Advanced Composites

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Lecture - 49
Anticlastic Curvature

Hello, welcome to advanced composites. Today is the start of the 9th week of this course and what we have done till so far is that we have finished, treatment of finite plates and, we will now, proceed into solving some more examples of finite plates, where the methods which we have discussed earlier, may not be necessarily applicable. Earlier, we have said that when we want to solve the problem of finite plates, we have to select, functions for u v and w such that they meet all the boundary conditions, associated with the plate and they also should satisfy the governing equations.

But usually it is not possible to, quickly arrived at such functions. So, when it is not possible to arrive at such functions what do we do and that context we, we will be introducing the notion of virtual work, and we will see that how the principle of virtual work can be used to solve the problem of finite plates, when the displacement functions for u v and w are not easily available in the sense that these functions exactly satisfies all the boundary conditions as well as the governing differential equation.

Before, we embark on this particular journey, we will have a small diversion and we will discuss what is, quite often known as anticlastic curvature and we will discuss this effect or the phenomena of anticlastic curvature in context of the semi infinite beam case, specifically case a, which we had studied and we will see, what this anticlastic curvature is.

The same concept though, we are going to explain in context of semi infinite beam, the same concept is also applicable to other plates, also which may not be necessarily semi infinite in geometry, but they may be finite plate. So, we will first revisit the case a for the semi infinite beam, in context of anticlastic curvature. So, let us first very quickly recap the case of the semi infinite beam.

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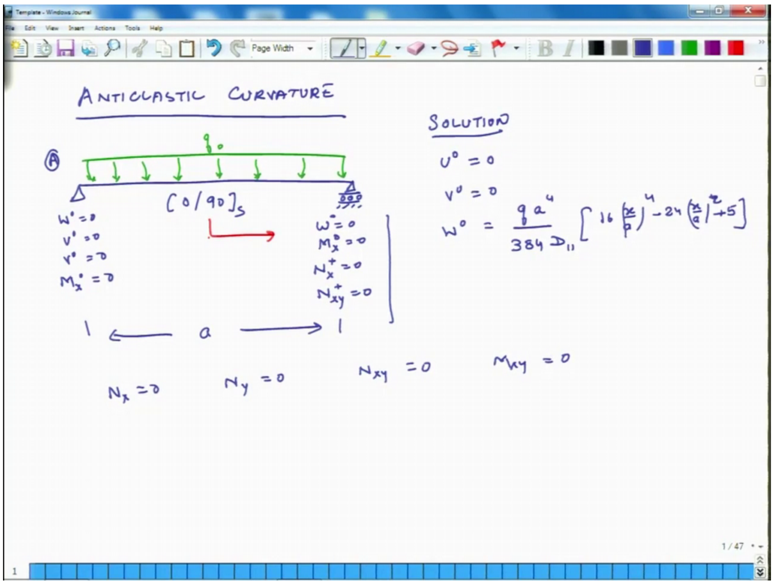

So, this is case a and here the beam lamination sequence was 0 90 symmetric and the boundary conditions at the pin end were w naught; naught is equal to 0 u naught naught is equal to 0 v naught is equal to 0 and M x naught equals 0 and at the roller end, the boundary conditions were w naught; naught equals 0 M x equals 0 and then, because it is free to move, because of roller support N x plus is equal to 0 and N x y plus equals 0.

And also I just write this lamination sequence below. So, the lamination sequence 0 90 symmetric, the plate was also uniformly loaded with the load, intensity of q or actually q naught. So, it is uniformly distributed load, for such the problem we had generated the solution, for this problem and the solution was that u naught equals 0 v naught equals 0 and w naught was found to be q a to the power of 4 over 384 D 11 into 16 x over a 4.

So, this entire length is a and a coordinate system is in the middle. So, 16 x over a whole 4 minus 24 x over a square plus 5, this was the expression for w and now, that we have u v and w, we also calculated N x was found to be 0 N y was found to be 0 N x y was found to be 0 M x y was found to be 0.

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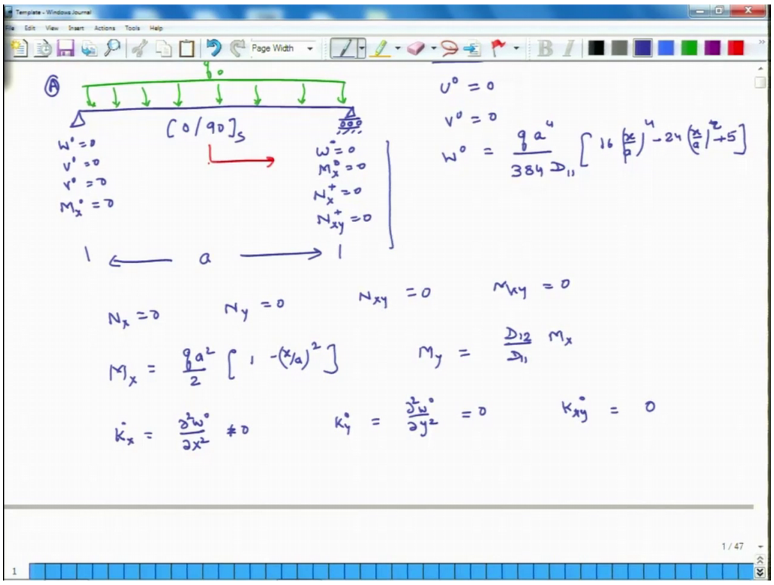

And M x was determined to be q a square over 2 1 minus x over a whole square and then M y equals D 12 over D 11 M x and if we have to calculate curvatures, K x is equal to del 2 w naught over del x square. So, this is not equal to 0 K y, because w is not a function of y and K y is defined at del 2 over del y square. So, this is equal to 0 and K x y is also equal to 0. So, this is the overall solution.

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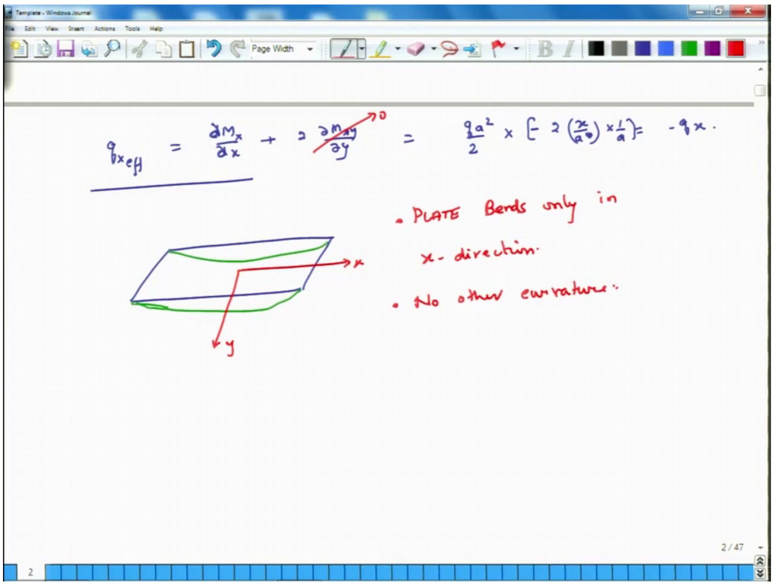

Now, and finally, we will also write an expression for q x effective. So, we had said that q x effective is defined as d M x over d x are actually partial derivative plus 2 del M x y over del y and, because M x y is 0, this term is 0 and M x is q a square times 1 minus x by a whole square. So, if I differentiate this in x, I get minus q a square over 2 into minus twice of x over a, twice of x over into 1 over a. So, what I get is minus q x. So, that is my actually, this is; so, this negative should not be there and I get minus q of x.

So, this is the overall thing now, let us look at the curvatures of this plane. So, we have said that the curvature of the plate in the y direction is 0. This is what the exact solution is telling us in the x direction, it is not zero and the twisting curvature K x y is not 0 so, the plate if I look at it. So, let us say this is the mid plane of the plate, this is the mid plane of the plate, in the deform position in the undeformed position and when the plate deforms, because the curvature is only in the x direction is not in the y direction, the plate will bend something like this.

So, this green line is the reflects the curvature of the plate. So, this is my x axis, this is my y axis and vertical is the z axis. So, the plate bends only x direction, no other curvature. Now, we will look at so, this is how the plate is going to bend, this is what we assured expect and this is what actually happens as observed in physical systems.

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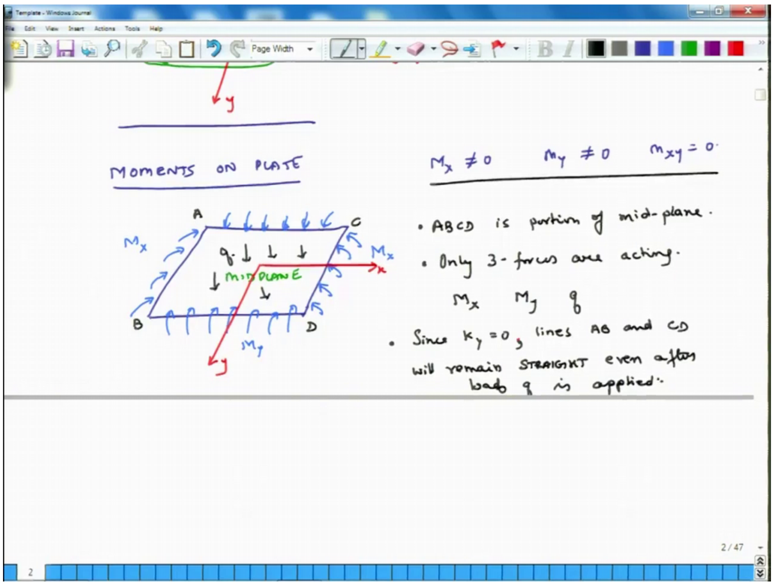

Now, let us look at the movements on the plate. So, if I take the plates mid plane, we have seen that M x is not equal to 0, M y is not equal to 0 and M x y is equal to 0. This is what we have computed M x is equal to q a square times 1 minus x over whole square and M y is D 12 divided by D 11 times M x.

So, M x is not equal to 0. So, if I have to; so, if this is the mid plane of the plate, the mid plane of the plate will experience M x and M y. So, this is how the M x will get. This is the positive M x on a small infinite, small mid plane surface area of the plane. So, this is M x and also there is M y and of course, M x and M y they vary with respect to position, but they are not 0 along the entire length. So, this is M y. So, let us mark this points A B C D, A B C D.

So, we will make some comments. So, the first thing is A B C D is portion of mid plane, then only 3 forces are acting. So, we should not call forces (Refer Time: 12:55) literally force, but only three things are acting on the plane, 3 force. So, what are those three things? M x, because it is computed to be nonzero M y, we computed to be nonzero M q, which is the vertical, because the plate is transversely loaded. So, that is q; q is there, then we will make some more comments, we say that since K y equals 0.

So, what is K y? K y is the curvature in the y direction. So, this is x this is y. So, K y is equal to 0 and if K y is equal to 0, it means the curvature along the line A B and C D that curvature is 0 and if that is going to be 0 lines A B and C D will remain straight, even after load q is applied. More observations is, K y is 0 and that implies that this line, this line will remain straight and also this line will remain straight, if it was straight to begin with, it will also remain straight, because the curvature in the y direction is 0.

The other thing is there is a vertical force q, acting on the plate the force, the force is acting like this vertical force and what balances that, that force; if there was nothing to balance it the plate will just drop right. So, what balances it? There is a moment like this M x moment like this M x and M y on these two edges. So, M x and M y are responsible for balancing the force against external load q.

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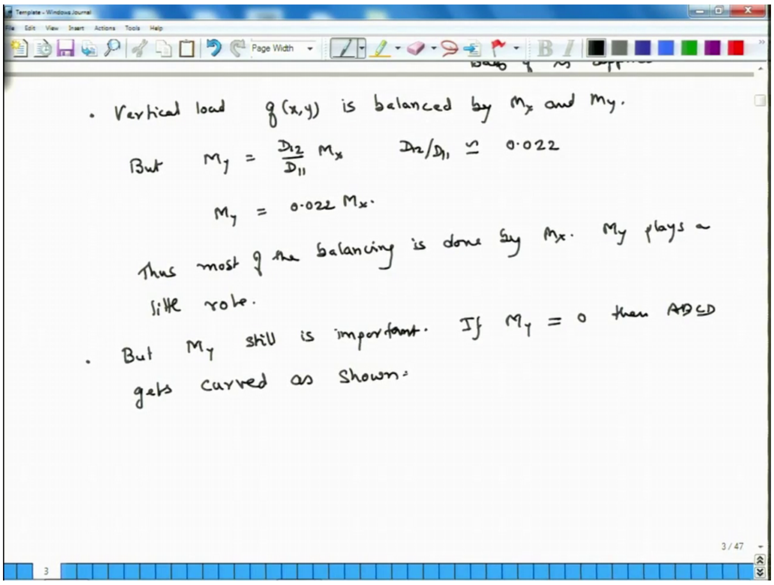

So, let us write that down, vertical force q x y, actually I should not call force, it is load and it is a distributed load is balanced by M x and M y, but M y equals D 1 2 over D 1 1 M x and typically for the composite system, which we are discussing D 1 2 over D 1 1 is approximately equal to 0.022.

So, what that means, is that M y is equal to 0.022 M x. So, M y is very small compared to M x. So, most of the balancing is being done by actually M x. Thus, so for balancing, M y plays a little role. So, M y it does balance, but it's role is not significant, but M y still is important and that we are going to see in a moment. If M y was equal to 0.

So, what will happen if M y was equal to 0? This plate suppose, this is x axis this so, the x axis along the length of this, paper x and this is y axis if M y was is equal to 0. So, what is M y M y is acting like this no M y is acting like this and M x is acting like this M x is acting like this, M y is acting like this. If M y was not zero then what would happen is that the plate will develop a curvature like this, it will develop a curvature in the y direction, it will develop a curvature like this. So, if M y then A B C D gets curved as shown.

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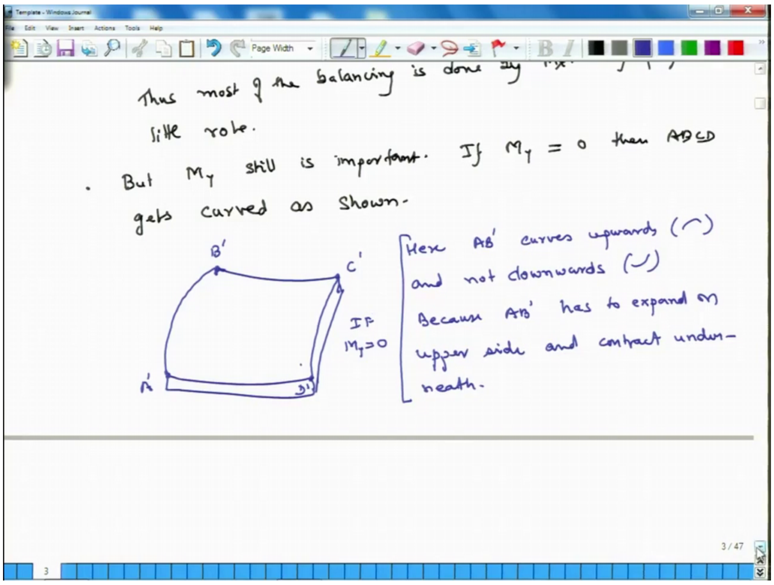

So, let us draw that. So, this is A, this is B. So, this is the deformed position, A prime and B prime and this is C prime, this is D prime. So, this is how the plate bends. So, here A B prime curves upwards and not downwards it (Refer Time: 19:48) to this, because A B prime has to expand on upper side, upper side remember, because the load is acting and we have seen that it has to expand on the upper side and contract underneath.

So, this was, happened if. So, this is what will happen if M y was equal to 0. So, this curvature, this is anticlastic curvature. So, this curvature will develop in absence of M y now, what does M y do? M y acts in a way that it opposes this curvature.

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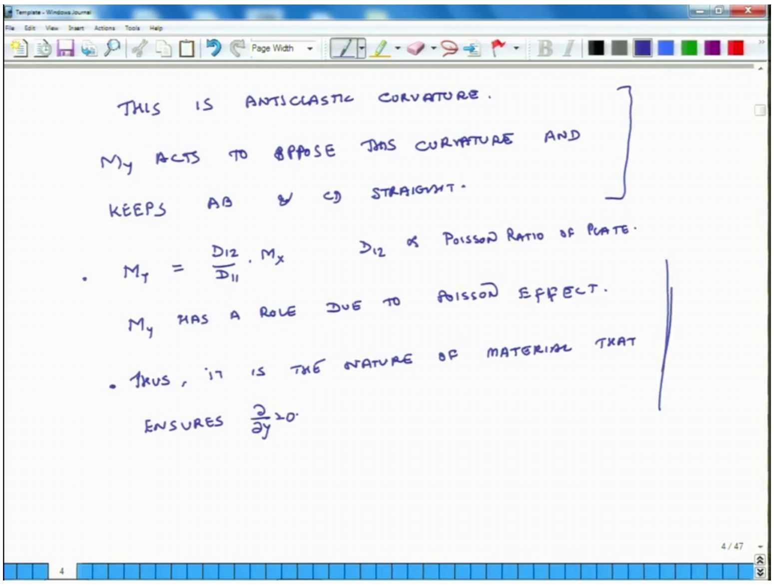

So, M y acts to oppose this curvature. So, M y acts to suppose, this curvature and keeps A B and C D is straight, let me shows that A B and C D are straight ok. So, this is the role of M y, if M y was not there then the plate would bend as per anticlastic curvature.

Now, what is M y M y equals D 1 2 over D 1 1 times M x and what is D 1 2, D 1 2 is directly proportional to Poisson ratio of the plate. So, M y has a role, due to Poisson effect. Thus, it is the nature of the material that ensures del over del y equals 0 ok. So, this curvature has got second derivative of w with respect to y. So, it is the nature of material that ensures del over del y is 0 and, because it is related to Poisson effect ok.

So, this is what is all about anticlastic curvature and you will see the same effects in other plates, but it is comes out very clearly, because in finite plates, things become very complicated. So, you cannot separate out different effects, but here, because things are simple, you can exactly see what is happening and you see that in absence of M y the plate would develop a curvature in y direction and it is M y, which ensures that plate does not develop a curvature and the curvature, which it opposes is known as anticlastic curvature.

So, that is what I wanted to discuss today starting. Tomorrow, we will start talking about, the principle of virtual work and how it can be used to solve again some more and a different type of different class of, finite plate problems. So, that is all for today and I look forward to seeing you tomorrow.

Thank you.