Advanced Composites
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Lecture- 05
Strength of single layer continuous fibre composites Part-I

Hello, welcome to Advanced Composites. Today is the 5th day, the first week of this course. And we have been discussing how to calculate different properties of single layer continuous fibre composites. In last class we had discussed how to calculate the Young's modulus of this type of composite materials in the longitudinal direction. Today, we will discuss the about the strength of such composites in the longitudinal direction when they are subjected to longitudinal tensile stresses.

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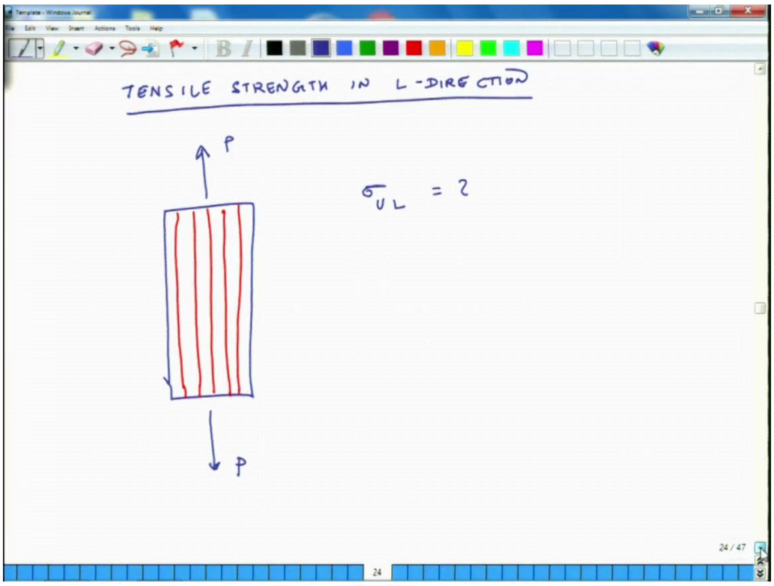

So, what we will discuss is tensile strength; tensile strength in L direction. This is what we want to discuss. So, to illustrate, you have a piece of composites and it is having fibres in the L direction running across the entire length of composite, and this composite is subjected to a tensile load of P.

So, remember we are discussing tensile strength in L direction. We are not talking about compressive strength or shear strength, but tensile strength in L direction. So, what we want to understand this that how do we calculate this ultimate tensile strength. So, ultimate means sigma U, and in the L direction. So, how do we find out this value?

Now, in our introductory course we had discussed several scenarios. And based on those scenarios, we had discussed that the tensile strength depends on the volume fraction, whether it crosses a minimum threshold or a critical threshold or not. So, first we will discuss those situations, and we will not go into the mathematics of that because the mathematics has already been discussed in the earlier class.

But let us consider those scenarios.

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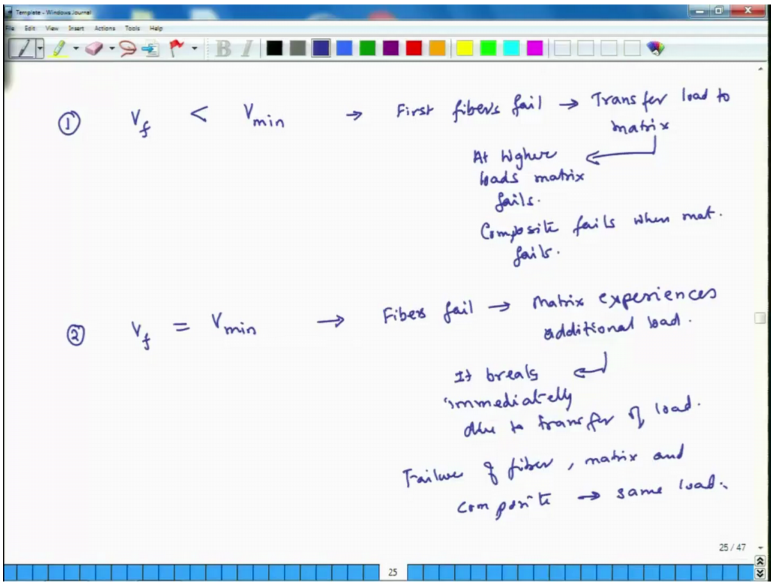

So, case one is when the volume fraction of fibre is less than a particular number and let us call it V min. So, if the volume fraction of the fibre is less than that of this V min value, then what happens? As I increase the load on the composite; initially, because fibres cannot take a lot of strain, they break.

And once they break whatever load they are carrying that load gets transferred to the matrix material. And because the matrix material is present in a very large amount so, it has no problem in absorbing that excessive load, which it receives because of breakage of fibres. And then I have to pull the fire composite even further to ensure that the composite fails.

So, in this case the breakage of fibre does not mean that the composite will fail. Rather, I have to keep on pulling the composite further and further and only once the matrix feels then done entire composite fails. So, here what is the failure process?

First fibre fields so, first fibres fails then they transfer load to matrix, then at higher loads matrix fails. And composite fails then matrix fails, composite fails and matrix fails. So, this is when V equals V min or V the fibre fraction is fibre volume fraction is less than V min. The second case is V f equals V min, in this case fibres fail matrix experiences additional load the additional stress load. And at this time at this point it breaks immediately due to transfer of load, ok. It breaks immediately due to transfer of load.

So, what happens is fibres fail, and suppose fibres are taking a load of 1000 newtons that 1000 newtons gets shifted to matrix. And all of a sudden matrix experiences 1000 extra newtons. And it is not strong enough to bear that extra load so, it also fails at the same time. So, failure fibre matrix and composite happens at same load.

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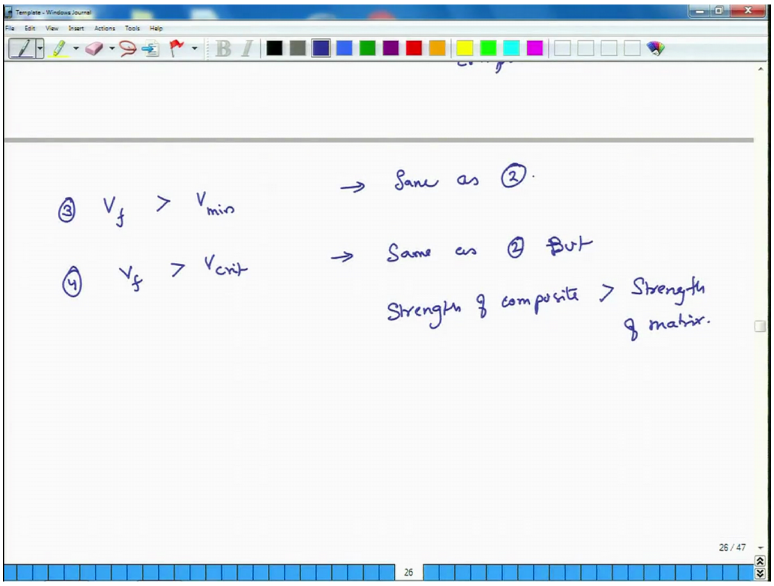

And the third case is so, this happens for V f is equal to V min. And also for all volume fractions of fibre when V f exceeds V min this the same thing happens. Then the third one is V f is more than V min. It is the same mechanism as above, same as 2, nothing new about that. But in all these cases, cases 1, cases 2, cases 3, case 3 in all these cases the overall strength of the composite will still be lesser than the strength of the matrix because there are very little number of fibres.

So, the overall strength of the composite is less than the strength of the matrix. But we want a composite which should be stronger than matrix material, right and that happens when V f is greater than another number called V crit. It is a critical volume fraction. So, in this case same as 2, but a strength of composite is more than strength of matrix.

So, if we are designing a composite material, we have to make sure that our volume fraction should exceed valuation for fibres it should exceed the critical volume fraction.

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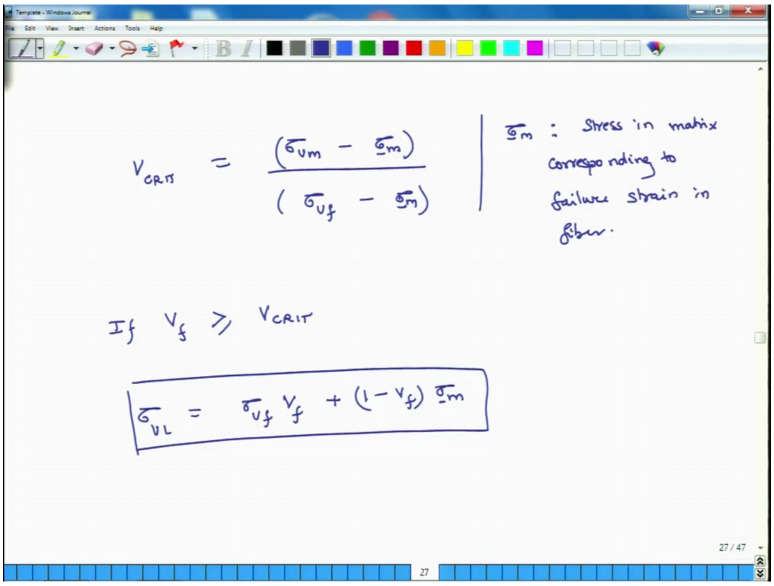

So, the next thing what I am going to do is I am going to provide you these numbers, ok. So, the crit critical volume fraction is if you want to find out what is the critical volume fraction. So, that is equal to ultimate tensile strength of matrix minus sigma m underscore ok, and we will explain what; that means, divided by ultimate tensile strength of the fibre minus sigma m under bar. And what is sigma m under bar? This is stress in matrix corresponding to failure strain in fibre.

What does this mean in plain English? That suppose fibre fails at one percent strain, so what will be the value of sigma m under bar? You pull a piece of sigma m matrix material pure matrix material and subjected to one percent strain, and monitor the stress level that stress is sigma m under bar so that is there and if V f is equal to or greater than V crit then the tensile strength of the composite material sigma L.

Now, what did we say? Sigma UL is defined as, or it can be calculated by sigma ultimate tensile strength of fibre times volume fraction of the fibre, plus 1 minus V f times sigma m underscore. So, this expression gives us the tensile strength of the composite material, if volume fraction of fibre is more than critical value.

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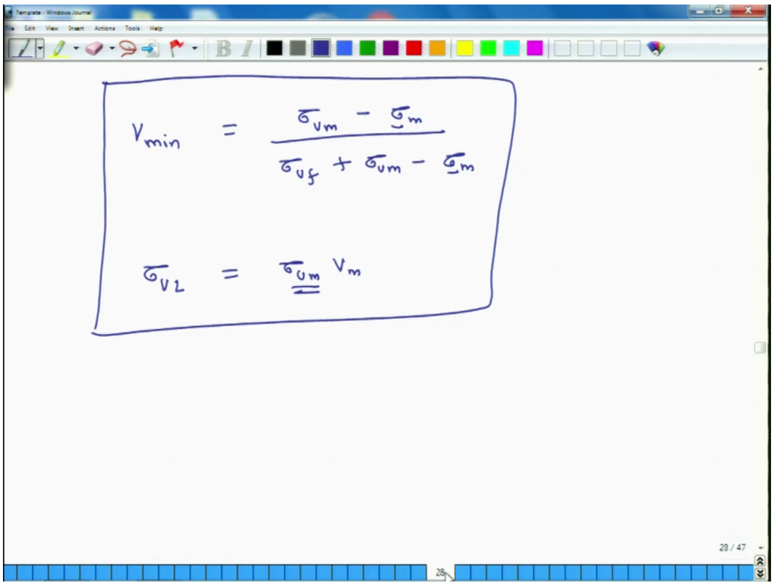

And for the other case if it is not the case and let us also find out what is the value of minimum volume fraction V min. And V min is sigma U m minus sigma m underscore which we have already explained, divided by ultimate tensile strength of fibre plus ultimate tensile strength of matrix minus sigma m bar. And in such a case, the ultimate tensile strength of composite is equal to sigma U m times V m, which is what? Sigma U m is ultimate tensile strength of the matrix material. So, these are the 2 scenarios. And for both these scenarios we have defined the tensile strength of composite in the longitudinal direction.

The next thing we will cover so, we have discussed longitudinal modulus and longitudinal strength.

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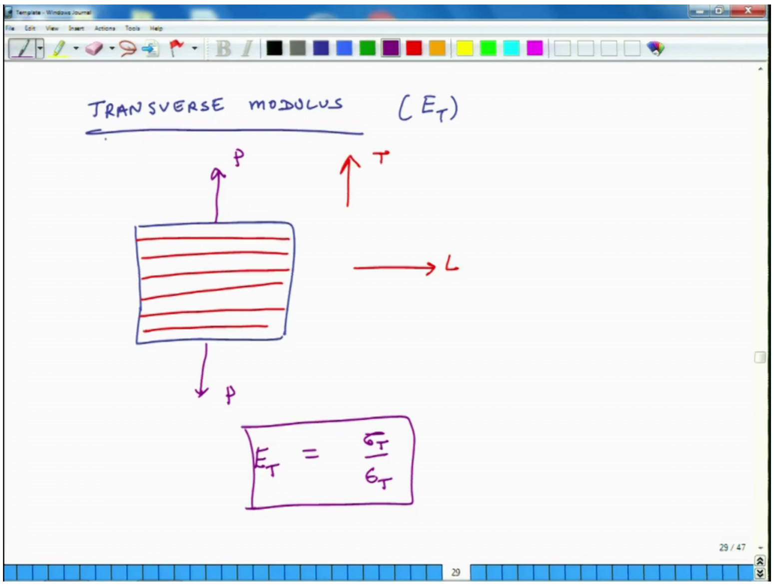

The next thing we will discuss is transverse modulus. So, just to we will start by illustrating. So, what this transverse modulus mean? That suppose I have a sample of composite material, and the fibres are again running like this. So, this is my L direction that is my T direction.

So, here I am interested in finding E T. And in this case, it means that I am subjecting my sample to a load P, but this p is aligned with that T direction, ok. So, E T equals stress in the transverse direction divided by strain in the transverse direction. This is assuming the material is linearly elastic. So, if I plot a stress and strain it if it is a straight line then this is the expression for E T. So, first we will make a course model like we developed for longitudinal modulus.

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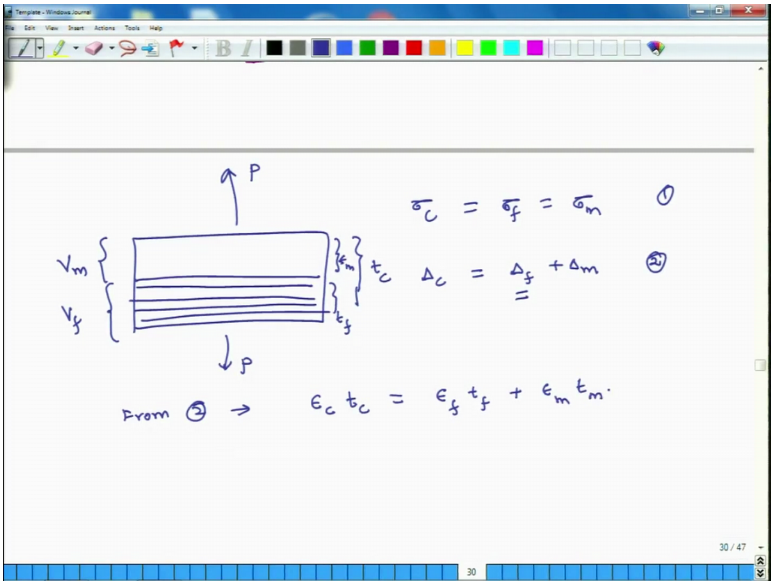

So, suppose I have a piece of composite and it has some number of fibres and the remaining part is material matrix. So, let us say that I designate this part, all this is fibre. So, this thickness corresponds to volume fraction of the fibre and this thickness corresponds to volume fraction of matrix material. So, when I am looking at it transversely I can I am just moving and collecting all the fibres in one side and matrix is in the other side. And I am subjecting it to load P. So, this is one way to get an idea what will be the value of E T.

So, in this case, the stress in composite will be same as stress in fibre, and it will be same as stress in matrix. Also when I pull the sample the overall sample will stretch by some distance delta c, and this stretching will be what? It will be the sum of stretching of fibre, plus stretching of matrix, ok. So, this is expression 1 and this is expression 2. So, from 2 what do we get? We can expand this stretching.

So, expansion in the composite is nothing but the strain in composite in the transverse direction times the thickness of composite. So, this is my thickness total t c, this is t m and this is t f. So, delta c is strain in the composite times thickness of the composite. And that equals the strain in fibre times thickness of fibre, because that is what delta f is, plus strain in matrix times thickness of matrix.

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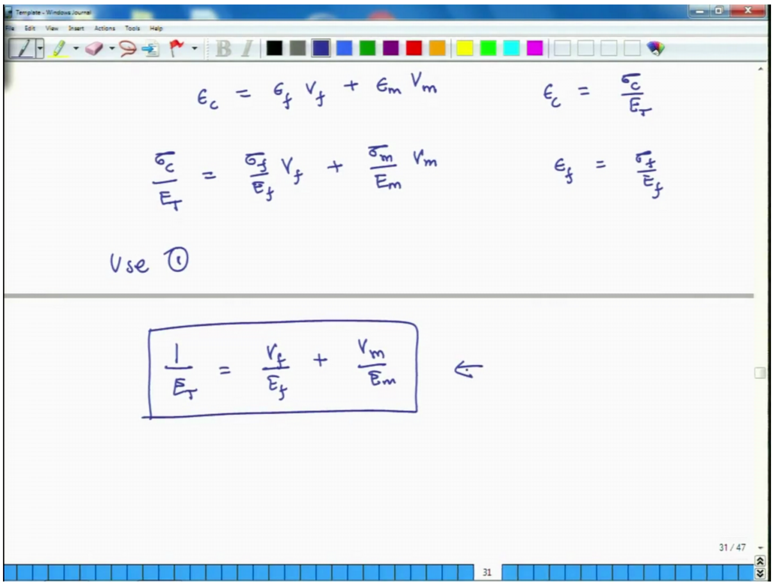

And now what I do is I divide this entire expression by t c. So, I get epsilon c is equal to epsilon f. And what is t f over t c? It is the volume fraction of fibre. So, V f plus epsilon m plus and t m over t c is volume fraction of matrix.

Now, what is epsilon c? Epsilon c is sigma c divided by E T, right? It is the stress in composites sigma c divided by the transverse modulus of the composite. Epsilon f is sigma f divided by E f and so on and so forth. So, I plug these things in this equation. So, I get sigma c by E T is equal to sigma f over E f V f plus sigma m over E m V m. And now we use equation one which says that sigma c sigma f and sigma m are same so, these guys go away.

And I am left with is 1 over E T is equal to V f over E f, excuse me, plus V m over E m, this is there. So, this is one expression for transverse modulus using the assumptions listed here in equations 1 and 2.

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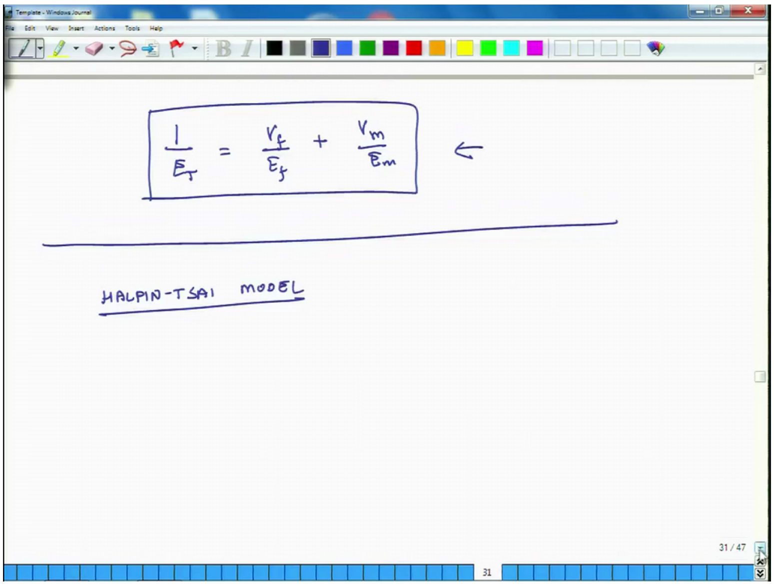

And then once again we ask the question is does this equation really work. And to find that out people did experiments and what they found that this expression does not work in a large number of cases it has limited accuracy it has limited accuracy. And there are several reasons for it is inaccuracies which we will not go into, but they have been discussed in the introductory course. So, people built more sophisticated models. And one model which provides very good results is I will explain that, and that is known as the Halpin-Tsai model.

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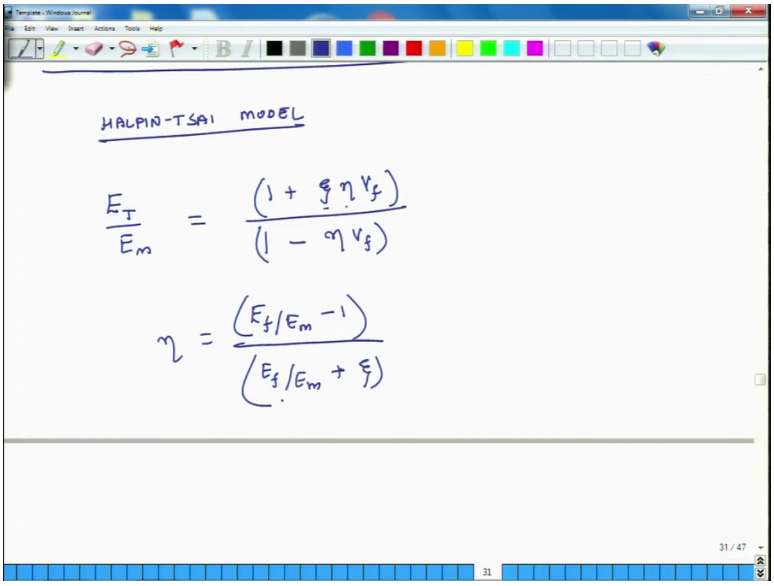

So, I am going to just write down the result directly from there. So, if I have used if I have to compute transverse modulus of a unidirectional fibrous composites. Then the relation for that using Halpin-Tsai relation is this that E T divided by E m equals 1 plus zeta eta times volume fraction of fibre, divided by 1 minus eta times V f which is again volume fraction of the fibre. And you would ask what are these things zeta and eta.

So, eta equals E f over E m minus 1 divided by E f over E m plus zeta, ok.

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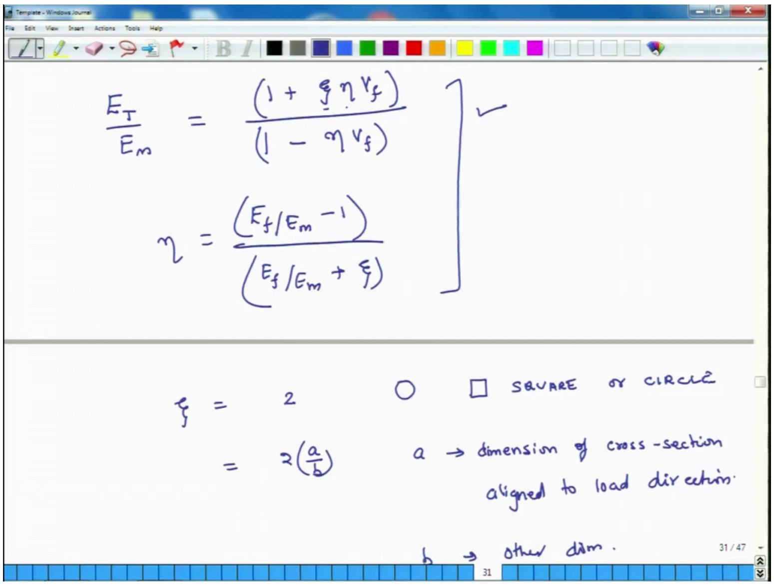

And zeta it depends on the type of fibre which we use. So, if its value is 2 if the fibre is circular in cross section, or it has a square cross section. But if it is a different cross section, let us say rectangular or elliptical. So, in that case, it is twice of a over b, and what is a? So, suppose you have so, a is the dimension of cross section aligned to load direction and b is the other dimension and b is the other dimension.

So, this is how you compute the transverse modulus. And this particular relation the help inside relation provides fairly good results, which are so good results in the sense that they are fairly consistent with lot of experimental data. So, if you want to quickly compute, the transverse modulus of a unidirectional fibrous composite, then if we use this relation we will be in good hands. So, this is what I wanted to discuss today. Tomorrow we will continue this discussion and we will talk about some other material properties and hope fully close this discussion related to prediction of properties of unidirectional composite materials.

Thank you.