Advanced Composites
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Lecture - 48
Example Based On a Finite Rectangular Plate (Part-III)

Hello, welcome to Advanced Composites. Today is the last day of this course, for the current week that is the 8th week and yesterday, we just developed a solution for out of plane deflection of a composite plate, which is laminated symmetric plate, rectangular plate, and the plate is simply supported on all the four sides.

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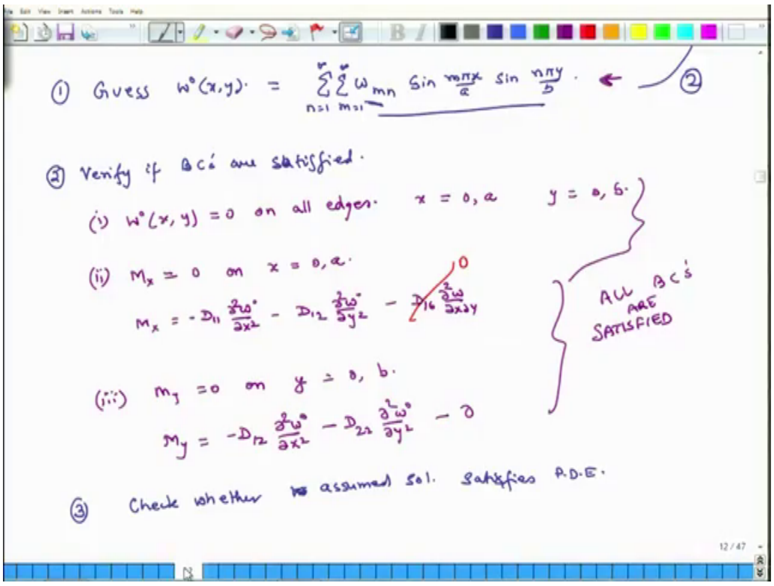

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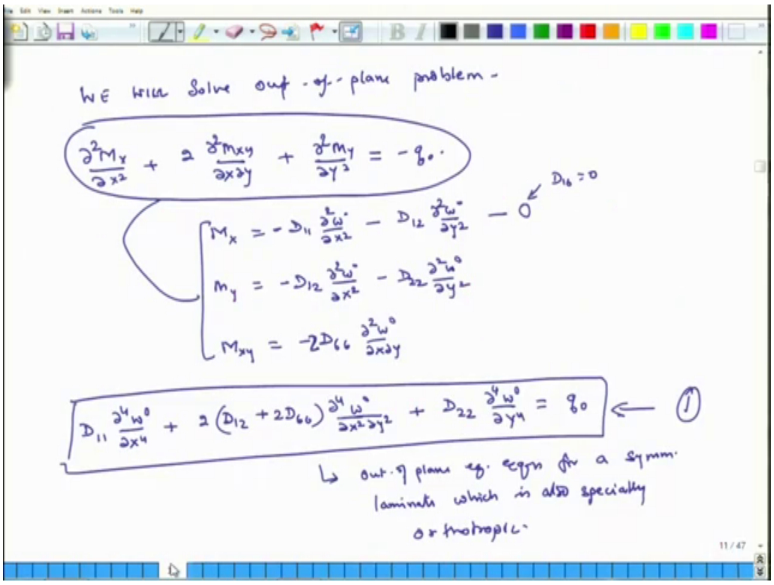

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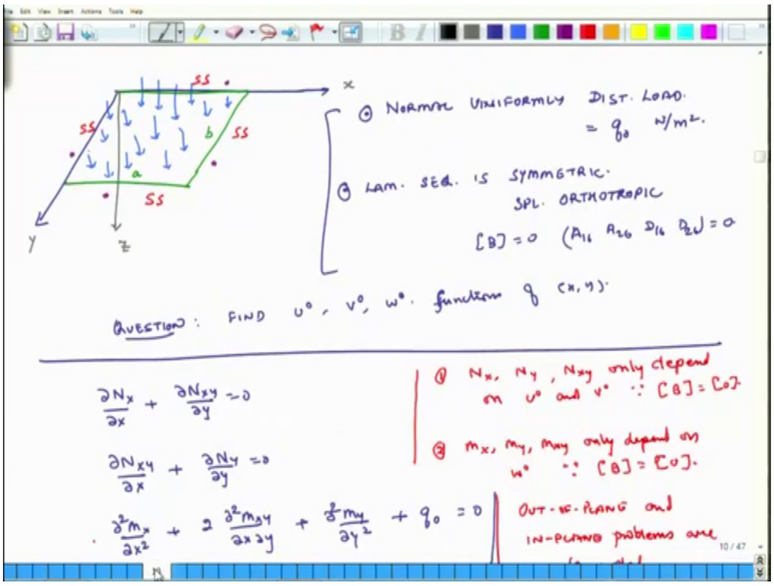

So, this is the definition of plate and the other thing about this plate is that it is transversely loaded by a uniformly distributed load of q Newton's per square meter. So, the way we solved this problem was that we assumed the solution and then we tried to show whether that this solution satisfies all the boundary conditions and also the governing differential equation.

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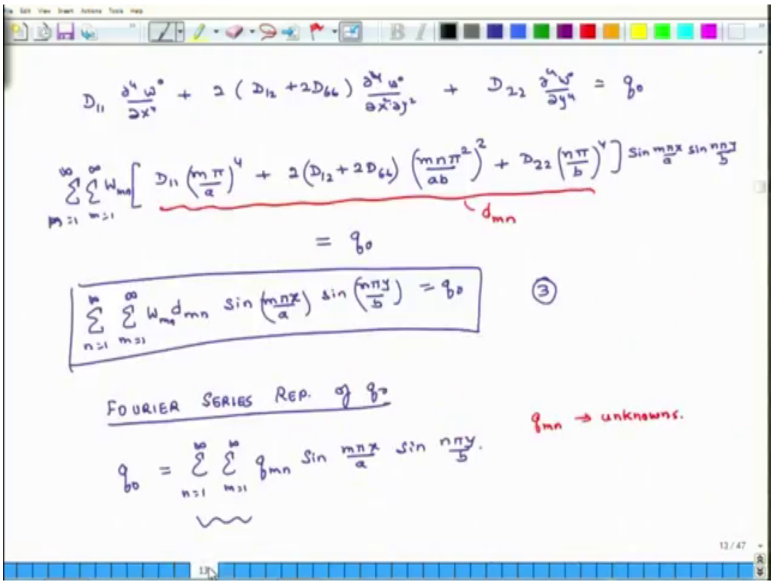

This procedure for solving for symmetric especially orthotropic plate, which is simply supported on all four sides is fairly general, because if we know q naught then we can always resolve that q naught in terms of its Fourier series expansion and if the Fourier series expansion has only sine terms, not cosine terms then this approach works out pretty well. So, then it solves our problem. So, we can solve the problem, but if the Fourier series expansion has sine terms and cosine terms then we cannot use the solution, because on the left side of the equation, we end up with as you see here only sine terms, but if on the right side, you have sine as well as cosine terms then you cannot compute w m n.

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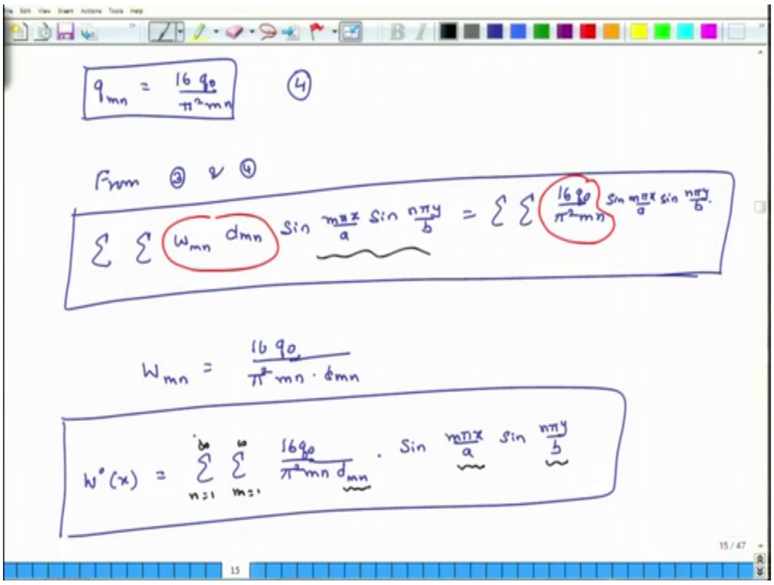

So, this is something important to understand. I will also, like to talk a little bit about convergences in context of this plate. So, what we mean by convergences how accurate are solution is and our solution becomes more and more accurate as we in keep on increasing the number of terms in our solution, but then I want to discuss this a little bit further. So, this is m is equal to 1 to infinity and this is n is equal to 1 to infinity ok.

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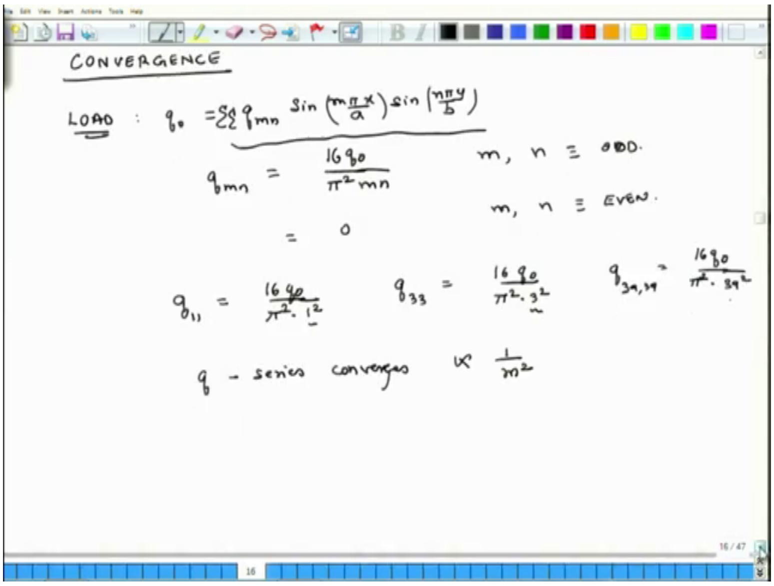

So, we will spend next 5 10 minutes about one convergence and this discussion is fair has significant implications not only in context of this particular plate problem, but in general about whenever we end up with finite element analysis or series solutions for any complex problem. So, let us look at the load.

So, the load in this plate was what, q naught and we said that q naught is equal to q m n sin m pi x over a sin n pi y over b and of course, there is a double sum here and what was q m n? q m n we had defined was 16 q naught divided by pi square m n and these m and n should be odd, if they are even then this q m n is 0. So, this is equal to 0 if m and n are even.

So, once again as I keep on increasing the number of terms in the Fourier series, the overall value of left side of this equation, right side of this equation will become closer and closer to q naught, but how fast does it become close to q naught. So, let us look at so, q 1 1 is equal to 16 q naught by pi square times 1 square q 3 3, which is the second term in the series or the, or the fourth term in the series.

This is equal to 16 q naught by pi square times 3 square of course, there will be cos terms also q 1 3 q 31 one, but I am just going when m and n are of the same value. Let us talk about the 20th term. So, that will be q 39 39. So, this will be 16 q naught over pi square times 39 square. So, what we see is that q series, it converges approximately as 1 over m square right.

So, as m increases it converges in a 1 over m square fashion, because we have 1 square 1 square 3 square 39 square and so on and so forth. So, so, it is not like 1 over m convergence, but it is faster than 1 over, 1 over m square convergence. So, this how load convergence. So, load convergence in a 1 over m square fashion in this problem.

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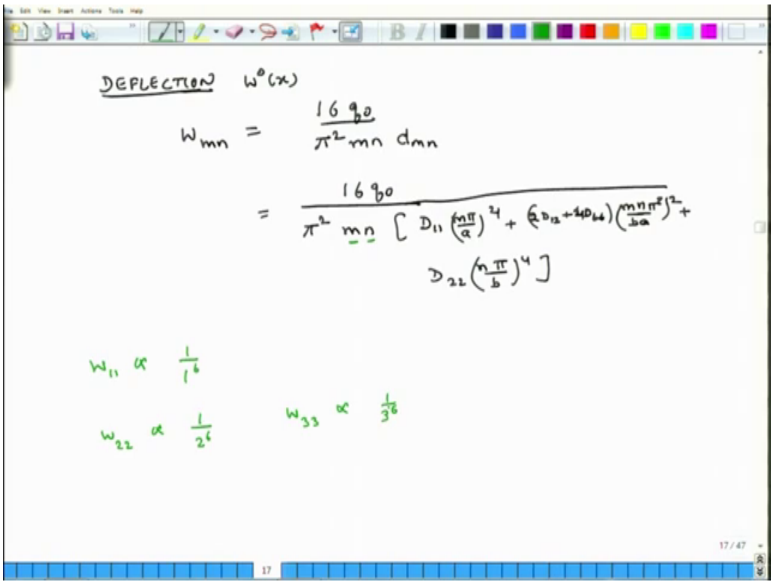

Now, let us look at deflection. So, deflection is w naught x and that; so, let us look at the w m n term. So, what is w m n? We have seen that it is 16 q naught divided by pi square times m n. So, it is equal to 16 q naught divided by pi square m n d m n and what is d m n. So, let us write down this 16 q naught over pi square m n times D 1 1.

This is the long expression m pi over a this is to the power of 4 plus 2 D 1 2 plus 2 D 6 actually it is 4 m n over b pi square this whole thing square plus D 2 2 n pi over b 4. So, this is the d m n right this is d m n ok. So, we have expanded this. So, essentially what it means is so, you; so, what will be w 1 1, w 1 1 will be something, will be proportional to 1 over 1 square 1 to the power of 4.

No, I am sorry, it will be one over 1 to the power of 6. Why I, why did I say 1 to the power of 6; you have one m another n. So, 1 1 then you have m to the power of 4 right. So, it is 1 to the 6 w 2 2 is proportional to roughly 2 to the power of 6 w 3 3 is proportional to 1 over 3 to the power of 6 ok.

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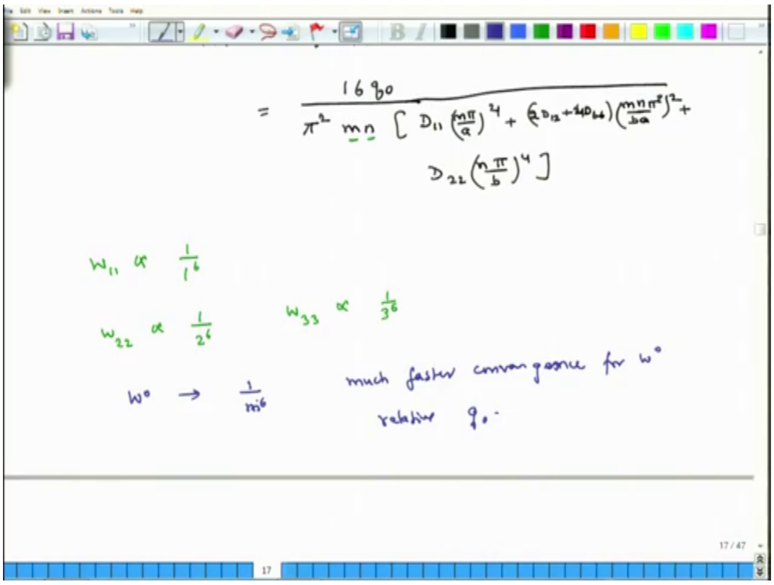

So, which means that w naught, it is convergence is over 1 to the power of 6 power of m. So, it is much faster, much faster convergence for w naught relative to q naught relative to q naught. So, if you are trying to solve a problem and you are interested in finding w naught, then maybe you need to use only few terms, but if you are trying to get a very good representation of q naught then you need more terms in the solution ok. So, will look at some other things also curvature.

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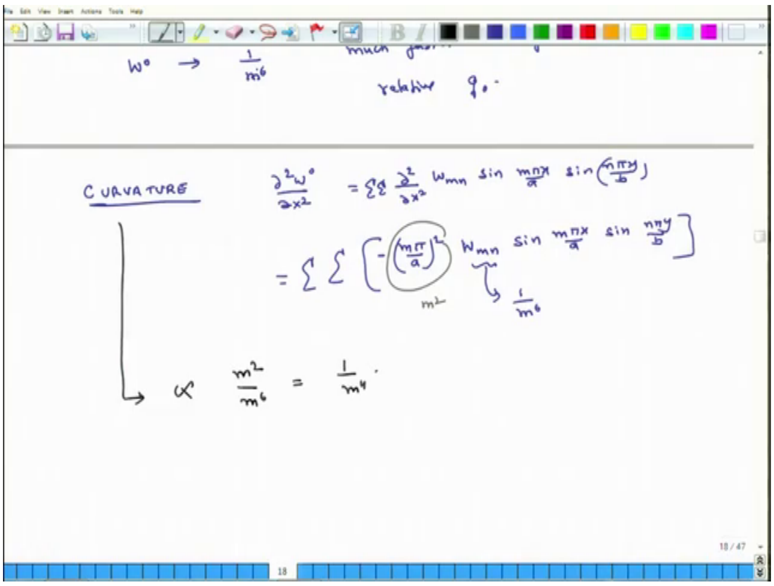

So, what is curvature let say it is del 2 w over del x square suppose, this is there. So, it is a second derivative. So, when we have second derivative then it means and so, this is equal to del 2 of w m n sine m pi x over a sine n pi y over b del with respect to del x square and I have to sum it on both the indices ok. And when I differentiate it twice essentially what I get is m pi over a whole square and w m n and then sign m pi x over a sine n pi y over b and of course, there is a negative sign here and this entire thing summed twice. So, that is curvature in the x direction right.

Now, w m n convergence is 1 over m to the power of 6, but I also have this term. So, this is in the numerator. So, convergence of curvature is proportional to m square over m 6. So, that is 1 over m to the power of 4 ok. So, curvature can convergence faster than load, but less faster than deflection 2, more things stress ok.

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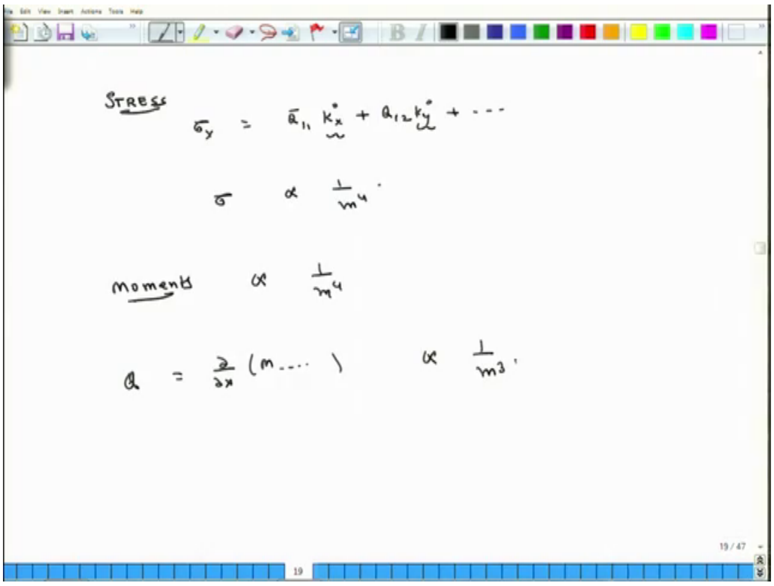

So, what is in a symmetric laminate what is the stress sigma is equal to k x. So, let us say sigma x is equal to k x curvature times D 1 1 and other terms also right sigma x. No, I am sorry. So, sigma x for a particular layer is q bar 1 1 k x plus Q bar 1 2 k y plus and so on and so forth right.

So, these curvatures have a 1 over m 4 convergence. So, sigma it converges in a 1 over m to power of 4 sense moments similarly, you can show that this is also converges in 1 over m to the power of 4 sense and then the last one is shear out of plane shear and that is related to partial derivate with respect to x of moments or partial derivative.

So, of moments; so, it is partial derivative with respect to a space term. So, this converges in a 1 over m cube sense. So, very quickly we will compile all this. So, w it converges as 1 over m to the power of 6, then we have curvature, they have 1 over m to the power of 4, stress is 1 over m to the power of 4. What is do we have? Moments, moments is 1 over m to the power of 4 and shear is 1 over m to the power of 3 and finally, we have q is 1 over m to the power of 2.

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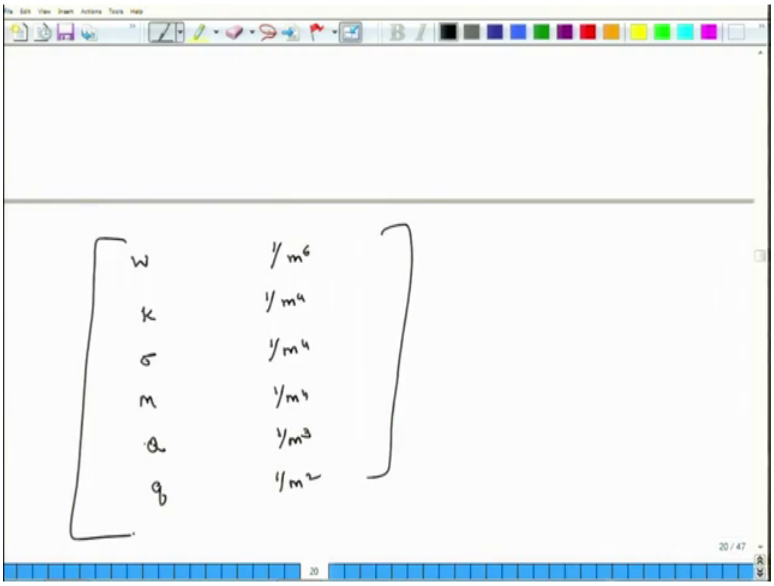

So, just the fact that are solution is converging in one parameter does not mean that we have a good solution for other parameters, this is very important to understand. So, we should have some understanding of how fast convergence happens on different parameters and accordingly if we are using finite element method, we have to increase the number of elements or if we are using a series solution, we have to increase the number of series solutions accordingly.

So, that concludes our discussion for today. We Will continue this discussion for finite plates in the next week also and may be in the week after that also, because this is a very wide area and I would like you to have some idea as to, how to negotiate finite plates, because these are realistic plates, in context of composite laminates. So, that is the plan for next couple of weeks and then if we are done with these plates then will move to some other topic.

So, thank you very much and I look forward to seeing you in next week bye.