributed, but does not have to be this is a very general equation ok. Now, we will try to solve it in context of the boundary conditions which we discussed which is that the plate is simply supported on all the four edges and also the other thing is that q is not a variable, but q is a constant.

So, our next goal is to solve this equation. So, how do we solve this equation? So, this is a fourth order differential equation in a single variable w naught. And the way we solve this equation is by making a guess. So, we guess an equation an expression for w naught. And the guess should be such that it should satisfy all the boundary conditions and it should also satisfy the differential equation. So, if it satisfies all the boundary conditions and it also satisfies the differential equation, then our guess is correct ok, so that is how we solve it.

So, this is the exercise we will undertake tomorrow. And hopefully we will arrive at the solution which will meet all the boundary conditions and also it will satisfy the differential equation exactly. So, that is all I wanted to discuss today and I look forward to seeing you tomorrow.

Thank you.